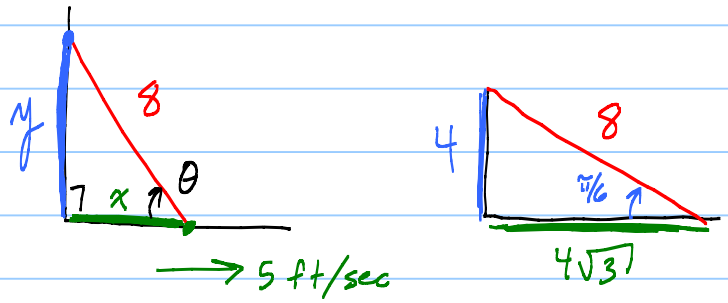
Lesson 22 on 3.9 Related rates, part 2.

Exam 2 – Thurs. 8-9 pm (see email)
Lessons 11-20

Prob: 8 ft ladder against wall on a slippery floor. Bottom slides out at 5 ft/sec. How fast is top sliding down at moment of 30° incline?



Know: $\frac{dx}{dt} = 5$

Want: $\frac{dy}{dt}$ when $\theta = \frac{\pi}{6}$

$x = 4\sqrt{3}$

$\sin\frac{\pi}{6} = \frac{\text{blue}}{8} = \frac{1}{2}$

$\cos\frac{\pi}{6} = \frac{\text{green}}{8} = \frac{\sqrt{3}}{2}$

$$x^2 + y^2 = 8^2$$

Method A

$$2x\cdot\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$x = 4\sqrt{3}$, $\frac{dx}{dt} = 5$, $y = 4$

$$\frac{dy}{dt} = \frac{-(4\sqrt{3})\cdot 5}{4} = -5\sqrt{3} \approx -8.66 \text{ ft/sec.}$$

Ans: Top sliding down at $5\sqrt{3}$ ft/sec

Method B

$$y = \sqrt{64 - x^2} = (64 - x^2)^{1/2}$$

$$\frac{dy}{dt} = \frac{1}{2}(64 - x^2)^{-1/2}\left(0 - 2x\frac{dx}{dt}\right)$$

derivative of inside

$$\frac{dy}{dt} = \frac{-x}{\sqrt{64 - x^2}}\frac{dx}{dt}$$

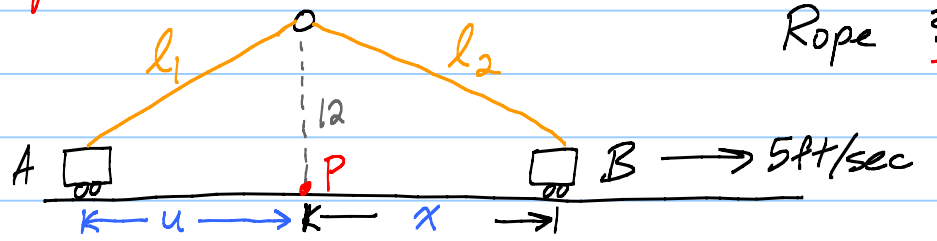
Get same answer.

$$\frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$$

$\cot\theta = \frac{x}{y}$ Ouch! at $\theta = 0$!

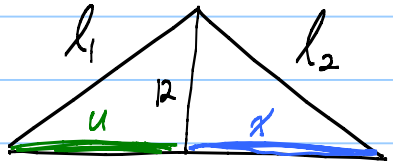
Prob:

Rope 39 ft long



Know: $\frac{dx}{dt} = 5$, $\ell_1 + \ell_2 = 39$ ←

Want: $\frac{dy}{dt}$ when B is 16 ft from P. ← Given.



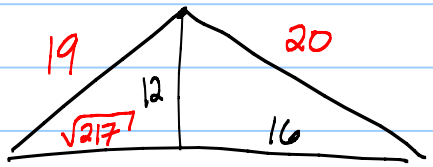
$u^2 + 12^2 = \ell_1^2$ $\quad x^2 + 12^2 = \ell_2^2$

$\ell_1 = \sqrt{u^2+144}$ $\quad \ell_2 = \sqrt{x^2+144}$

Key: $\ell_1 + \ell_2 = 39$

$$\sqrt{u^2+144} + \sqrt{x^2+144} = 39$$ ← Diff wrt t now.

$$\frac{1}{2}(u^2+144)^{-1/2}(2u)\frac{du}{dt} + \frac{1}{2}(x^2+144)^{-1/2}(2x)\frac{dx}{dt} = 0$$



$$\frac{u}{\sqrt{u^2+144}}\frac{du}{dt} + \frac{x}{\sqrt{x^2+144}}\frac{dx}{dt} = 0$$ ←

$$\frac{\sqrt{217}}{19}\cdot\frac{du}{dt} + \frac{16}{20}(5) = 0$$

$\frac{dy}{dt} = \frac{-x}{u}\frac{\sqrt{u^2+144}}{\sqrt{x^2+144}}\frac{dx}{dt}$

$\sqrt{585}$ $\quad$ 12 $\quad$ 27 $\quad$ 27 $\quad$ 5

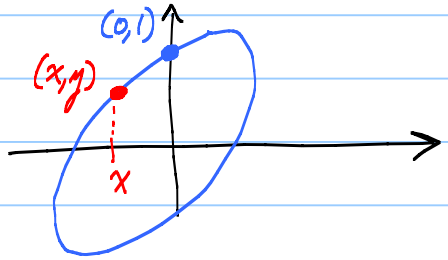
$u \to 0!$

Whaam! When $u=0$,

$$\frac{dy}{dt} = \frac{-19}{\sqrt{217}}\cdot\frac{4}{5}\cdot 5 = \frac{-76}{\sqrt{217}} \approx -5.16 \text{ ft/sec.}$$

Ans: Car A is moving $\frac{76}{\sqrt{217}}$ ft/sec to the right when $x=16$.

Prob: Roller coaster car towed in a such a way that $\frac{dx}{dt} = 5$ mph.



$$2x^2 - xy + y^2 = 1$$

↑ Diff wrt t.

What is $\frac{dy}{dt}$ when $x=0, y=1$.

$$2(2x)\frac{dx}{dt} - \underbrace{\left[\frac{dx}{dt}y + x\frac{dy}{dt}\right]}_{\text{prod rule}} + (2y)\frac{dy}{dt} = 0$$

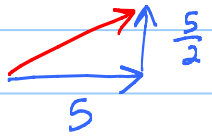
Know: $\frac{dx}{dt} = 5$

Want: $\frac{dy}{dt}$ when $x=0, y=1$.

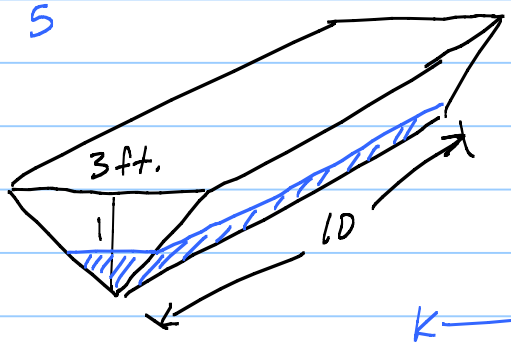
$$2\cdot 2\cdot 0\cdot 5 - \left[5\cdot 1 + 0\cdot\frac{dy}{dt}\right] + 2\cdot 1\cdot\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{5}{2} \text{ mph}$$

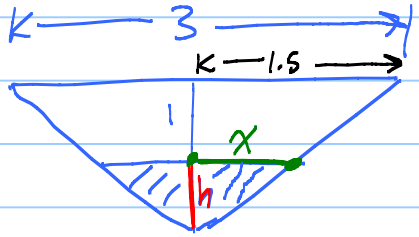
Velocity:



WebAssign:



Trough



Know $\frac{dV}{dt}$ = Const. Given.

$$\frac{1.5}{1} = \frac{x}{h}$$

Get rid of x! $x = 1.5h$

$$\text{Area} = \frac{1}{2}(2x)h$$

$$V = \text{Volume} = (\text{Area})(\text{length}) = \left(\frac{1}{2}(2x)h\right)\cdot 10$$

Want $\frac{dh}{dt}$ when $h=1$ ← overflow moment.