Lesson 12 on 3.2 Product & Quotient Rules

Exam 1 on Thursday, Sept. 22 8:00 – 9:00 pm in various rooms.
Know FOUR digit section #. PUID = $\underbrace{00xxx\ xxxxx}_{10\ \text{digits}}$

Constant Rule: $\frac{d}{dx}\left[c f(x)\right] = c f'(x)$

Sum Rule: $\frac{d}{dx}\left[f(x)+g(x)\right] = f'(x)+g'(x)$

Product Rule: $(fg)' = \cancel{f'g'} = f'g + fg'$

$$DQ = \frac{f(a+h)\,g(a+h) - f(a)g(a)}{h}$$

$$= \frac{\left[\left(f(a+h) - f(a)\right) + f(a)\right] g(a+h) - f(a)g(a)}{h}$$

$$= \frac{f(a+h)-f(a)}{h}\cdot g(a+h) + f(a)\,\frac{g(a+h)-g(a)}{h}$$

as $h \to 0$

$$= f'(a)\cdot g(a) + f(a)\cdot g'(a) \quad \checkmark$$

EX: $\frac{d}{dx}\left[\underset{f}{x}\,\underset{g}{e^x}\right] = (1)\cdot e^x + (x)(e^x) = e^x + xe^x$

$= f'\cdot g + f\cdot g'$

$e^{2x} = e^x \cdot e^x$

EX: $\frac{d}{dx}\left[e^{2x}\right] = \frac{d}{dx}\left[\underset{f}{e^x}\cdot \underset{g}{e^x}\right] = (e^x)(e^x) + (e^x)(e^x)$

$f'\cdot g + f g'$

$$= 2e^x \cdot e^x = 2e^{2x}$$

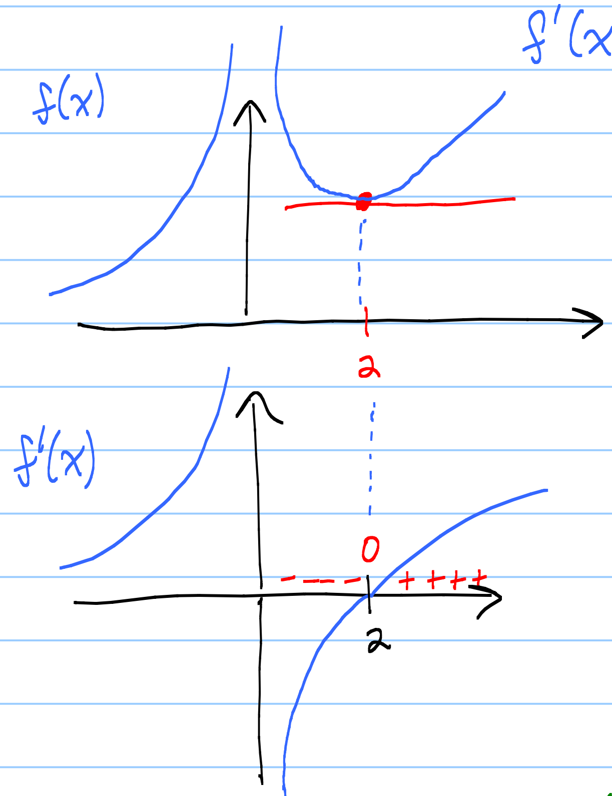
EX: $$\frac{d}{dx}\left[\frac{e^x}{\underbrace{x^2}_{f(x)}}\right] = \frac{d}{dx}\left[x^{-2}\underbrace{e^x}_{g}\right]$$

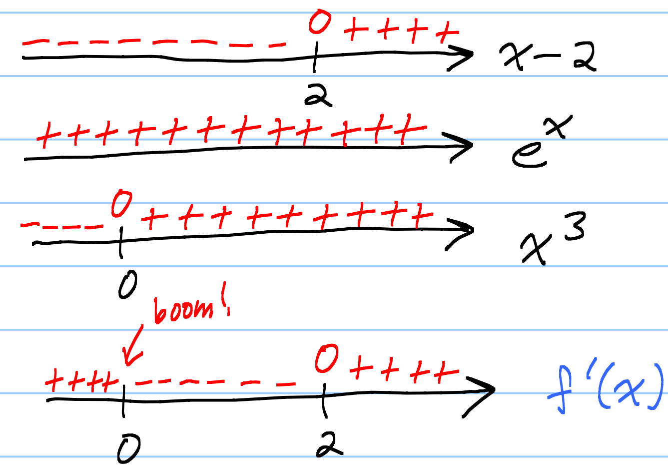
g'

$$= \left(-2x^{-2-1}\right)\cdot e^x + \left(x^{-2}\right)\left(e^x\right)$$

$$= -2x^{-3}e^x + x^{-2}e^x$$

$$f'(x) = \frac{-2e^x}{x^3} + \frac{e^x}{x^2}\cdot\frac{x}{x} = \frac{(x-2)e^x}{x^3}$$





Quotient Rule: $$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$ ← Provided that $g(x) \neq 0$.

$$\left[f\cdot(g)^{-1}\right]' = ? \quad \text{Need } \frac{d}{dx}\left[(g)^{-1}\right] = ?$$

Same idea: $$\frac{\dfrac{\left(f(a+h) - f(a)\right) + f(a)}{g(a+h)} - \dfrac{f(a)}{g(a)}}{h}$$

$= \ldots$ algebra ... bang!

EX: $\dfrac{d}{dx}\left[\underbrace{\dfrac{x \leftarrow f}{1+x^2 \leftarrow g}}_{F(x)}\right] = \dfrac{f'g - fg'}{g^2}$

$= \dfrac{(1)(1+x^2) - (x)(0+2x)}{(1+x^2)^2}$

$= \dfrac{1+x^2-2x^2}{(1+x^2)^2} = \dfrac{1-x^2}{(1+x^2)^2}$ $F'(x)$

$F(x)$ has no vertical asymp.

$F(x) = \dfrac{x \leftarrow \text{boss}}{x^2+1}$ boss

$= \dfrac{x\,(1)}{x^2(1+1/x^2)} = \dfrac{1}{x}\left[\dfrac{\text{close to}}{1}\right] \to 0$

both as $x \to \infty$ and $x \to -\infty$.

Horizontal tangents at $x = \pm 1$.

One horizontal asymp.

Mega Product Rule: $(fgh)' = f'gh + fg'h + fgh'$

EX: $\dfrac{d}{dx}[x^4] = \dfrac{d}{dx}[x \cdot x \cdot x \cdot x] =$

$= 1 \cdot x \cdot x \cdot x + x \cdot 1 \cdot x \cdot x + x \cdot x \cdot 1 \cdot x + x \cdot x \cdot x \cdot 1$

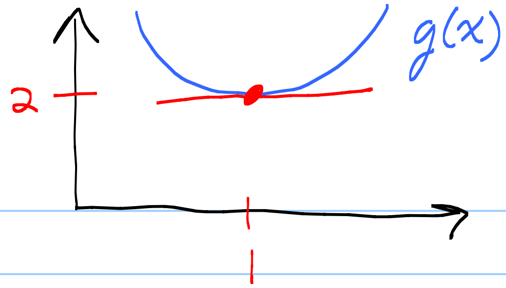
$= 4x^3$ ✓

Fact: If f and g have horizontal tangents at $x = a$, then so does fg. And so does $\frac{f}{g}$ if $g(a) \neq 0$

$(fg)'(a) = \underbrace{f'(a)}_{0}\, g(a) + f(a)\, \underbrace{g'(a)}_{0} = 0$

$\left(\dfrac{f}{g}\right)'(a) = \dfrac{\overbrace{f'(a)}^{0}\, g(a) - f(a)\, \overbrace{g'(a)}^{0}}{g(a)^2} = 0$

EX:



and tangent to $y=f(x)$ has slope 3 at $x=1$.

What is $\left(\frac{f}{g}\right)'(1)$?

$$= \frac{\overset{3}{f'(1)}\,\overset{2}{g(1)} - \overset{(?)}{f(1)}\,\overset{0}{g'(1)}}{\underbrace{g(1)^2}_{2^2}}$$

$$= \frac{3\cdot 2 - 0}{2^2} = \frac{3}{2}$$

Collect: $g(1)=2$

$g'(1)=0 \leftarrow$ horiz tang.

$f(1)=?$

$f'(1)=3$

EX: $\frac{d}{dx}\left(\sqrt[3]{x^2}\; e^{2x}\right) =$

$\sqrt[3]{x^2} = (x^2)^{1/3} = x^{2/3}$

$$\frac{d}{dx}\left[x^{2/3}\; e^{2x}\right] = \left(\frac{2}{3}x^{\frac{2}{3}-1}\right)e^{2x} + x^{2/3}\left(2e^{2x}\right)$$

$$= \frac{2}{3}x^{-1/3}e^{2x} + 2x^{2/3}e^{2x}$$

EX: $$\frac{d}{dx}\left[\frac{1-xe^x}{1+xe^x}\right] = \frac{\left[0-(1\cdot e^x + x\cdot e^x)\right](1+xe^x) - (1-xe^x)\left[0+(e^x+xe^x)\right]}{(1+xe^x)^2}$$

Home Page: Archives of old exams.

Piazza: $\frac{1}{\ln(\cos^2 x)}$ Domain?

Bad things: ln of zero and negative not defined.

$\cos x = 0$ is bad.

2) $\ln(\text{~~~}) = 0$ is bad!

$\text{~~~} = 1$ $\qquad \ln 1 = 0$