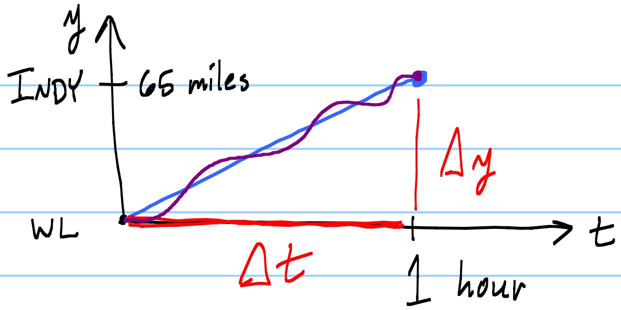
Lesson 4 on 2.1 Tangents & Velocity

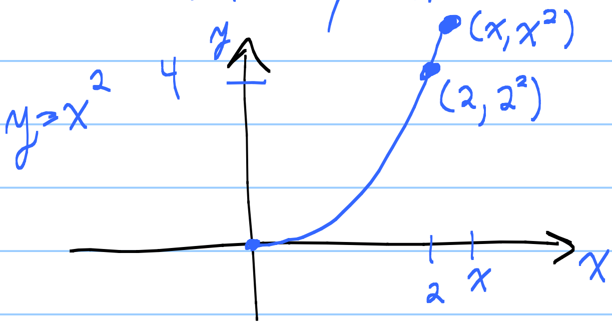


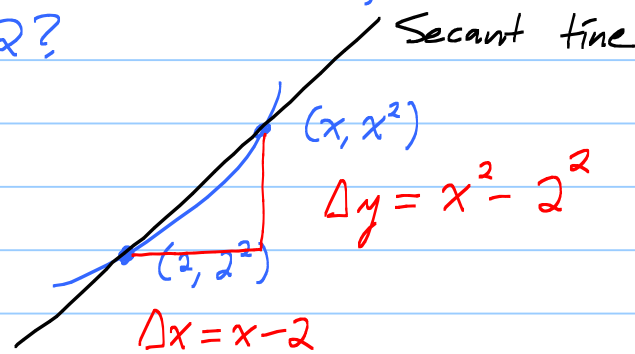
$$65 \text{ mph} = \frac{\Delta y}{\Delta t} = \frac{65 \text{ miles}}{1 \text{ hour}}$$

$$= \text{slope}$$

$$\text{Ave speed} = \frac{\Delta y}{\Delta t} = \frac{65}{1} = 65$$

EX: Distance function $y = x^2$. (x = time) What is my velocity at time $x = 2$?



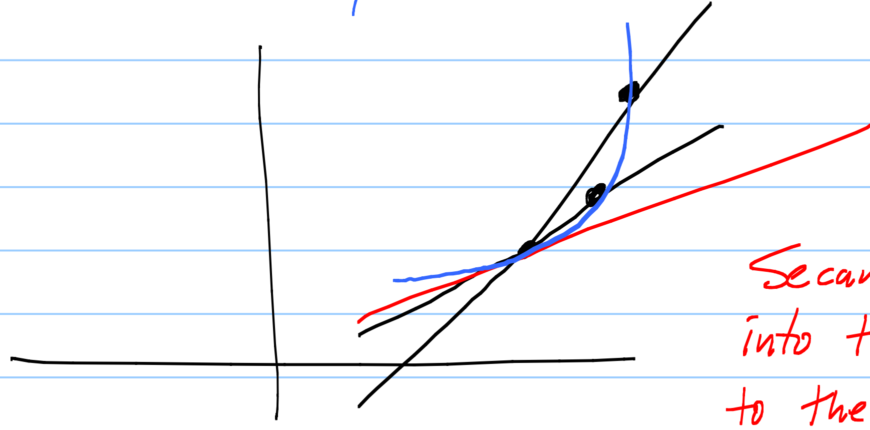


$$\text{Velocity} \approx \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = \frac{x^2 - 2^2}{x - 2} = \frac{(x-2)(x+2)}{x-2}$$

$$= x + 2 = \underbrace{(2 + \text{tiny})}_{x} + 2$$

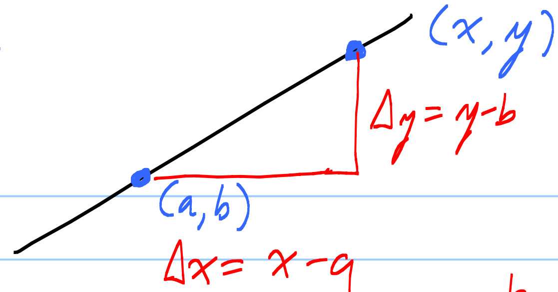
$$= 4 + \text{tiny} \longrightarrow 4 \quad \text{as tiny} \to \text{tinier!}$$

The "Instantaneous velocity" at time $x = 2$ is 4.



Secant lines slide into the "Tangent line" to the graph at $x = 2$.

Point-slope form of a line:



Slope $= m$

$$\boxed{m = \frac{y-b}{x-a}}$$

$$y = b + m(x-a)$$

Tangent line $y = x^2$ at $x = 2$:

$$\boxed{\frac{y-2^2}{x-2} = 4}$$

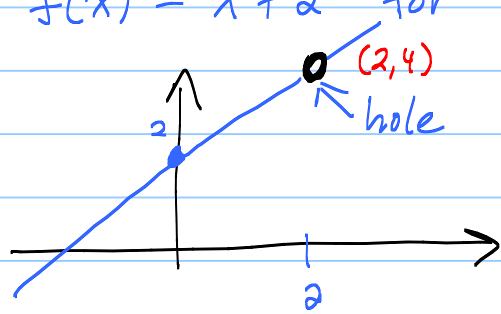
$$y = 4 + 4(x-2)$$

$$y = 4x - 4$$

<u>Ex</u>: $f(x) = \dfrac{x^2-4}{x-2}$ ← looks like $\frac{0}{0}$ at $x=2$

$f(x)$ is <u>not</u> defined at $x = 2$

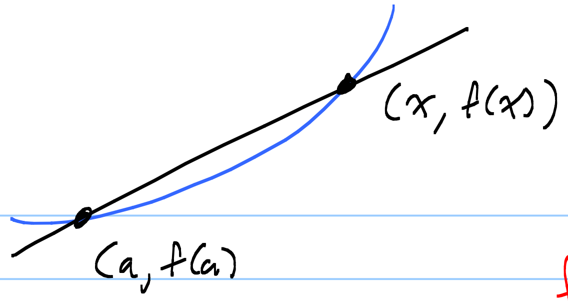
But $f(x) = x + 2$ for $x \neq 2$ by algebra.



$$\boxed{\lim_{x \to 2} f(x) = 4}$$

<u>Defn</u>: $\lim_{x \to a} f(x) = L$ means: We can make $f(x)$ get <u>arbitrarily</u> close to L by taking x <u>sufficiently</u> close to a with $x \neq a$.

<u>Math language</u>: Given $\varepsilon > 0$ ←arbitrarily close (no matter how small), there is a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ when $|x - a| < \delta$ ←sufficiently close, $x \neq a$.

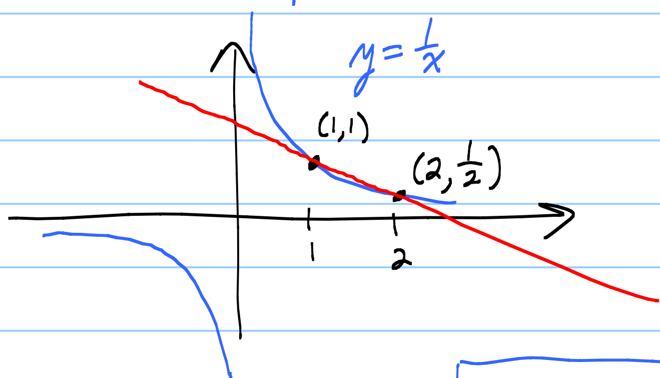


Slope of secant line

$= \underbrace{\text{"Difference quotient"}}_{DQ}$

$$DQ = \frac{f(x) - f(a)}{x - a}$$

Prob: Find equ of secant line:



$$\text{Slope of secant} = \frac{\left(\frac{1}{2} - \frac{1}{1}\right)}{2 - 1}$$

$$= -\frac{1}{2} \leftarrow \text{slope}$$

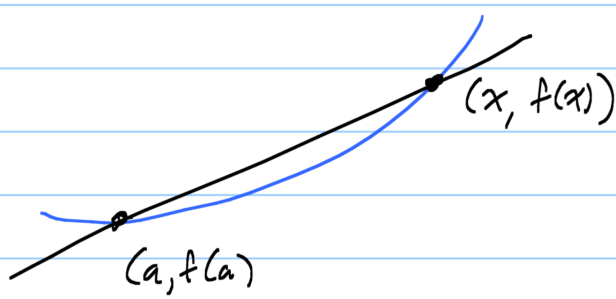
$\uparrow$ m

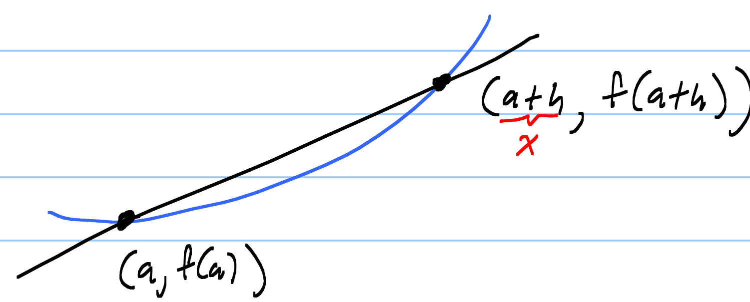
Point: $(2, \frac{1}{2})$

$\uparrow$ a $\quad$ $\uparrow$ b

$$\frac{y - b}{x - a} = m \qquad \boxed{\frac{y - \frac{1}{2}}{x - 2} = -\frac{1}{2}}$$

Two ways to think about DQ:





$$DQ = \frac{f(x) - f(a)}{x - a} = \frac{f(a+h) - f(a)}{h}$$

EX: $f(x) = x^2$ again.
$$\frac{f(a+h) - f(a)}{h} = \frac{(a+h)^2 - a^2}{h}$$

$$= \frac{[\cancel{a^2} + 2ah + h^2] - \cancel{a^2}}{h} = \frac{2ah + h^2}{h} =$$

$$= 2a + \underline{\underline{h}} \leftarrow h = \text{tiny}. \qquad \text{Let } h \to 0.$$

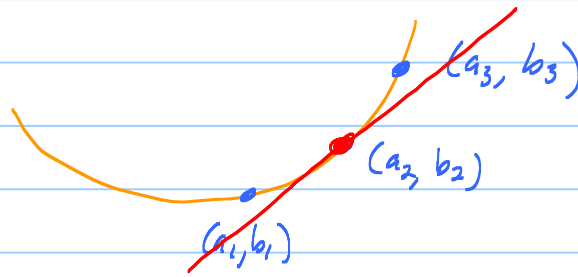
Instantaneous velocity at time $x=a$ is $2a$!

Def^n: DQ = "Average velocity" over interval $(a, a+h)$.

[or (a, x)]

EX:

x	$f(x)$
a_1	b_1
a_2	b_2
a_3	b_3



Ave velocity on (a_2, a_3): $\dfrac{b_3 - b_2}{a_3 - a_2} = m_2$

Ave velocity on (a_1, a_2): $\dfrac{b_2 - b_1}{a_2 - a_1} = m_1$

Engineering idea: Good approx for instantaneous velocity at $x = a_2$ is $\frac{1}{2}(m_1 + m_2)$ [average]

EX: $f(x) = \dfrac{x-1}{x+1}$ Prob: Calculate the $DQ = \dfrac{f(x) - f(2)}{x-2}$

$$\frac{\left(\frac{x-1}{x+1} - \frac{2-1}{2+1}\right)}{x-2} = \frac{\left(\frac{x-1}{x+1} - \frac{1}{3}\right)}{x-2}$$

$$\frac{\left(\frac{x-1}{x+1} - \frac{1}{3}\left(\frac{x+1}{x+1}\right)\right)}{x-2} = \frac{\left[\frac{x - \frac{1}{3}x - \frac{4}{3}}{x+1}\right]}{x-2}$$

$$\left[\frac{2/3\, x - \frac{4}{3}}{x+1}\right] \Big/ (x-2) = \left[\frac{2/3\,(x-2)}{x+1}\right] \Big/ (x-2)$$

$$= \frac{2/3}{x+1} \qquad DQ = \frac{f(x) - f(2)}{x-2}$$

What happens as $x \to 2$. $DQ = \frac{0}{0}$ at $x=2$!

But $DQ = \frac{2/3}{x+1} \longrightarrow \frac{(2/3)}{3} = \frac{2}{9}$. Nice!