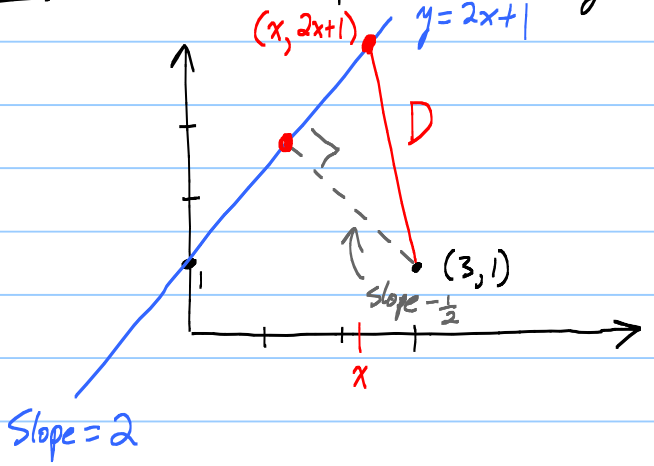
Lesson 31 on 4.7 Optimization problems, part 2.

Exam 3 Monday 8-9 pm
www.math.purdue.edu/MA161

Prob: Find closest point on line $y=2x+1$ to $(3,1)$.



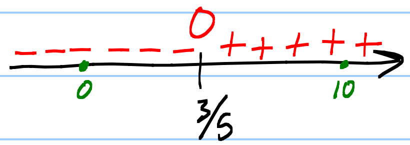
$$D(x) = \sqrt{(x-3)^2 + ((2x+1)-1)^2}$$

$$= \sqrt{x^2 - 6x + 9 + 4x^2}$$

$$= \left(5x^2 - 6x + 9\right)^{1/2}$$

$$\frac{dD}{dx} = \frac{1}{2}\left(5x^2-6x+9\right)^{1/2-1}\left[\underbrace{10x-6}_{=0}\right] = 0$$

$x = \frac{6}{10} = \frac{3}{5}$
crt. #

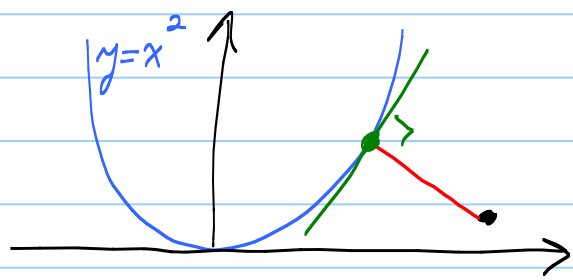


Sign of $\frac{dD}{dx}$

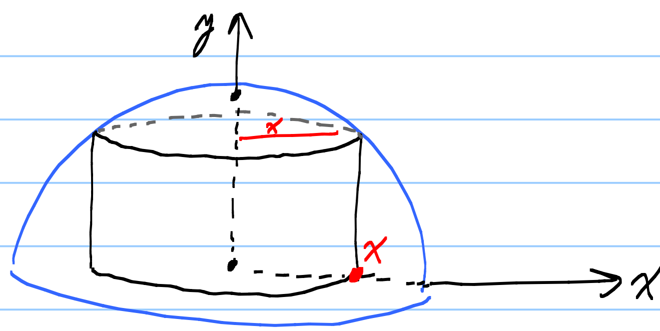
local min. Global too!

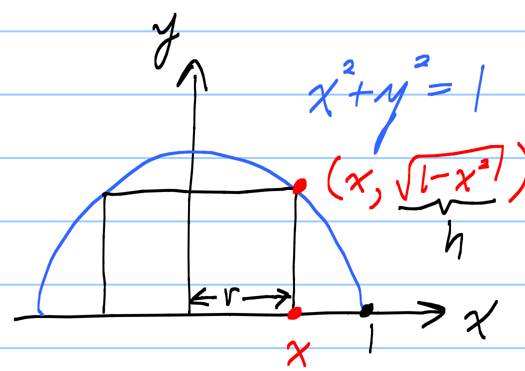
$x_{min} = \frac{3}{5}$, $y_{min} = 2 \cdot \frac{3}{5} + 1$ Closest point: $\left(\frac{3}{5}, \frac{11}{5}\right)$

HWK:



Prob: Cylindrical corn crib in a hemispherical dome. Maximize volume of the crib.





Volume: $V = \pi r^2 \cdot h = (\pi x^2) \cdot \sqrt{1-x^2}$ Note: $0 < x < 1$.

$$\frac{dV}{dx} = (2\pi x)\sqrt{1-x^2} + (\pi x^2)\,\frac{1}{2}(\underbrace{1-x^2}_{u})^{\frac{1}{2}-1}\cdot(\underbrace{-2x}_{\frac{du}{dx}})$$

$$= 2\pi x\sqrt{1-x^2} - \frac{\pi x^3}{\sqrt{1-x^2}} = 0$$

$$\pi x\left[\underbrace{2\sqrt{1-x^2} - \frac{x^2}{\sqrt{1-x^2}}}_{=0}\right] = 0$$

$x = 0$

straw case!

$$2\sqrt{1-x^2} = \frac{x^2}{\sqrt{1-x^2}}$$

$$2(1-x^2) = x^2$$

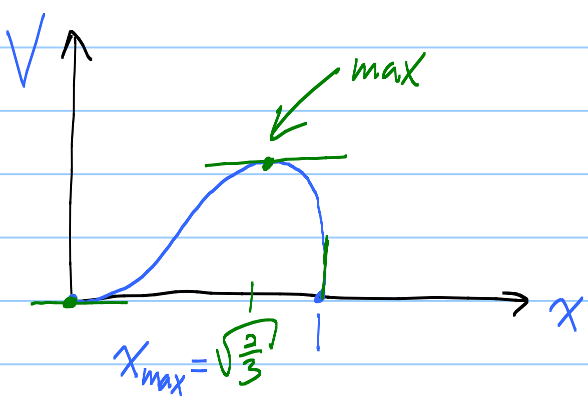
$$2 = 3x^2$$

$$x^2 = \frac{2}{3}$$

$$x = \pm\sqrt{\frac{2}{3}}$$

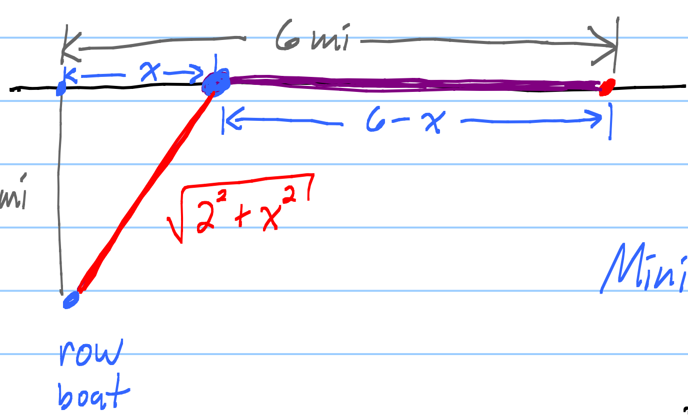
toss out − one

Crt. # $\boxed{x = \sqrt{\frac{2}{3}}}$



$$V_{max} = \pi\left(\sqrt{\frac{2}{3}}\right)^2\sqrt{1-\left(\sqrt{\frac{2}{3}}\right)^2} = \pi\,\frac{2}{3}\cdot\sqrt{\frac{1}{3}}$$

Prob:



row 2 mph

walk 5 mph

Minimize travel time

$$\text{Velocity} = \frac{\text{Distance}}{\text{Time}} \quad \text{so} \quad \text{Time} = \frac{\text{Distance}}{\text{Velocity}}$$

Total travel time: $\boxed{T = \frac{\sqrt{4+x^2}\ m}{2\ mph} + \frac{6-x}{5}}$ $0 \le x \le 6$

$$\frac{dT}{dx} = \frac{1}{2}\left[\frac{1}{2}(4+x^2)^{\frac{1}{2}-1}(2x)\right] + 0 - \frac{1}{5} = 0$$

x	$T(x)$
0	$\frac{2m}{2mph} + \frac{6m}{5mph} = 2.2$
$\frac{4}{\sqrt{21}}$	≈ 2.117 ← min
6	$\frac{\sqrt{40}}{2} + 0 \approx 3.162$

$$\frac{x}{2\sqrt{4+x^2}} = \frac{1}{5}$$

$$5x = 2\sqrt{4+x^2} \quad \leftarrow \text{square}$$

$$25x^2 = 4(4+x^2) = 16 + 4x^2$$

$$21x^2 = 16$$

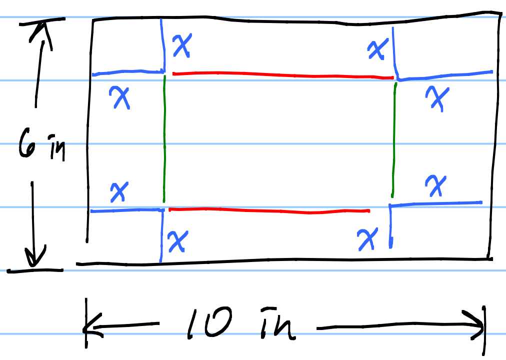
$$x^2 = \frac{16}{21}$$

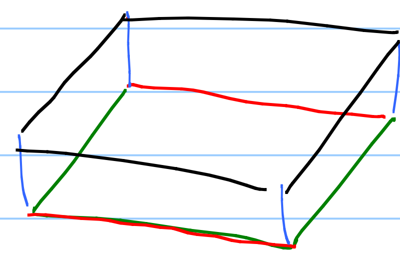
$$x = \pm\frac{4}{\sqrt{21}} \quad \leftarrow \text{toss out} < 0 \text{ one,}$$

$$\frac{4}{\sqrt{21}} \approx .87\ m$$

$$T\left(\frac{4}{\sqrt{21}}\right) \approx 2.117 \text{ hrs.}$$

Prob:





open box

Maximize Volume

$$V = l \cdot w \cdot h$$

$$V = (10-2x)(6-2x)\cdot x$$

$$V = 4x^3 - 32x^2 + 60x, \quad 0 \le x \le 3$$

$$\frac{dV}{dx} = 12x^2 - 64x + 60 = 0$$

$$3x^2 - 16x + 15 = 0$$

Quad formula: $x = \frac{16 \pm \sqrt{16^2 - 4\cdot 3\cdot 15}}{2\cdot 3} = \frac{16 \pm \sqrt{76}}{6}$

Ops! $\frac{16+\sqrt{76}}{6} > 3$, out

Max happens at $x = \dfrac{16-\sqrt{761}}{6} \approx 1.2137$ in

See p. 335. Store sells 100 TVs per week at \$200 each.
If increase price by \$10, sales drop by 15 TVs/wk.
If each TV costs the store \$150, what should prize be to maximize profits.

$$\text{Profit} = \underbrace{\left(\#\text{ TVs sold}\right)}_{x} \left[\underbrace{\text{Price}}_{P(x)} - \underbrace{\text{Cost}}_{150} \right]$$

x	Price
100	200
85	210
70	220 ← $p(x)$
⋮	⋮

$P(x)$ is the demand function.

Assume it is linear

$$P(x) = mx + b = -\tfrac{2}{3}x + \tfrac{800}{3}$$

solve
$$\begin{cases} P(100) = m\cdot 100 + b = 200 \\ P(85) = m\cdot 85 + b = 210 \end{cases}$$

get $m = -\frac{2}{3}$, $b = \frac{800}{3}$