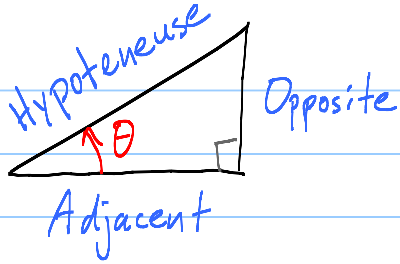
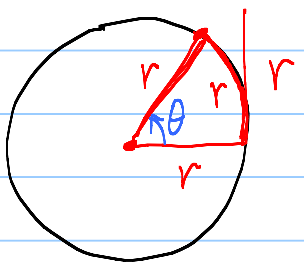
Lesson 13 on Appendix D and part of 1.5 pp. 63-66 Trig



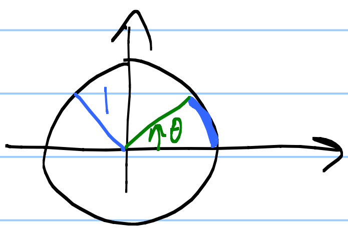
Why e? $\lim_{h \to 0} \frac{e^h - 1}{h} = 1$

So $\frac{d}{dx}\left(e^x\right) = e^x$

Why radians? $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ $\quad \frac{d}{dx}(\sin x) = \cos x$



$\theta = 1$ radian slightly $< 60°$



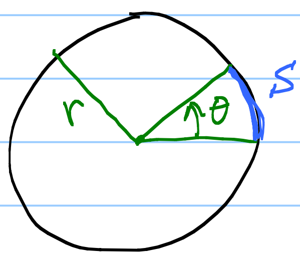
$\theta =$ arc length (radians)

$d =$ degrees

$$\frac{\theta}{2\pi \cdot 1} = \frac{d}{360}$$

$$\boxed{\theta = \frac{\pi}{180} d}$$

$$\boxed{d = \frac{180}{\pi}\theta}$$

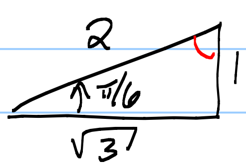


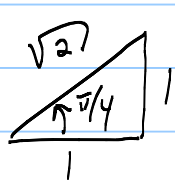
$$\frac{s}{2\pi r} = \frac{\theta}{2\pi}$$

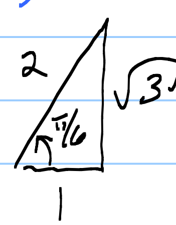
Arc length $\boxed{s = \theta r}$

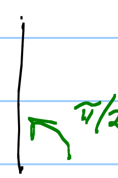
Important angles: $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$

$30°, 45°, 60°, 90°, 180°$



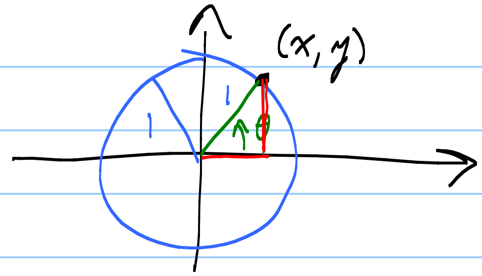






$\sin\theta = \frac{OPP}{HYP}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\sqrt{3}/2$	1
$\cos\theta = \frac{ADJ}{HYP}$	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta = \frac{OPP}{ADJ}$	$1/\sqrt{3}$	1	$\sqrt{3}$	Boom!

$\sec\theta = \frac{1}{\cos\theta}$, $\csc\theta = \frac{1}{\sin\theta}$, $\cot\theta = \frac{1}{\tan\theta}$



$\sin\theta = \frac{y}{1} = y$

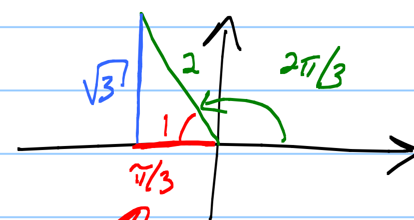
$\cos\theta = \frac{x}{1} = x$

$\tan\theta = \frac{y}{x}$

Pythagorean Thm: $x^2 + y^2 = r^2$

$$\boxed{\cos^2\theta + \sin^2\theta = 1}$$

EX: Find trig fcns at $\theta = 2\pi/3 \sim 120°$



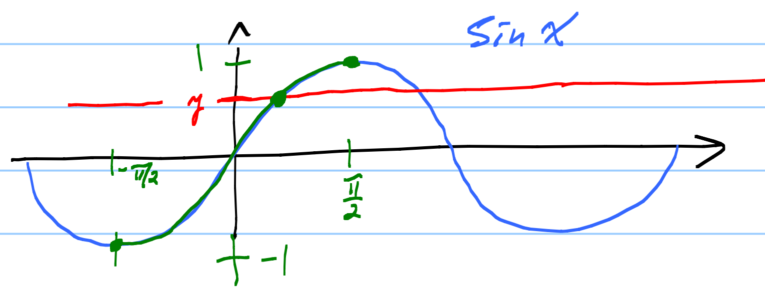
red side = −1

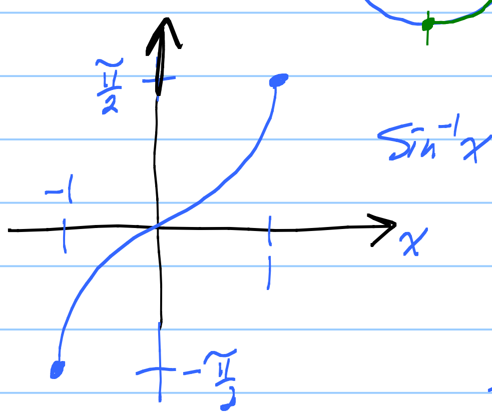
$\sin\theta = \frac{\sqrt{3}}{2}$

$\cos\theta = \frac{(-1)}{2}$

$\tan = \frac{\sqrt{3}}{(-1)}$

Inverse Trig Fcns :

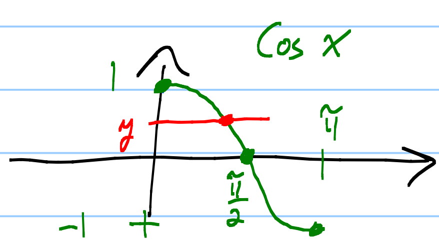


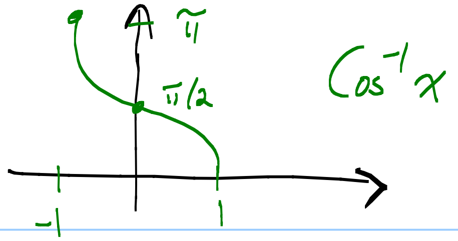


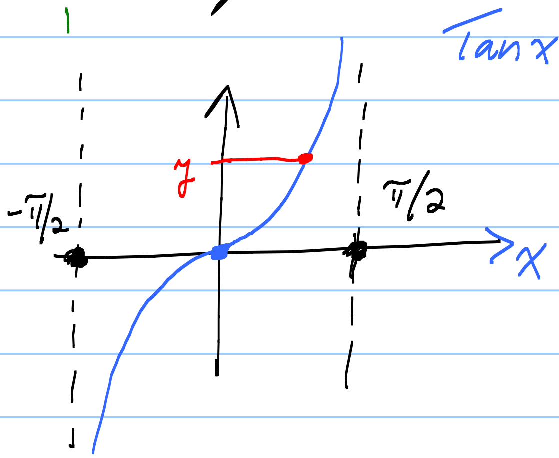
$\sin^{-1}x$ = the angle θ between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ such that $\sin\theta = x$

Domain of $\sin^{-1}$: $[-1, 1]$

Range of $\sin^{-1}$: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

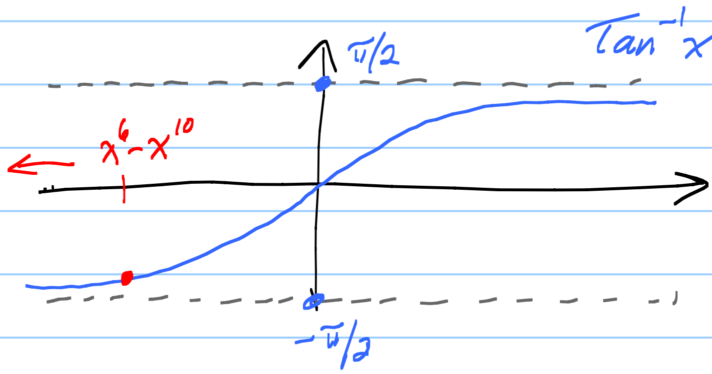






$$\lim_{x \to \frac{\pi}{2}^-} \tan x = \infty$$

$$\lim_{x \to -\frac{\pi}{2}^+} \tan x = -\infty$$



$$\lim_{x \to \infty} \tan^{-1} x = \frac{\pi}{2}$$

$$\lim_{x \to -\infty} \tan^{-1} x = -\frac{\pi}{2}$$

Prob: What is $\lim_{x \to \infty} \tan^{-1}(x^6 - x^{10})$ $= -\frac{\pi}{2}$

boss

$$x^6 - x^{10} = x^{10}\left(\frac{1}{x^4} - 1\right) = x^{10} \cdot (\text{Close to } -1)$$

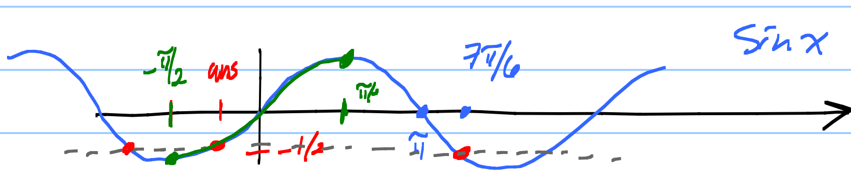
$$\longrightarrow -\infty \quad \text{as} \quad x \to \infty.$$

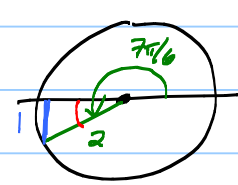
Prob: What is $\sin^{-1}\left(\sin \frac{7\pi}{6}\right)$?

Hmmm. $\sin^{-1}(\sin x) = x$, right?

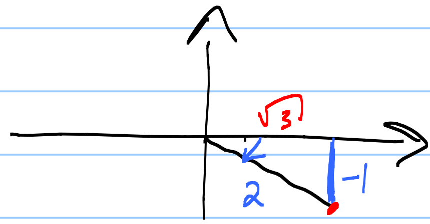
Yes, when $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

$$\sin \frac{7\pi}{6} = \frac{(-1)}{2}$$





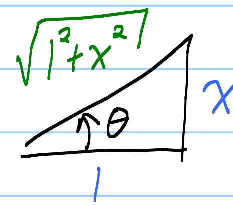
$\sin^{-1}\left(-\frac{1}{2}\right)$ = the angle θ between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$
where $\sin\theta = -1/2$



$$\sin\left(-\frac{\pi}{6}\right) = \frac{(-1)}{2} = -\frac{1}{2}$$

Ans: $\sin^{-1}\left(\underbrace{\sin\frac{7\pi}{6}}_{-1/2}\right) = -\frac{\pi}{6}$

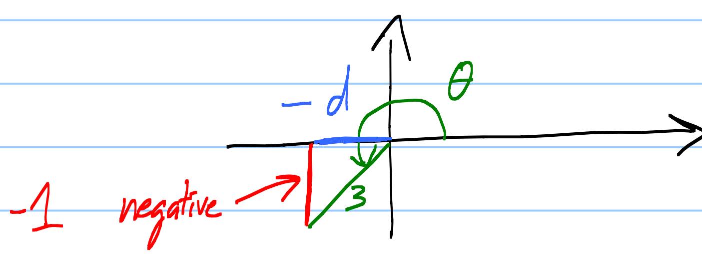
Prob: Simplify $\sin\left(\underbrace{\tan^{-1}x}_{\theta}\right)$



$$\tan\theta = x = \frac{x}{1}$$

Aha! $\sin\theta = \frac{x}{\sqrt{1+x^2}}$ ← Ans.

Prob: If $\sin\theta = -\frac{1}{3}$ and $\pi < \theta < \frac{3\pi}{2}$, what is $\tan\theta$?



$$1^2 + d^2 = 3^2$$

$$d = \sqrt{9-1} = \sqrt{8}$$

$$\sin\theta = \frac{(-1)}{3}$$

$$\tan\theta = \frac{(-1)}{(-d)} = \frac{(-1)}{(-\sqrt{8})} = 1/\sqrt{8}$$