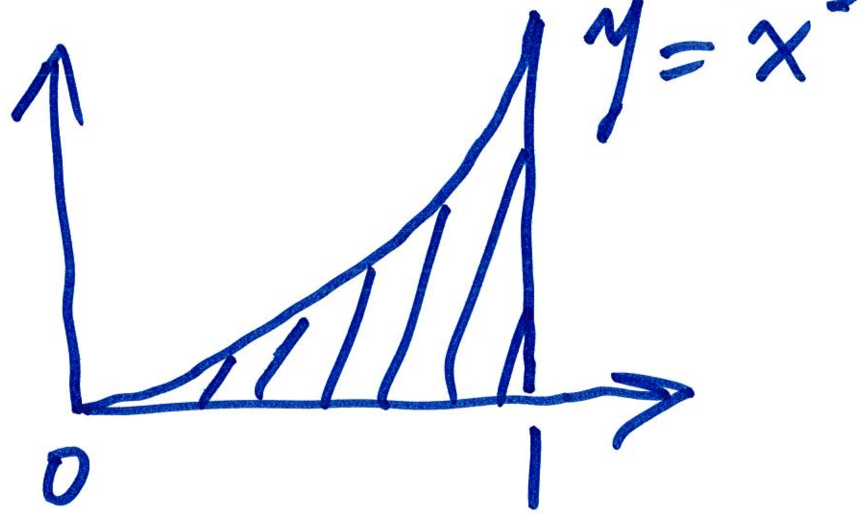
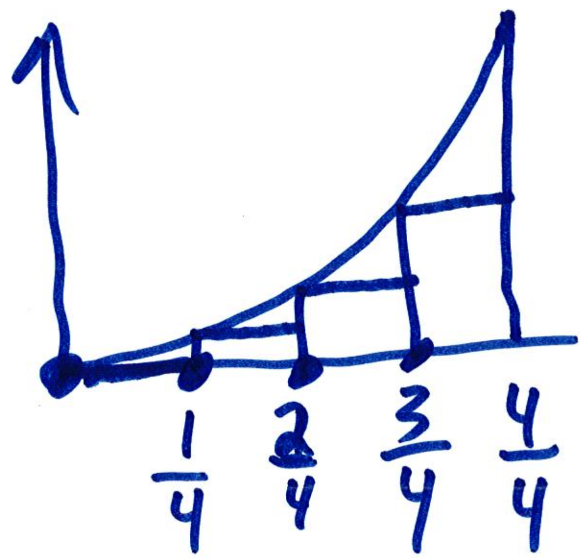
Lesson 33 on 5.1 & 5.2

Areas, distance, integrals

Prob:

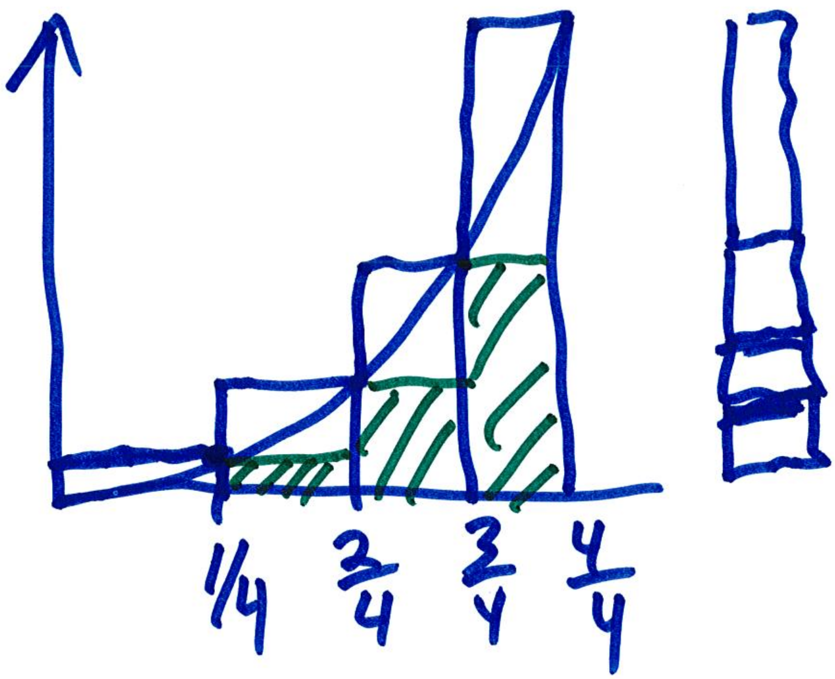




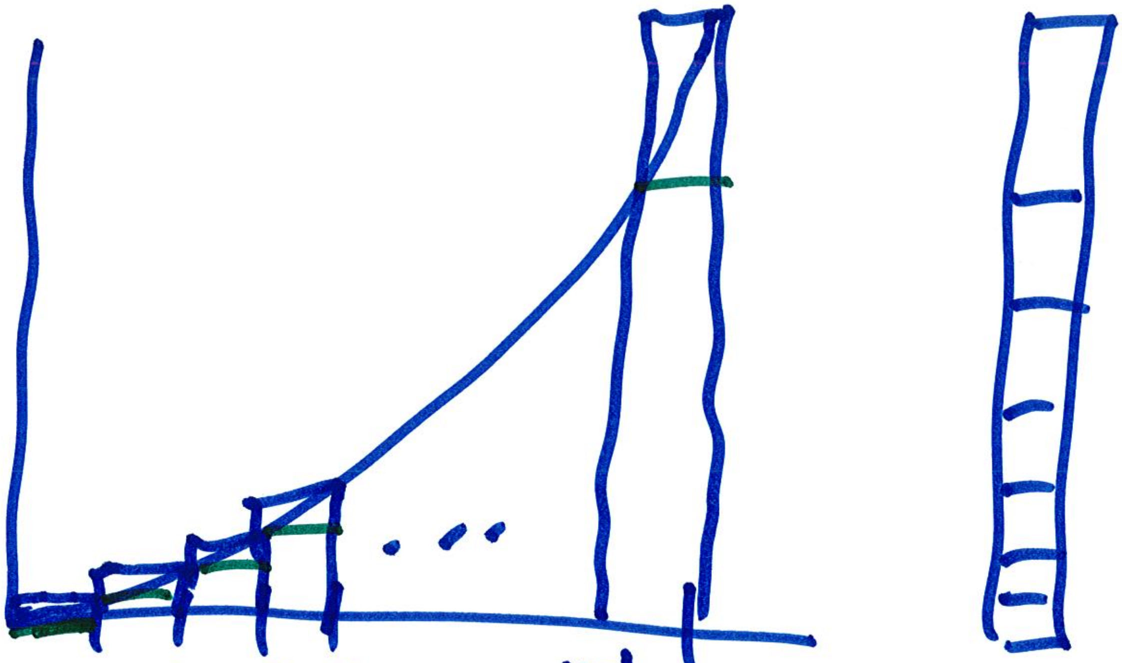
$$A_{\text{too small}} = 0^2 \cdot \frac{1}{4} + \left(\frac{1}{4}\right)^2 \cdot \frac{1}{4} + \left(\frac{2}{4}\right)^2 \cdot \frac{1}{4} + \left(\frac{3}{4}\right)^2 \cdot \frac{1}{4}$$

left endpoint

$$A_{too\ big} = \left(\frac{1}{4}\right)^2\cdot\frac{1}{4} + \left(\frac{2}{4}\right)^2\cdot\frac{1}{4} + \left(\frac{3}{4}\right)^2\cdot\frac{1}{4} + \left(\frac{4}{4}\right)^2\cdot\frac{1}{4}$$



$$A_{too\ b} - A_{too\ s} = \frac{1}{4}\cdot 1 = \frac{1}{4}$$



$$A_{too\ b} = \left(\frac{1}{n}\right)^2 \cdot \frac{1}{n} + \left(\frac{2}{n}\right)^2 \cdot \frac{1}{n} + \cdots \left(\frac{n}{n}\right)^2 \cdot \frac{1}{n}$$

$$A_{too\ s} = \left(\frac{0}{n}\right)^2 \cdot \frac{1}{n} + \left(\frac{1}{n}\right)^2 \cdot \frac{1}{n} \cdots \left(\frac{n-1}{n}\right)^2 \cdot \frac{1}{n}$$

$$A_{too\ b} - A_{too\ s} = \frac{1}{n} \cdot 1 = \frac{1}{n}$$

Math induction:

$$1+2+3+\cdots+N=\frac{N(N+1)}{2}$$

$$1^2+2^2+\cdots+N^2=\frac{N(N+1)(2N+1)}{6}$$

$$A_{\text{too } b}=\left(\frac{1}{n}\right)^2\cdot\frac{1}{n}+\left(\frac{2}{n}\right)^2\cdot\frac{1}{n}+\cdots+\left(\frac{n}{n}\right)^2\cdot\frac{1}{n}$$

$$=\frac{1}{n^3}\left[\underbrace{1^2+2^2+\cdots+n^2}\right]$$

$$=\frac{1}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$$

$$= \frac{\frac{n}{n} \cdot \frac{n+1}{n} \cdot \frac{2n+1}{n}}{6}$$

$$= \frac{1 \cdot \left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6}$$

$$\to \frac{1 \cdot 1 \cdot 2}{6} = \underline{\underline{\frac{1}{3}}}$$

the Area!

$$A_{\text{too s}} = \left(\frac{0}{n}\right)^2 \cdot \frac{1}{n} + \cdots + \left(\frac{n-1}{n}\right)^2 \cdot \frac{1}{n}$$

$$= \frac{1}{n^3}\left[1^2 + \cdots + (\underbrace{n-1}_{N})^2\right]$$

$$= \frac{1}{n^3}\left[\frac{(n-1)\left[(n-1)+1\right]\left[2(n-1)+1\right]}{6}\right]$$

$$\xrightarrow{\text{also}} \frac{1}{3} \checkmark$$

Calculus notation:

$$\int_0^1 x^2\,dx = \text{Area} = \frac{1}{3}$$

$$A_{\text{too big}} = \text{Sum}\ \underbrace{\left(\frac{k}{n}\right)^2}_{f(x_k)} \cdot \underbrace{\frac{1}{n}}_{\Delta x} \leftarrow \begin{matrix} k=1 \\ \text{to} \\ n \end{matrix}$$

x_k = right endp.

$$= \sum_{k=1}^{n} f(x_k)\,\Delta x$$

$$\longrightarrow \int_0^1 f(x)\,dx$$

Def^{n}: Definite integral

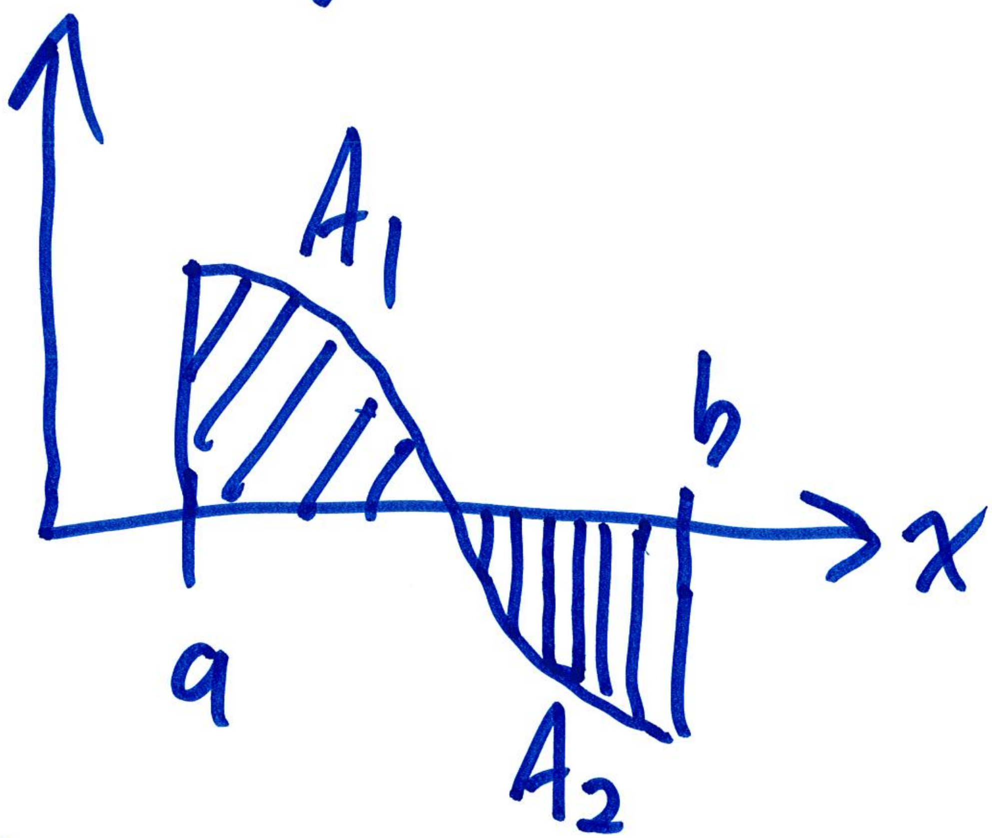
$$\int_a^b f(x)\,dx =$$

$$\lim_{\Delta x \to 0} \sum_{k=1}^{n} f(x_k)\,\Delta x$$

same for $x_k = \left.\begin{matrix}\text{right}\\ \text{left}\end{matrix}\right\}$ ends

Works for f continuous.

Warning:



$$\int_a^b f(x)\,dx = A_1 - A_2$$

$=$ (Area above x-axis)

$-$ (area below)

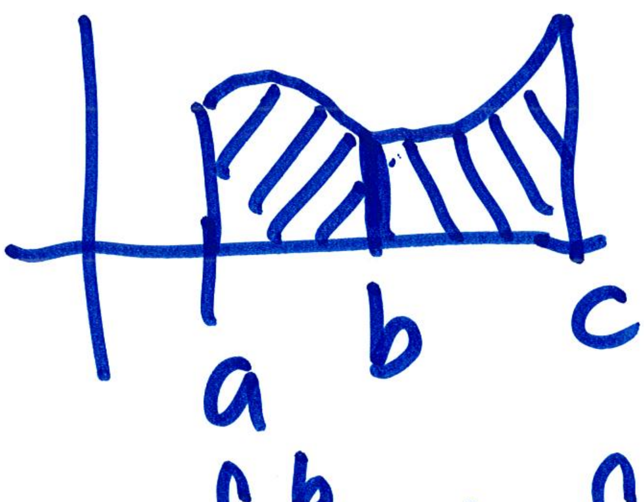
~~Prob~~: Area facts:

1) $$\int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx$$

2) $$\int_a^b f(x)+g(x)\,dx$$

$$= \int_a^b f\,dx + \int_a^b g\,dx$$

3)

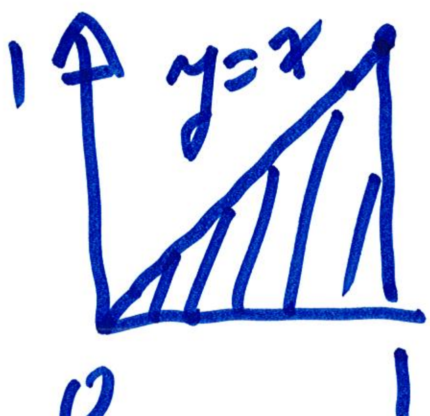


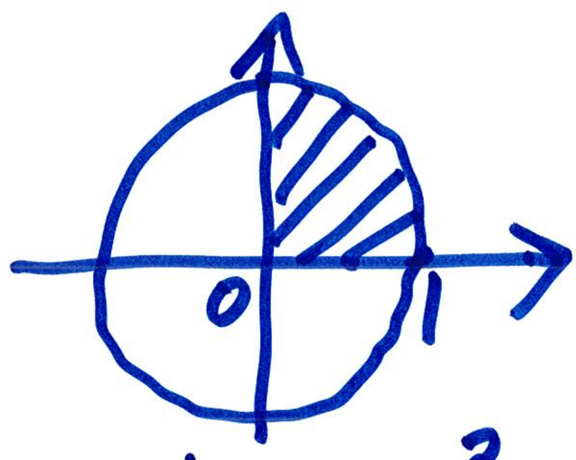
$$\int_a^c = \int_a^b + \int_b^c$$

<u>Prob</u>: Compute

$$I = \int_0^1 x - 2\sqrt{1-x^2}\,dx$$

$$= \underbrace{\int_0^1 x\,dx}\; - 2\underbrace{\int_0^1 \sqrt{1-x^2}\,dx}$$





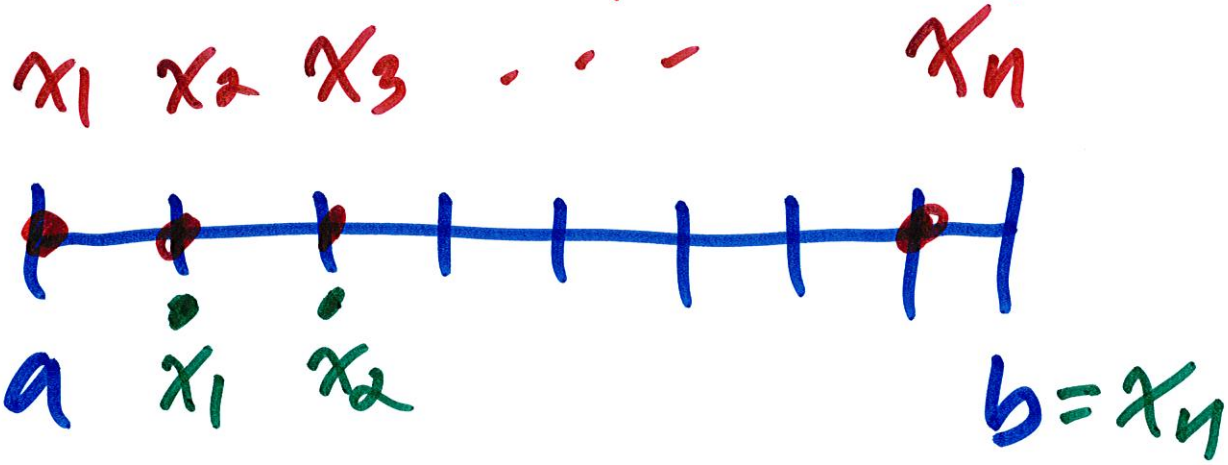
$\frac{1}{2}\cdot 1 \cdot 1$ $\qquad\qquad$ $\frac{1}{4}\cdot \pi \cdot 1^2$

$$= \left(\frac{1}{2}\right) - 2\left(\frac{1}{4}\pi\right) = \frac{1}{2} - \frac{\pi}{2}$$

Riemann sums

$$\sum_{k=1}^{n} f(x_k)\,\Delta x$$

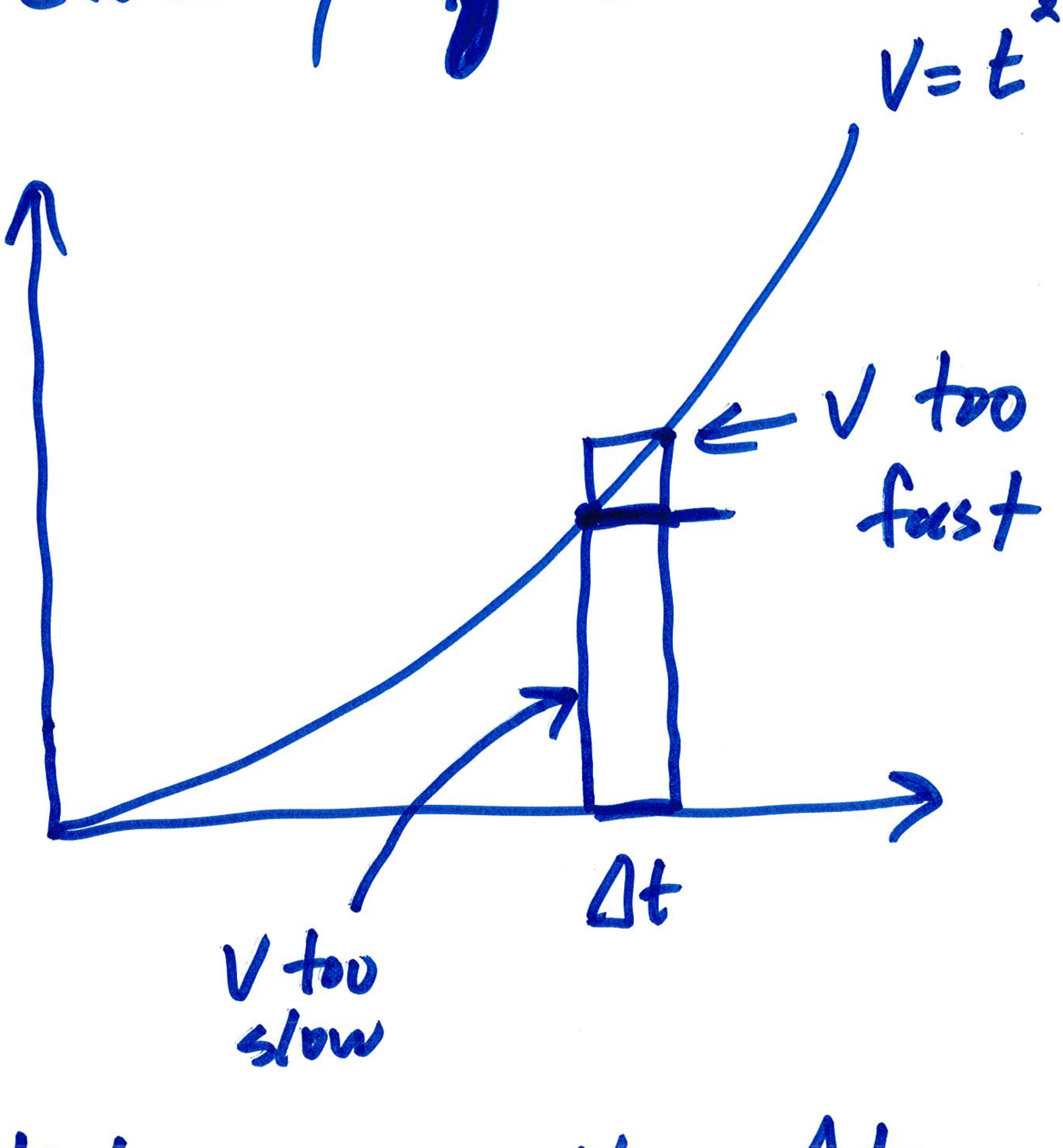
left endpts.



right ends

$$\Delta x = \frac{b-a}{n}$$

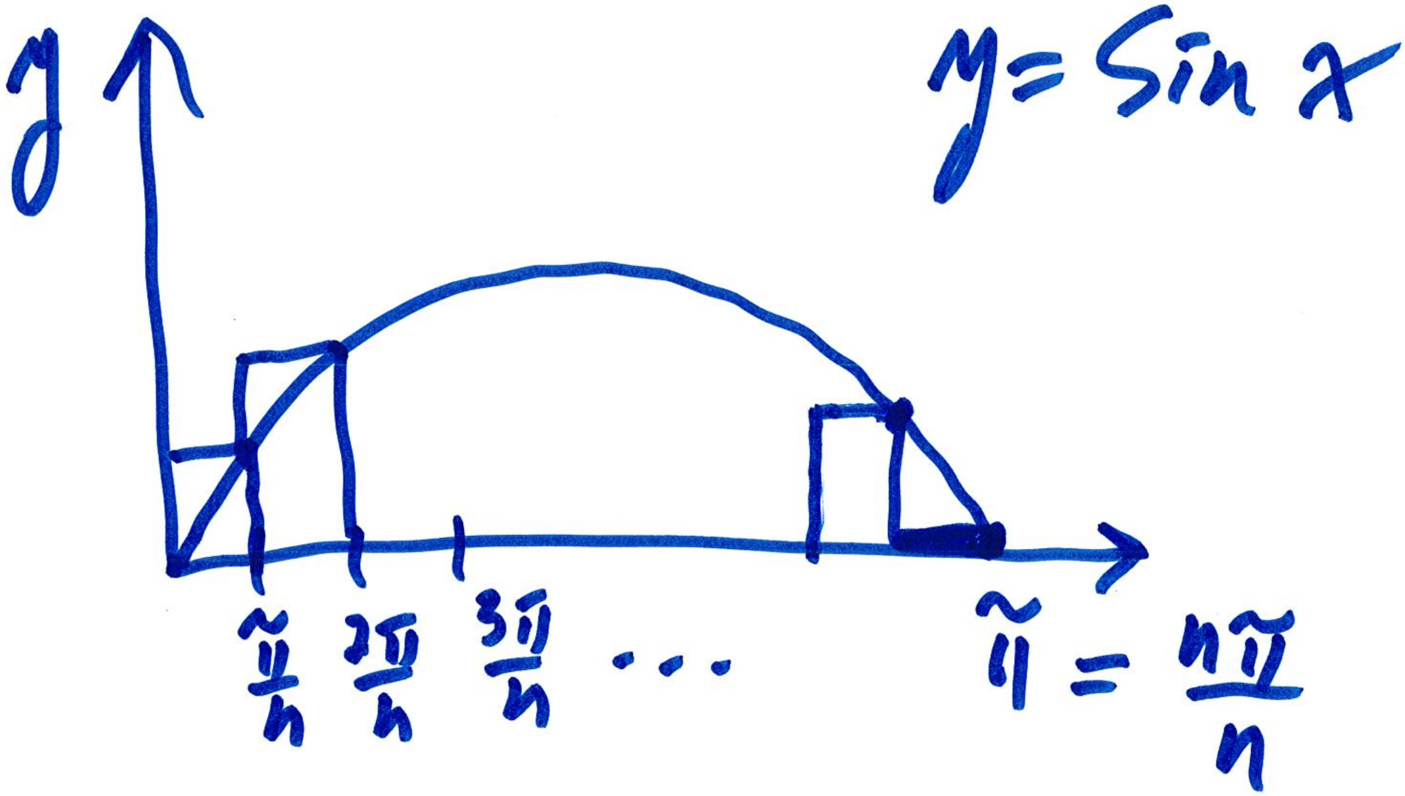
Velocity $v = t^2$



$$\text{dist}_{\text{too short}} = v_{\substack{\text{too}\\ \text{slow}}} \Delta t$$

$$\int_0^1 t^2\, dt = \text{Distance traveled!}$$

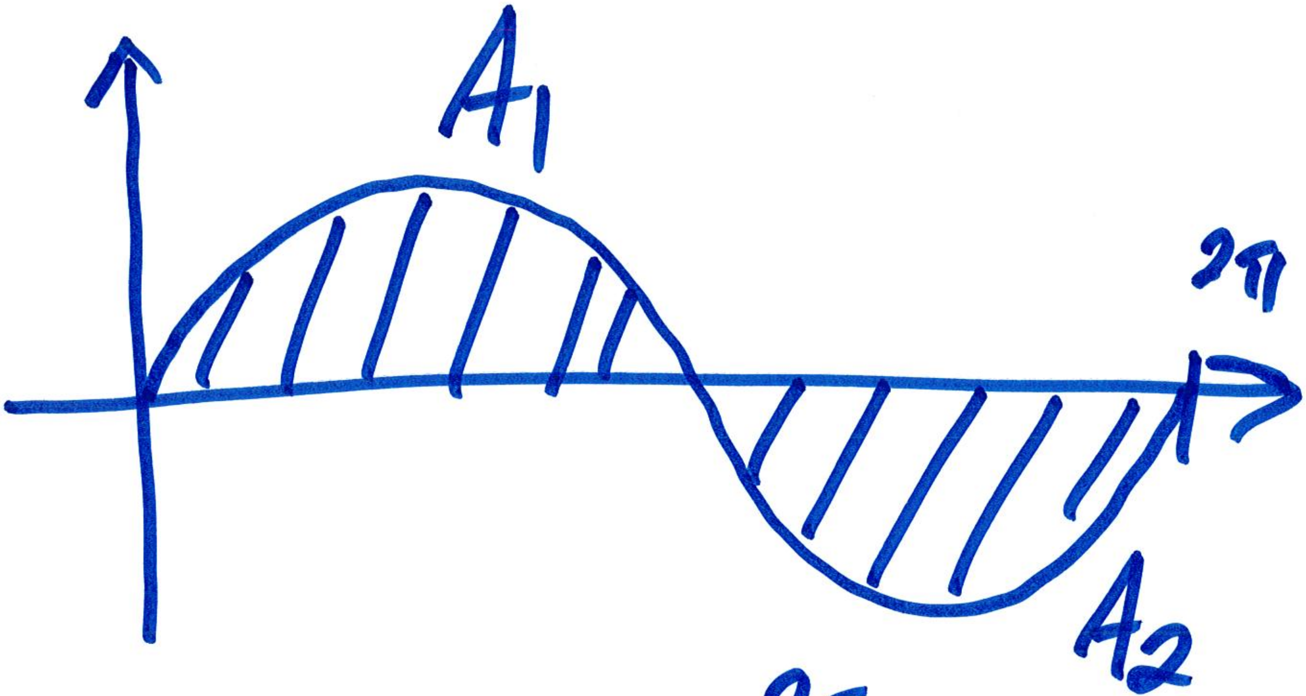
dist just right.



$$A \approx \left(\sin \frac{\pi}{n}\right)\cdot \frac{\pi}{n} + \left(\sin \frac{2\pi}{n}\right)\cdot \frac{\pi}{n}$$

$$+ \cdots + \left(\sin \frac{n\pi}{n}\right)\cdot \frac{\pi}{n}$$

$$\rightarrow 2$$



What is $\int_0^{2\pi} \sin x \, dx$?

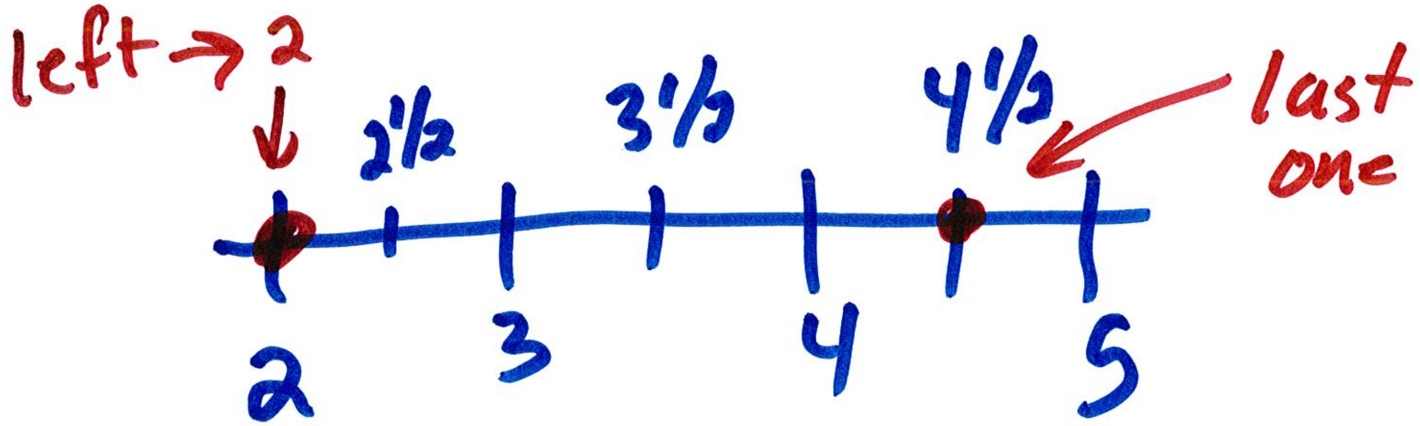
$$= A_1 - A_2 = 0$$

$$=$$

Calculate Rieman sum using <u>left</u> endpoints for $y=\sqrt{x}$, $n=6$.

$$\int_2^5 \sqrt{x}\,dx$$

$$\Delta x = \frac{b-a}{n} = \frac{5-2}{6} = \frac{1}{2}$$



$$A \approx \sqrt{2} \cdot \left(\frac{1}{2}\right)$$

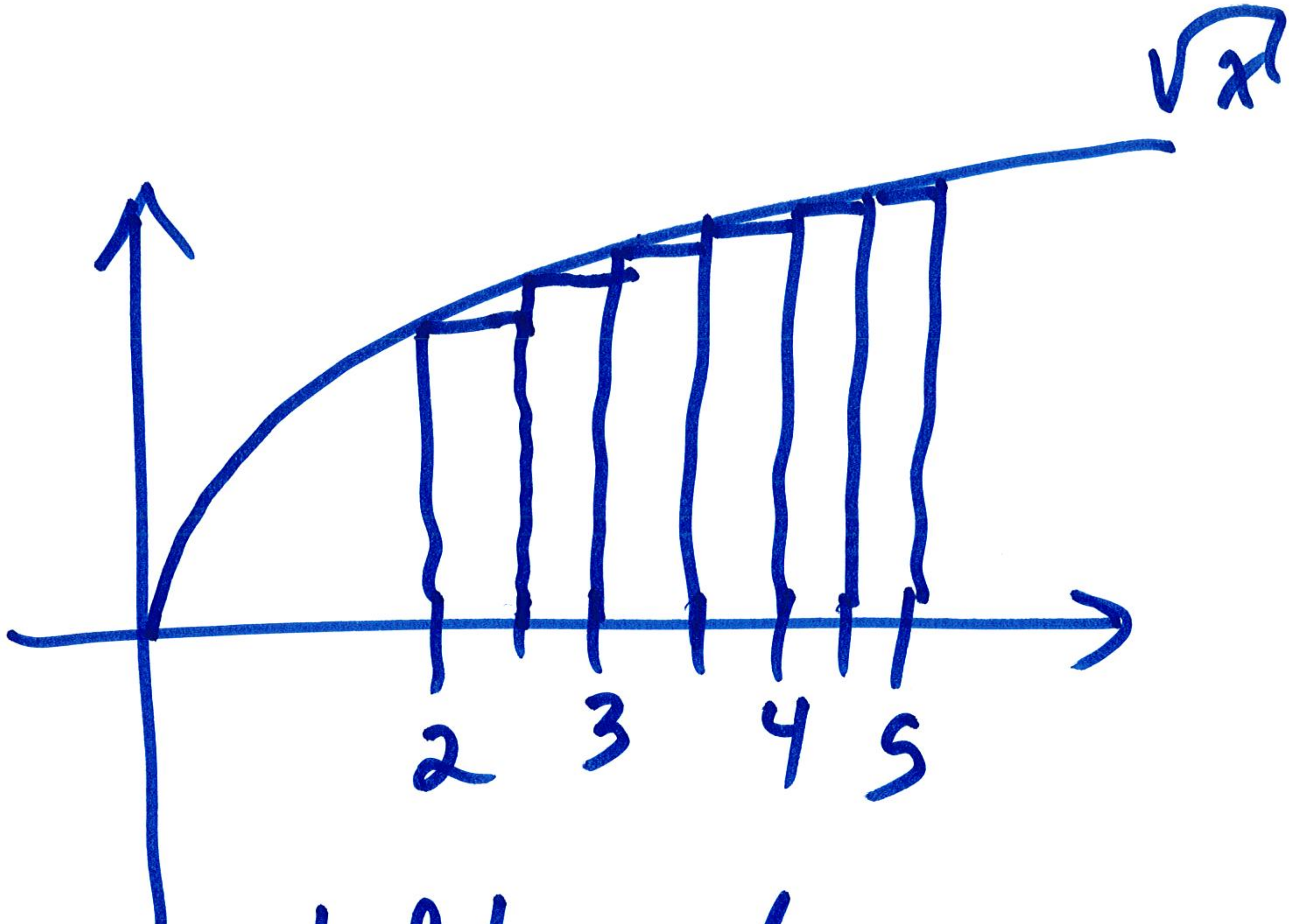
$$+ \sqrt{2\tfrac{1}{2}} \cdot \left(\frac{1}{2}\right)$$

$$+ \sqrt{3} \cdot \left(\frac{1}{2}\right)$$ too big?

$$+ \sqrt{3\tfrac{1}{2}} \cdot \left(\frac{1}{2}\right)$$ too small?

$$+ \sqrt{4} \cdot \left(\frac{1}{2}\right)$$

$$+ \sqrt{4\tfrac{1}{2}} \cdot \left(\frac{1}{2}\right)$$



left ends

put $\square$s under graph

R. Sum is too small