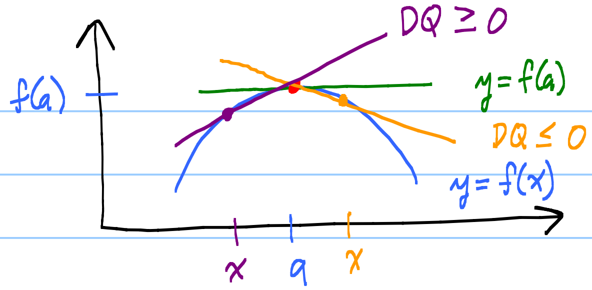
Lesson 24 on 4.1 Max & mins



Say f(x) has a local maximum at $x=a$ and f is diff'ble at a

[a local max means the top of a bump and global max means top of Mt. Everest.]

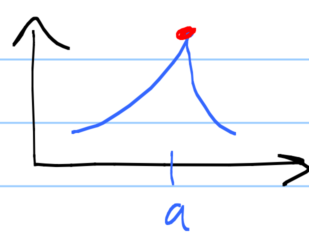
x on right: $DQ = \dfrac{f(x)-f(a)}{x-a} = \dfrac{(\le 0)}{(+)} \le 0$

x on left: $DQ = \dfrac{f(x)-f(a)}{x-a} = \dfrac{(\le 0)}{(-)} \ge 0$

Conclusion: $f'(a) = \underbrace{\lim_{x\to a^-} DQ}_{\ge 0} = \underbrace{\lim_{x\to a^+} DQ}_{\le 0}$

Aha! $f'(a)$ must be zero

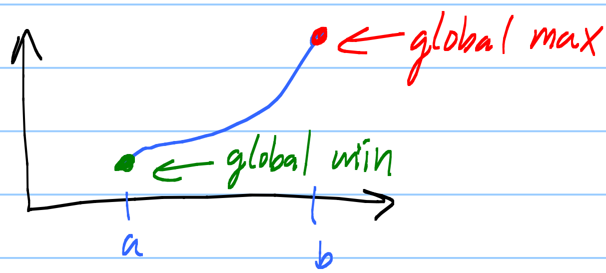
What if $f'(a)$ DNE?

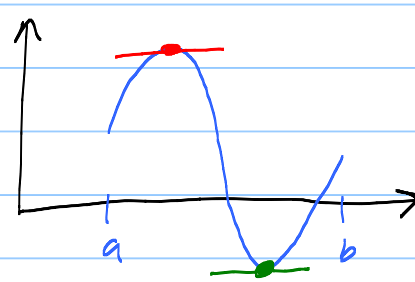


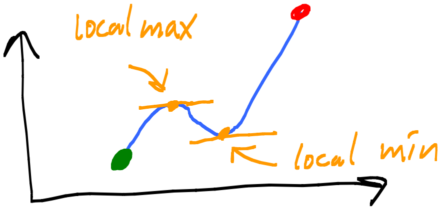
Could still be a max or min.

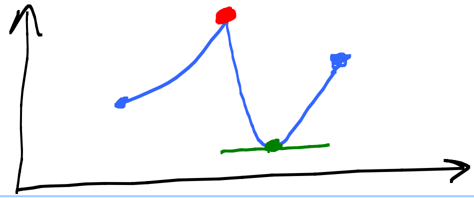
Same story for mins.

Prob: Given a continuous fcn f on a closed interval [a,b], find the Max and min of f on [a,b].









Fermat's Thm: If c is a point in $a < x < b$ and $f(c)$ is a global max (or min) on $[a,b]$, and f is diff'ble at c, then $f'(c) = 0$.

Critical Points: The critical numbers of f on (a,b) are the numbers c where
a) $f'(c)$ exists and $f(c) = 0$, or
b) $f'(c)$ DNE.

Prob: Find global max (or min) of f on $[a,b]$.

Sol^n: List the critical numbers and the endpoints a and b. Calculate $f(x)$ at those points.

x	$f(x)$
a	$f(a)$
c_1	$f(c_1)$
c_2	$f(c_2)$
b	$f(b)$

} Spot Max and min in this list.

Prob: Find max value of $f(x) = xe^{-2x}$ on $[0,10]$.

Step 1: No place where f' DNE. No crt #s where f' DNE.

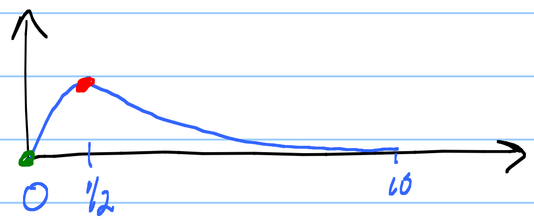
Step 2: $f'(x) = 1 \cdot e^{-2x} + x\left(e^{-2x} \cdot (-2)\right)$

$$= \underset{\text{never zero}}{e^{-2x}} \underbrace{(1 - 2x)}_{=0} = 0 \text{ at crt \#}$$

$x = \frac{1}{2}$ is a crt #.

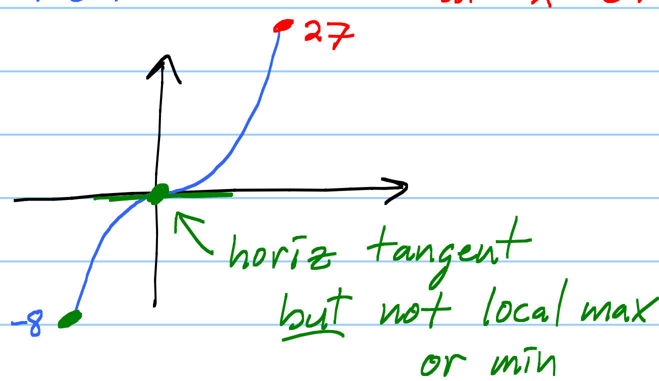
	x	$f(x)=xe^{-2x}$	
end →	0	0	← Global min
crt # →	$\frac{1}{2}$	$\frac{1}{2}e^{-2\cdot\frac{1}{2}} = \frac{1}{2e}$	← Global max
end →	10	$10e^{-20} = \frac{10}{e^{20}}$	>0 (tiny!)

Name for method: Extreme value theorem (EVT)

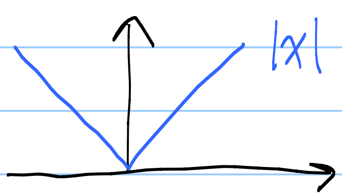


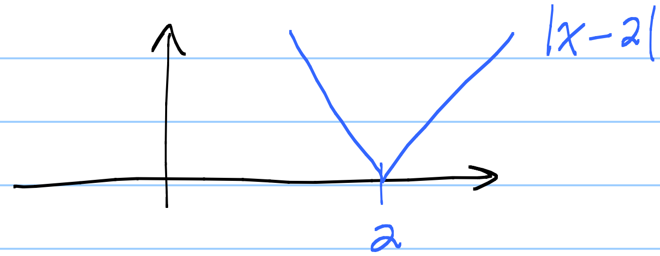
EX: $f(x) = x^3$ on $[-2,3]$ $\qquad f'(x) = 3x^2 = 0$ at $x=0$.

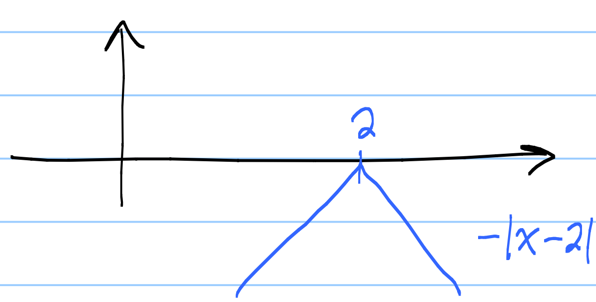
x	$f(x)$	
-2	-8	← min
0	0	
3	27	← max

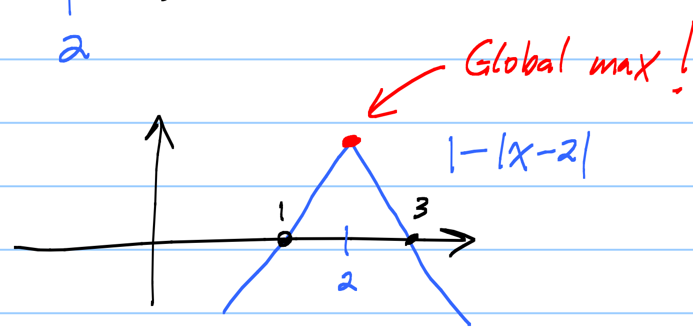


EX: $y = 1-|x-2|$ on $[0,3]$

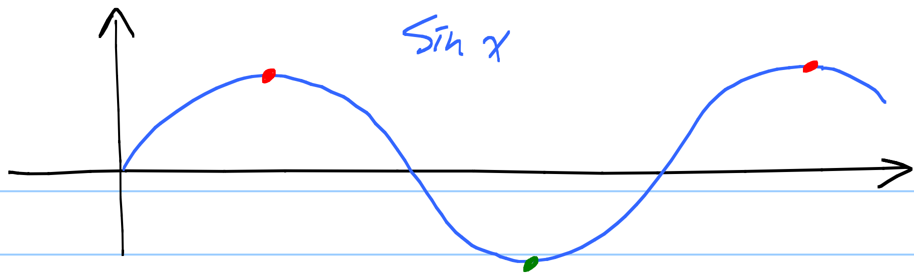






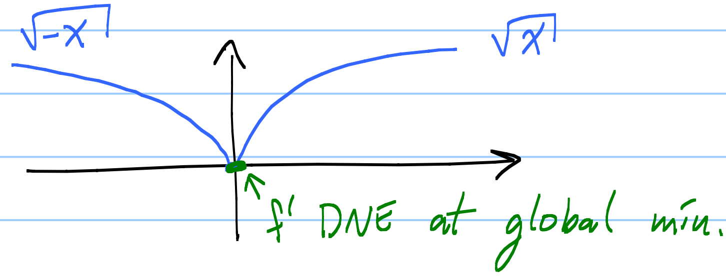


x	$f(x)$
0	-1
2	1
3	0



Multiple solⁿs possible.

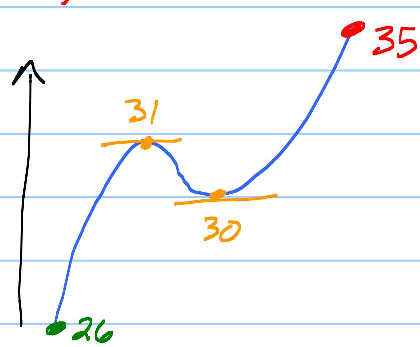
EX: $f(x) = \sqrt{|x|}$



Prob: $f(x) = 2x^3 - 15x^2 + 36x + 3$ on $[1, 4]$.

1. No crt #s wher f' DNE.
2. $f'(x) = 6x^2 - 30x + 36 = 6[x^2 - 5x + 6]$
   $= 6(x-2)(x-3) = 0$
   $x = 2, 3$ are crt #s.
3.

	x	$f(x)$	
end →	1	26	← min
crt #s →	2	31	
crt #s →	3	30	
end →	4	35	← max



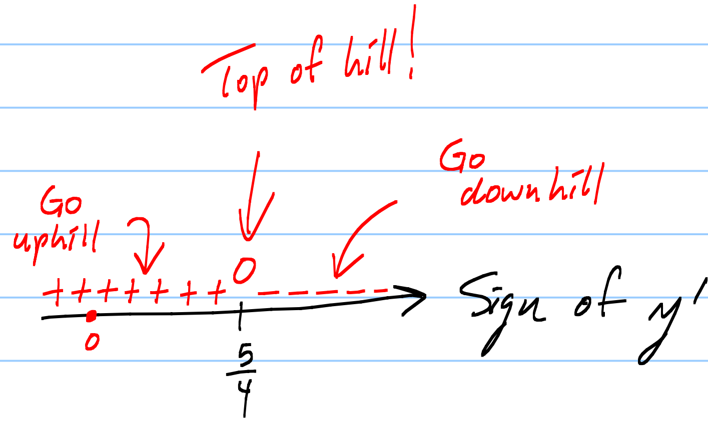
Prob: Throw rock up:

$y = 40t - 16t^2 + 4$

How high does it go?

$y' = 40 - 32t = 0$

$t = \frac{40}{32} = \frac{5}{4}$



So max happens at $x = \frac{5}{4}$. How high? $y = 40\left(\frac{5}{4}\right) - 16\left(\frac{5}{4}\right)^2 + 4$

$= 29$ ft.