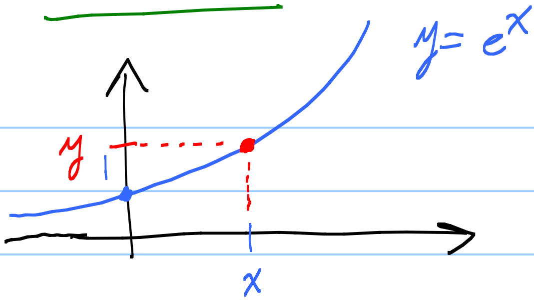
Lesson 3 on 1.5 Inverse Functions



$x = \ln y$ = the x such that $e^x = y$

domain of e^x : $(-\infty, \infty)$
range of e^x : $(0, \infty)$

Ln only makes sense for >0 numbers.

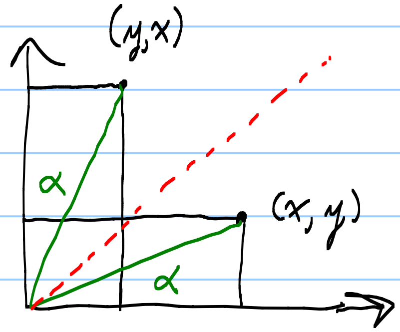
Ln and e^x are inverse fcns

$$\begin{cases} \ln e^x = x \\ e^{\ln x} = x \end{cases}$$

They "undo" each other

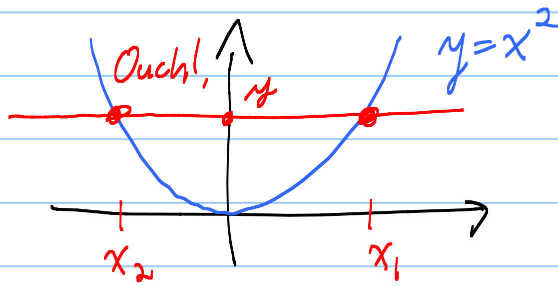
Notation: $y = f(x)$
$x = f^{-1}(y)$

[$y = \frac{1}{f(x)} = f(x)^{-1}$
Careful where you put -1.

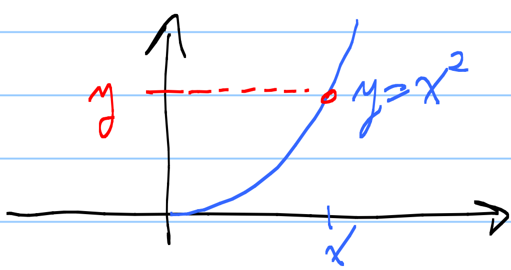


(x,y), (y,x) are symmetric about line $y = x$

EX:



Fails the "red line test"
[one point only allows an inverse fcn.]



$x = \sqrt{y}$ for $y > 0$

$\sqrt{\;}$ is inverse to x^2.

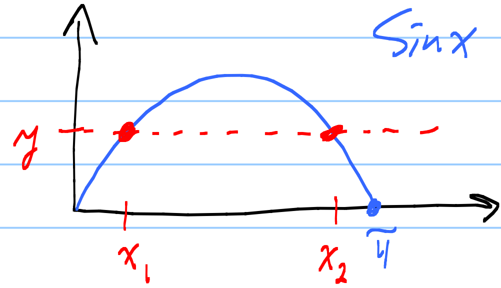
Common practice: Using x as var. for both f and f^{-1};

EX: "$\ln x$ is inverse of e^x"

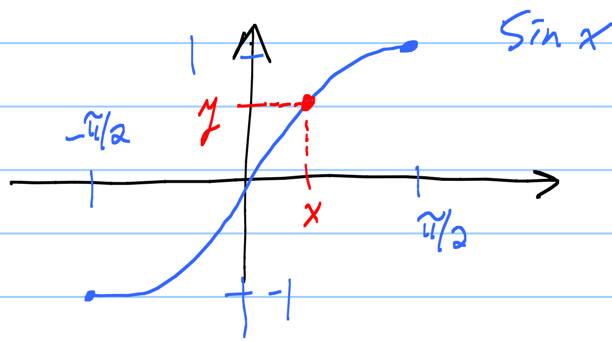
$$\begin{cases} y = e^x \\ x = \ln y \end{cases}$$

Def^n: $f(x)$ is one-to-one: If $f(x_1) = f(x_2)$, then x_1 must $= x_2$

EX:



EX:

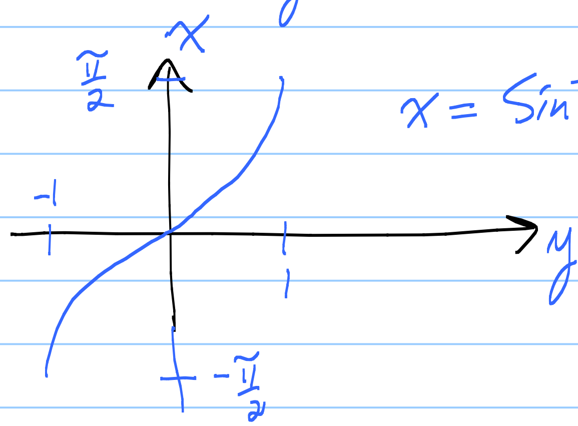


Domain $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Range $\underbrace{[-1, 1]}_{\text{domain of inverse}}$

$y = \sin x$

$x = \sin^{-1} y$ or $\text{ArcSin}\, y$



$x = \sin^{-1} y$

Note: $\sin^2 x = (\sin x)^2$

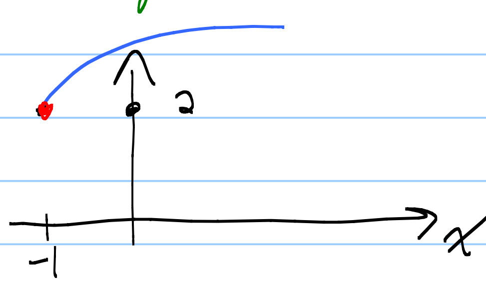
But $\sin^{-1} x \neq \dfrac{1}{\sin x}$

Algebra to find f^{-1}: Ex: $y = 2 + \sqrt{1+x}$ ← $x > -1$

↑ solve for x

$y - 2 = \sqrt{1+x}$

$(y-2)^2 = 1 + x$ ← $x = -1 + (y-2)^2$



domain $[-1, \infty)$

range $[2, \infty)$ ← domain of f^{-1}

Ex:

$$y = \frac{2x+1}{5x+3}$$

$$(5x+3)y = 2x+1$$

$$5xy + 3y = 2x+1$$

$$5xy - 2x = 1 - 3y$$

$$x(5y-2) = 1-3y$$

$$x = \frac{1-3y}{5y-2}$$

Rules of Ln:

$x, y > 0$

$$\begin{cases} \ln(xy) = \ln x + \ln y \\ \ln \frac{1}{x} = -\ln x \\ \ln \frac{x}{y} = \ln x - \ln y \end{cases}$$

$$e^{x+y} = e^x e^y$$

$$e^{-x} = \frac{1}{e^x}$$

$$e^{x-y} = \frac{e^x}{e^y}$$

$x > 0$ $\qquad \ln x^p = p \ln x \qquad -\infty < p < \infty$

Music: $\quad f = 440 \ 2^x$

$$\ln f = \ln(440 \cdot 2^x) = \ln 440 + \ln 2^x$$

$$= \underbrace{\ln 440 + x \ln 2}_{\text{Linear}}$$

Prob: Solve $y = 10e^{2x+1}$ for x [Find inverse]

$$\frac{y}{10} = e^{2x+1}$$

Undo the exp using Ln,
$\qquad f \qquad\qquad f^{-1}$

$$\ln \frac{y}{10} = \ln e^{2x+1} = 2x+1$$

So $\quad x = \frac{1}{2}\left[\ln \frac{y}{10} - 1\right]$

$$x = \frac{1}{2}\left[\ln y - \ln 10 - 1\right]$$

Note: $\ln e = 1$

$\ln 1 = 0$

Ex: $\ln \frac{1}{\sqrt{e}} = \ln 1 - \ln e^{1/2}$

$= 0 - \frac{1}{2}\underbrace{\ln e}_{1} = -\frac{1}{2}$

or $\ln e^{-1/2}$ = same thing

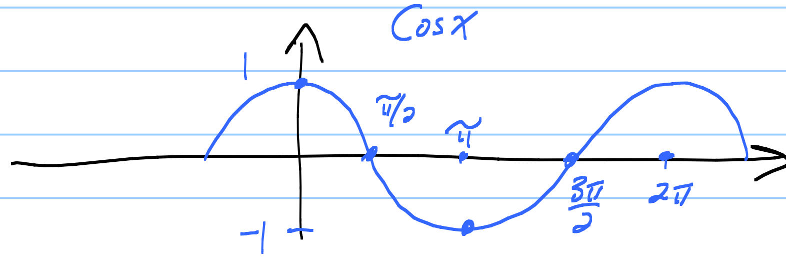
Other bases: $y = 10^x$ $\quad$ $x = \log_{10} y \leftarrow =$

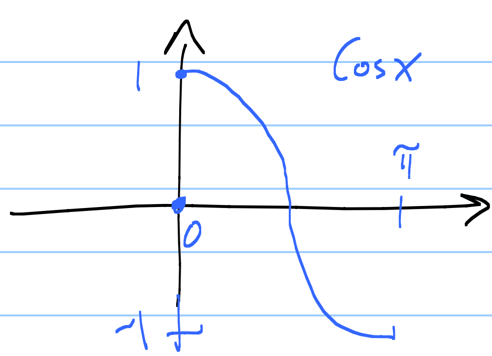
$\ln y = \ln 10^x = x \ln 10$ $\quad$ $x = \frac{\ln y}{\ln 10}$

Fact: 1) $\log_b a = \frac{\ln a}{\ln b}$

2) $\log_b x$ satisfies same rules as $\ln$

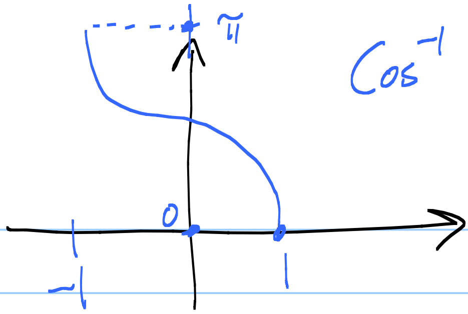
Inverse Trig Functions:

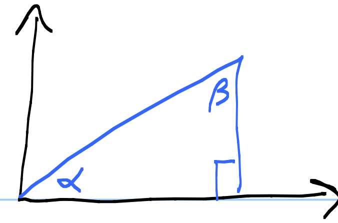




flip horizontal, Rotate 90°

$\cos^{-1}$

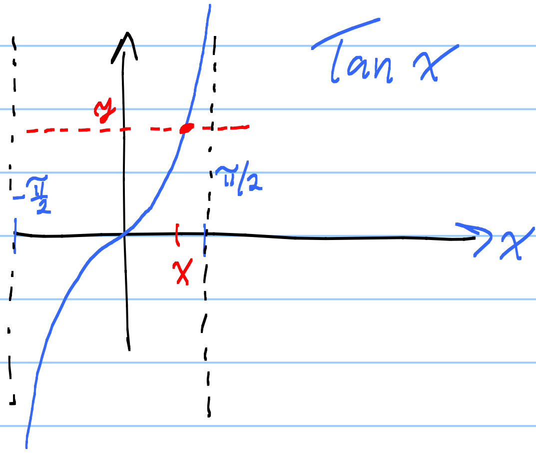




$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$$

who cares about $\cos^{-1}$?!

$\tan x$



$\tan^{-1}$