Lesson 21 on 3.9 Related rates

Exam 2 Thurs. Oct. 20
8:00–9:00 pm
Review: Wed. Oct. 19 (Lec)
Thurs. Oct. 20 (Rec)

Lessons 11–20

Prob: A balloon is being inflated at a rate of 5 ft^3/min. How fast is the radius growing when the balloon has radius 1 ft?

Step 1: Draw picture

V = volume

Step 2: Name variables

Step 3: How are vars related

$$V = \frac{4}{3}\pi r^3$$

Step 4: Write down what you know and what you want to know

Know: $\frac{dV}{dt} = 5\ ft^3/min$

Want: $\frac{dr}{dt}$ when $r = 1$ ft.

Step 5: Diff'tiate and use chain rule

$$\underbrace{\frac{d}{dt}V}_{=5} = \frac{d}{dt}\left[\frac{4}{3}\pi r^3\right] = \frac{4}{3}\pi\left(3\cdot r^2\right)\frac{dr}{dt}$$

$r=1$ ↑ ↑ want

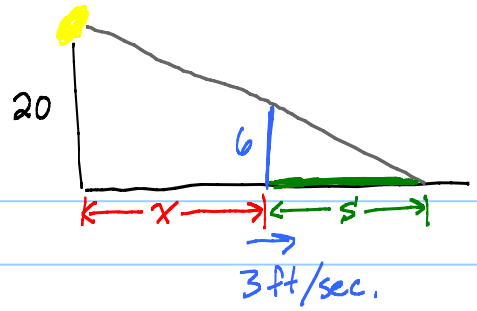
Step 6: Solve for rate you want:

$$\frac{dr}{dt} = \frac{5}{4\pi r^2} = \frac{5}{4\pi\cdot 1^2} \quad \text{when } r=1.$$

$$= \frac{5}{4\pi}\ ft/min \leftarrow \text{Ans.}$$

Remark: $\frac{dr}{dt}$ blows up when $r \to 0+$.

Prob: A 6 ft man walks away from from a 20 ft street light at 3 ft/sec. How fast is his shadow growing when he is 50 ft from pole?



Geometry! Similar Δ's:

$$\frac{s}{6} = \frac{s+x}{20}$$

Know: $\frac{dx}{dt} = 3$

Want: $\frac{ds}{dt}$ when $x = 50$

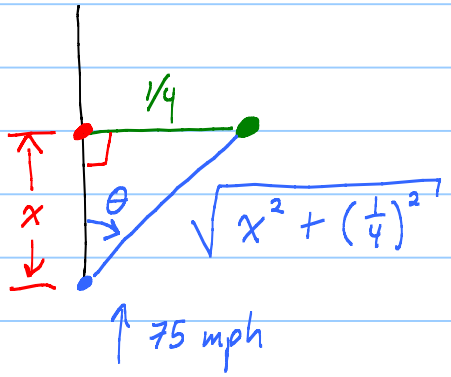
Solve for s: $\boxed{s = \frac{3}{7}x}$

$$\frac{ds}{dt} = \frac{3}{7}\frac{dx}{dt} = \frac{3}{7}(3) = \frac{9}{7} \text{ ft/sec.}$$

Prob: Driving down a road at 75 mph look at a silo a $\frac{1}{4}$ mile in. How fast head spinning when I pass it.

Know: $\frac{dx}{dt} = -75$ mph

Want: $\frac{d\theta}{dt}$ when $x = 0$



$$\tan\theta = \frac{(1/4)}{x}$$

$$\underbrace{\frac{d}{dt}(\tan\theta)}_{(\sec^2\theta)\frac{d\theta}{dt}} = \frac{d}{dt}\left[\frac{1}{4}x^{-1}\right] = \frac{1}{4}(-x^{-2})\frac{dx}{dt}$$

($\frac{dx}{dt} = -75$)

$$\frac{d\theta}{dt} = \frac{\frac{75}{4}\frac{1}{x^2}}{\sec^2\theta} = \frac{\frac{75}{4}\frac{1}{x^2}}{\left(\frac{\sqrt{x^2+\frac{1}{16}}}{x}\right)^2}$$

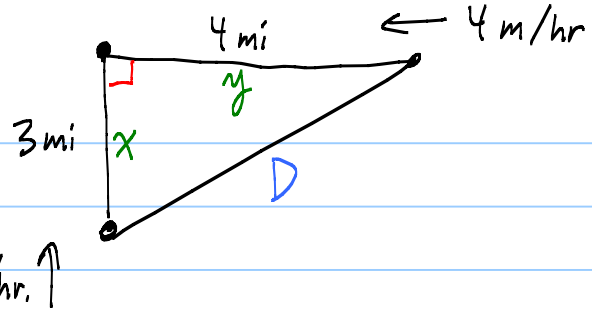
$$\frac{d\theta}{dt} = \frac{75/4}{x^2 + \frac{1}{16}} = \frac{75/4}{0 + \frac{1}{16}} = 300 \text{ rad./hr.}$$

when $x = 0$.

Fun fact: Replace $\frac{1}{4}$ m with distance d. Get $\frac{d\theta}{dt} = \frac{75}{d} \to \infty$

as $d \to 0$!

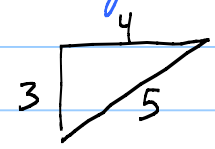
Prob:



Know $\frac{dx}{dt} = -3$, $\frac{dy}{dt} = -4$

Want $\frac{dD}{dt}$ when $x = 3$, $y = 4$.

$$x^2 + y^2 = D^2$$



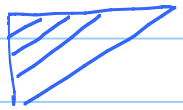
Take $\frac{d}{dt}$ of eqn and use Chain rule instead of solving for D.

$$2x\cdot\frac{dx}{dt} + 2y\cdot\frac{dy}{dt} = 2D\frac{dD}{dt}$$

(x: 3, $\frac{dx}{dt}$: −3, y: 4, $\frac{dy}{dt}$: −4, D: 5)

Get $\frac{dD}{dt} = \frac{-9-16}{5} = -5$ m/hr. ← Ans.

Does this rate change closer to collision time?

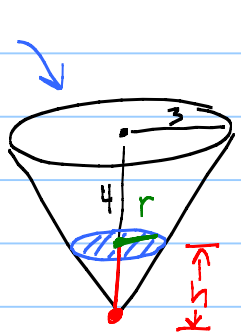


$$\frac{dD}{dt} = \frac{\left(\frac{x}{D}\right)\frac{dx}{dt} + \left(\frac{y}{D}\right)\frac{dy}{dt}}{D}$$

(x/D: const.; y/D: const.)

Crash at 5 mph.

EX:



Water hosed in at 5 ft^3/hr.

How fast is water rising when depth is 2 ft?

$$V = \frac{1}{3}\pi r^2 h$$

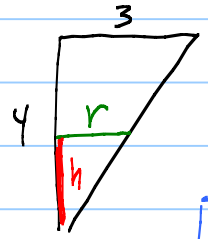
Know: $\frac{dV}{dt} = 5$.

Want: $\frac{dh}{dt}$ when $h = 2$.

$$V = \frac{1}{3}\pi\left(\frac{3}{4}h\right)^2 \cdot h$$

$$\boxed{V = \frac{3}{16}\pi h^3}$$

Slice:



$$\frac{r}{h} = \frac{3}{4}$$

$$\boxed{r = \frac{3}{4}h}$$

$$\frac{dV}{dt} = \frac{3}{16}\pi\left(3h^2\right)\frac{dh}{dt}$$

($\frac{dV}{dt}$: 5; h: when $h = 2$)

$$\frac{dh}{dt} = \frac{5}{\frac{9}{16}\pi \cdot 2^2} \quad \text{when } h = 2.$$

$$= \frac{20}{9\pi} \approx .707 \text{ ft/hr.}$$

Or $V = \frac{1}{3}\pi r^2 h$

$$\frac{dV}{dt} = \frac{1}{3}\pi \left[\left(2r \frac{dr}{dt}\right) h + r^2 \left(\frac{dh}{dt}\right) \right]$$

product rule.

Still need r and h prob.