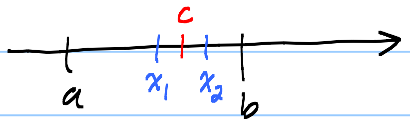
Lesson 26 on 4.3 Derivatives & the shape of graphs

How do we know what we think we know?

1. $f' > 0$ on (a,b) implies f is increasing on (a,b)



$$\boxed{\frac{f(x_2)-f(x_1)}{x_2-x_1} = f'(c)} > 0$$

MVT

Mult. by $x_2 - x_1 > 0$:

$f(x_2) - f(x_1) > 0$

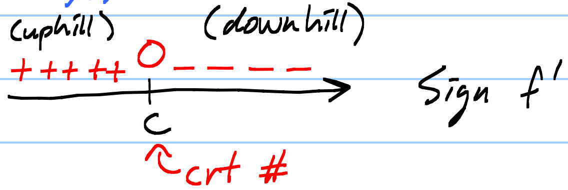
$f(x_2) > f(x_1)$.

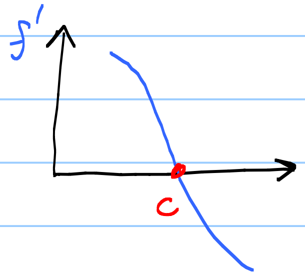
strictly increasing!

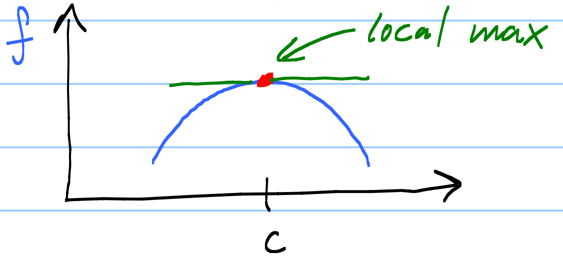
[$x_1 < x_2$ can chosen arbitrarily.]

2. $f' < 0$ on (a,b) implies f is decreasing on (a,b).

Sign of f':

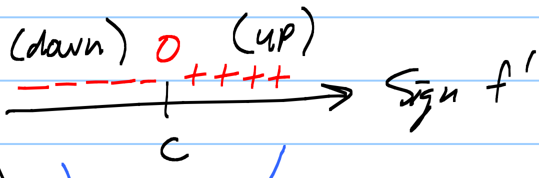


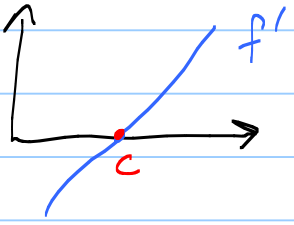


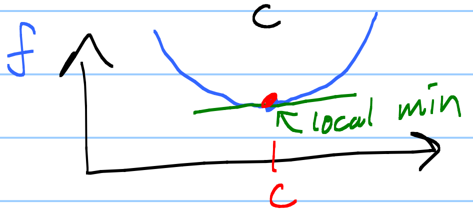


First Derivative Test (Part 1): If f' goes from being positive to negative at c, then f has a local max at $x = c$.

Part 2: If f' goes from neg to pos at c, then f has local min at c.

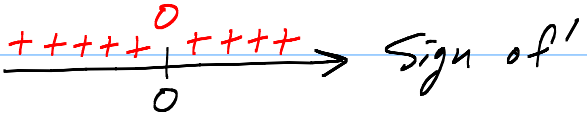


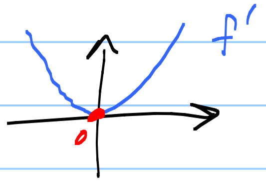


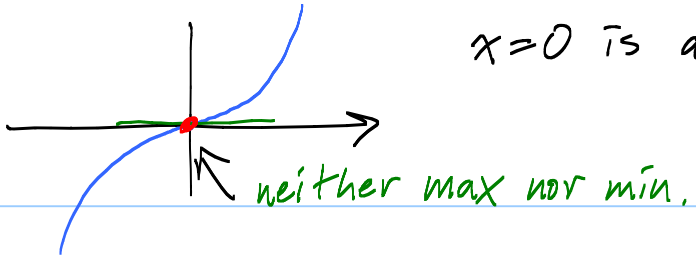


EX: $y = x^3$

$y' = 3x^2$



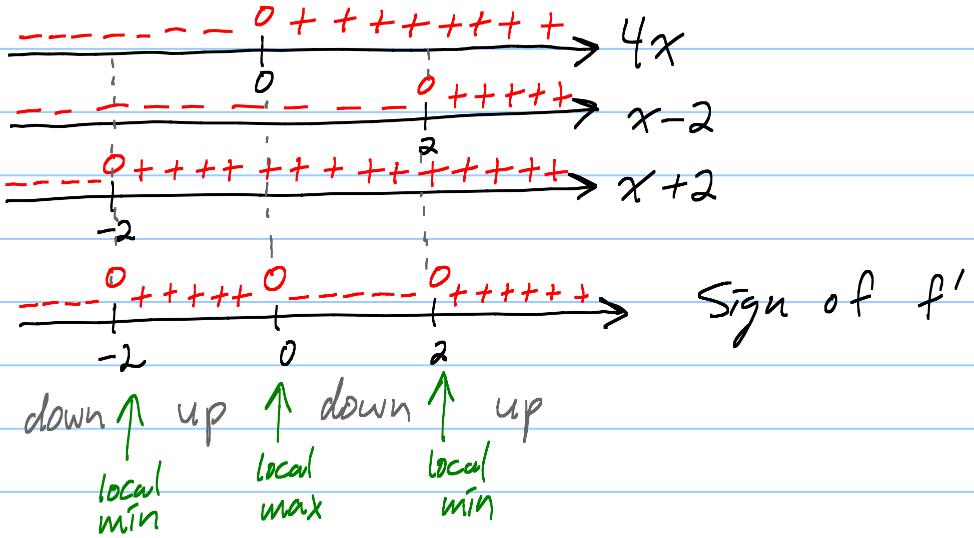


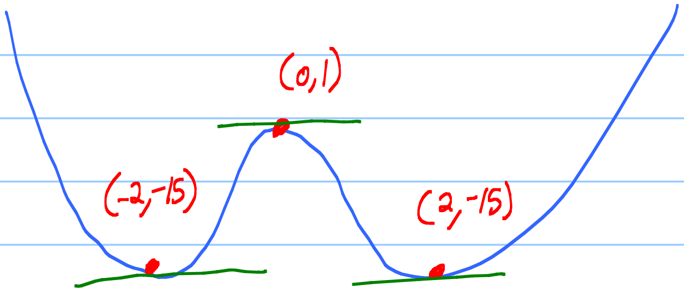


$x=0$ is a "resting spot" going uphill.

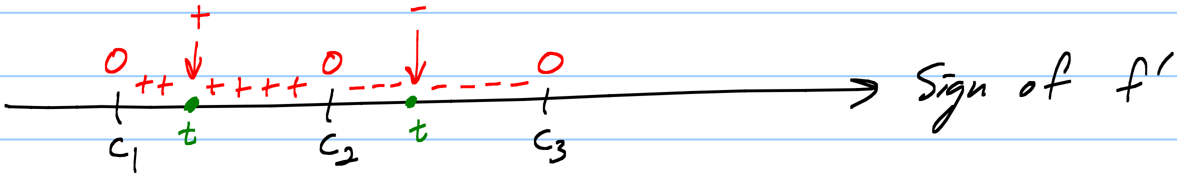
EX: Where is $f(x) = x^4 - 8x^2 + 1$ inc/dec? Find local extrema.

First: $f'(x) = 4x^3 - 16x = 4x(x^2-4) = 4x(x-2)(x+2)$





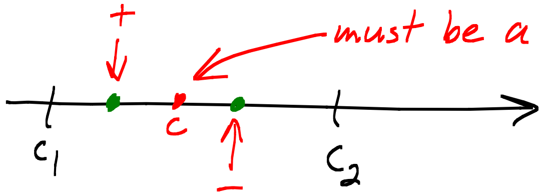
What to do if you can't factor f':



Find all crt #'s c_j.

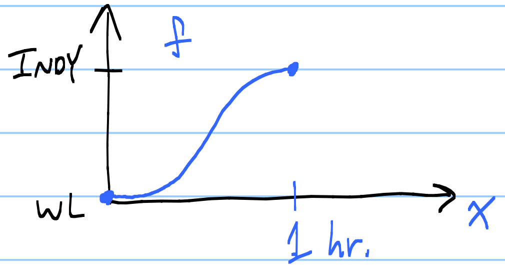
Pick "test point" t between two c's. If $f'(t) > 0$, then f' is > 0 on whole interval. If $f'(t) < 0$, then $f' < 0$ in whole interval.

Why: Intermediate Value Theorem!



must be a c where it is zero. Oops!
There is no c between c_1 and c_2.

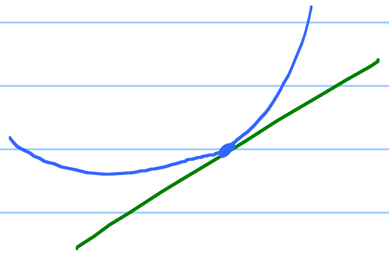
Meaning of sign of f'':



Velocity
$f' > 0$ ← going forward
$f' < 0$ ← going backwards
$f' = 0$ ← stop

Accel
$f'' > 0$ ← stepping on gas
$f'' < 0$ ← slowing down
$f'' = 0$ ← coasting (on cruise control.)

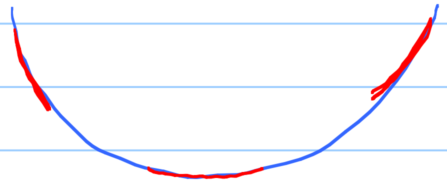
$f'' > 0$



tangent lines peel up off a tangent line

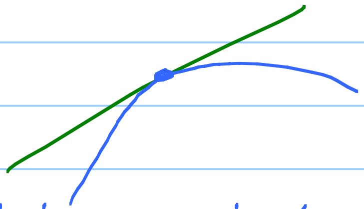
CONCAVE UP

Graph of f lies above all the tangent lines.



Think: Any piece of a bowl right side up.

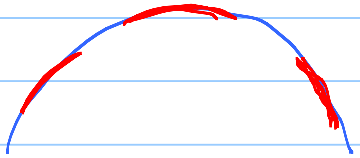
$f'' < 0$



tangent lines peel down off a tangent line

CONCAVE DOWN

Graph of f lies below all the tangent lines.

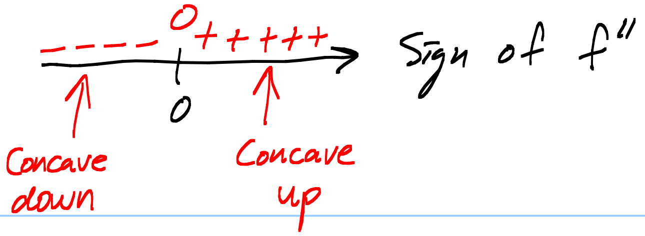


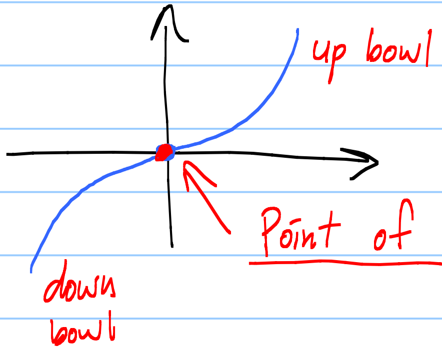
Think: Piece of upside down bowl.

EX: $f(x) = x^3$

$f'(x) = 3x^2$

$f''(x) = 6x$





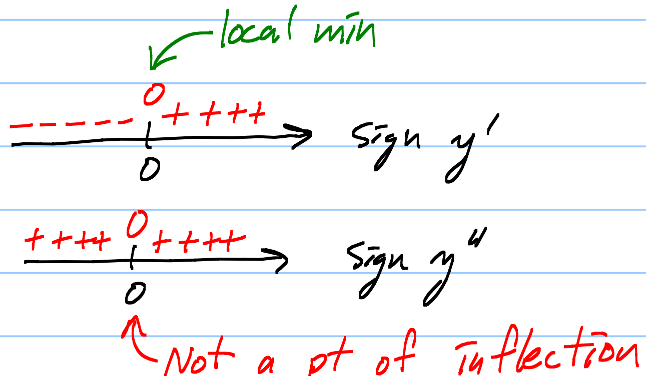
Point of Inflection: Where concavity changes.

(minus to plus
or
plus to minus)

EX: $y = x^4$

$y' = 4x^3$

$y'' = 12x^2$



local min is a global min!