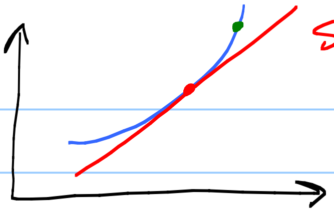
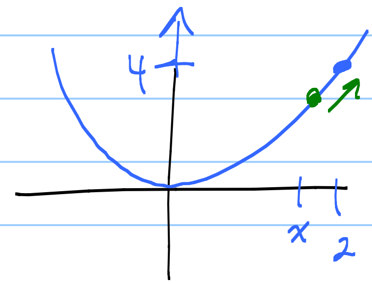
Leson 5 on 2.2 Limits



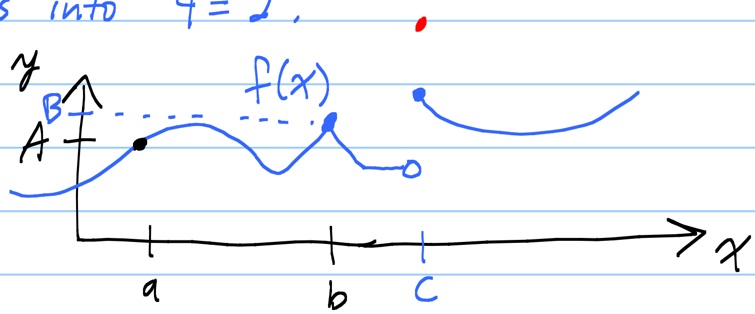
Slope Tangent = Limit DQ as we slide secant line down to the point.



EX: $\lim_{x \to 2} x^2 = 2^2 = 4$

Think: As x slides into 2, x^2 slides into $4 = 2^2$.

EX:



$\lim_{x \to a} f(x) = A$ Good at b too.

But $\lim_{x \to c} f(x)$ does not exist (DNE).

EX: $\dfrac{\sqrt{x^2+9} - 3}{x^2}$ ← looks like $\frac{0}{0}$ at $x=0$.

↑ Indeterminate form.

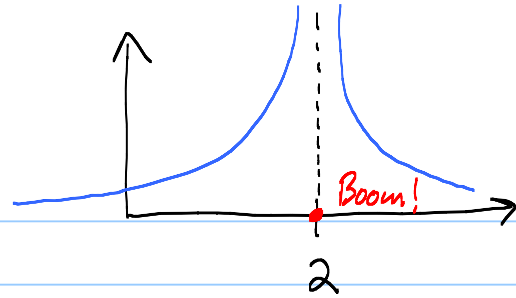
Algebra trick: "Rationalize the numerator"

$$\frac{\sqrt{x^2+9} - 3}{x^2} \cdot \frac{\sqrt{x^2+9} + 3}{\sqrt{x^2+9} + 3} = \frac{(\cancel{x^2} + \cancel{9}) - \cancel{3^2}}{\cancel{x^2}(\sqrt{x^2+9} + 3)}$$

$$= \frac{1}{\sqrt{x^2+9} + 3} \longrightarrow \frac{1}{\sqrt{0^2+9} + 3} = \underline{\underline{\frac{1}{6}}}$$

as $x \to 0$.

∞ limits



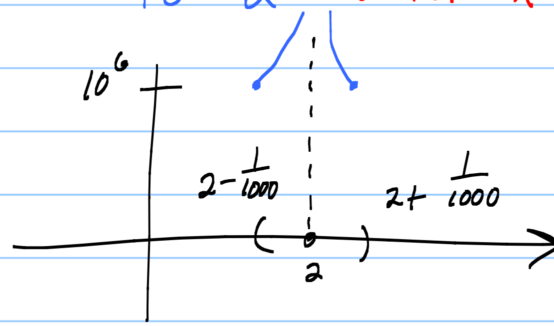
$$f(x) = \frac{1}{(x-2)^2}$$

$\lim_{x \to 2} \frac{1}{(x-2)^2} = \infty$ means: We can make $f(x)$ get arbitrarily large by taking x to be sufficiently close to 2 with $x \neq 2$.

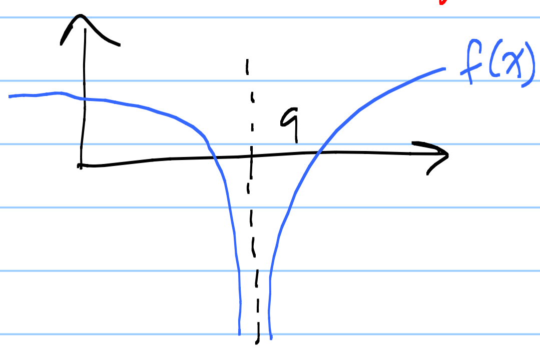
Think about: How close do we need to make x to 2 to have $\frac{1}{(x-2)^2} > 10^6$?

$$(x-2)^2 < 10^{-6}$$

$$\underbrace{|x-2| < \sqrt{10^{-6}} = 10^{-3}}_{\text{sufficiently close}}$$

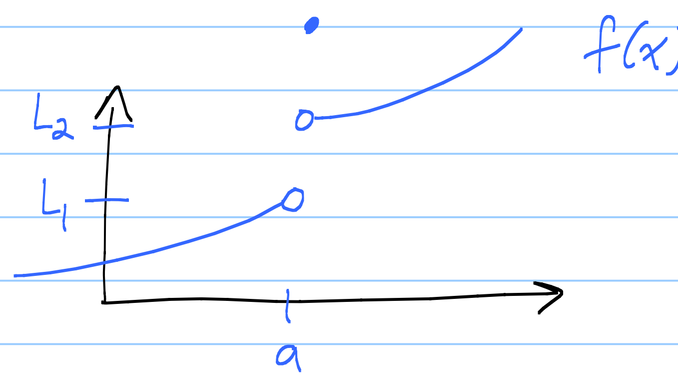


EX:



$$\lim_{x \to a} f(x) = -\infty$$

Left and right limits:



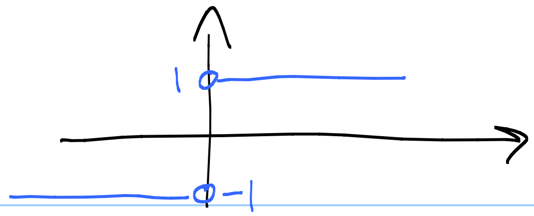
$$\lim_{x \to a-} f(x) = L_1$$

$$\lim_{x \to a+} f(x) = L_2$$

Who cares what $f(a)$ is?!

Maybe it is undefined.

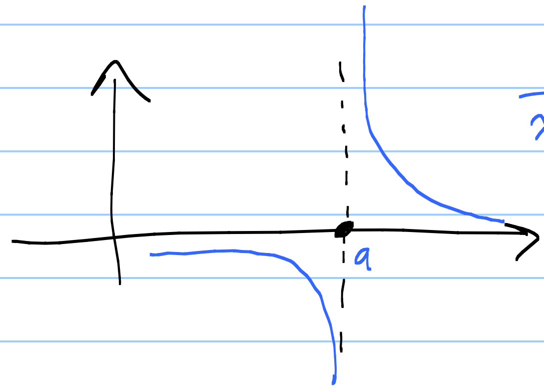
EX: $f(x) = \frac{x}{|x|} = \begin{cases} 1 & x > 0 \\ \text{Not defined at } x = 0 \\ -1 & x < 0 \end{cases}$



$$\lim_{x \to 0+} \frac{x}{|x|} = 1$$

$$\cdots \; 0- \; \cdots = -1$$

EX:

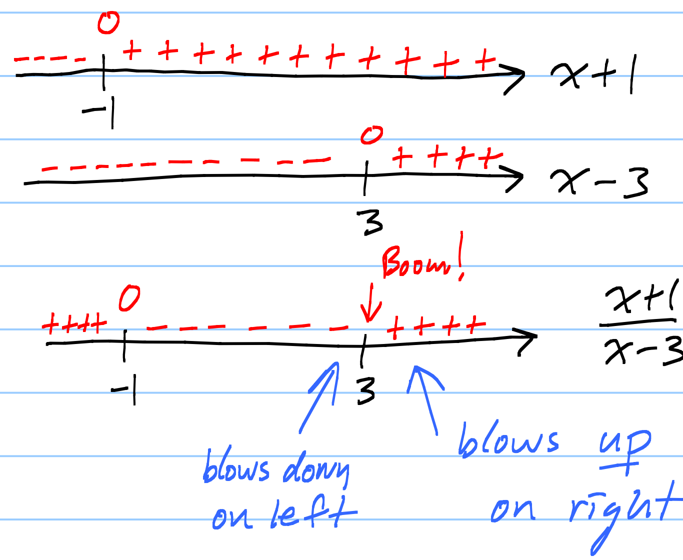


$$\frac{1}{x-a}$$

$$\lim_{x \to a+} \frac{1}{x-a} = \infty$$

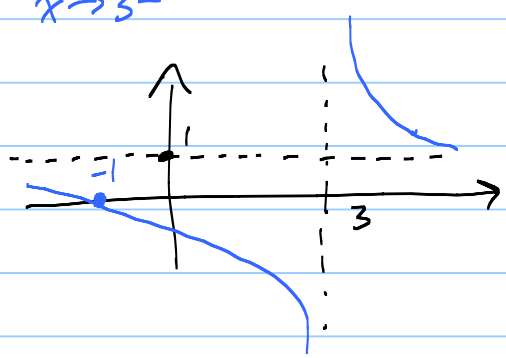
$$\lim_{x \to a-} \frac{1}{x-a} = -\infty$$

EX: Analyze $f(x) = \dfrac{x+1}{x-3}$



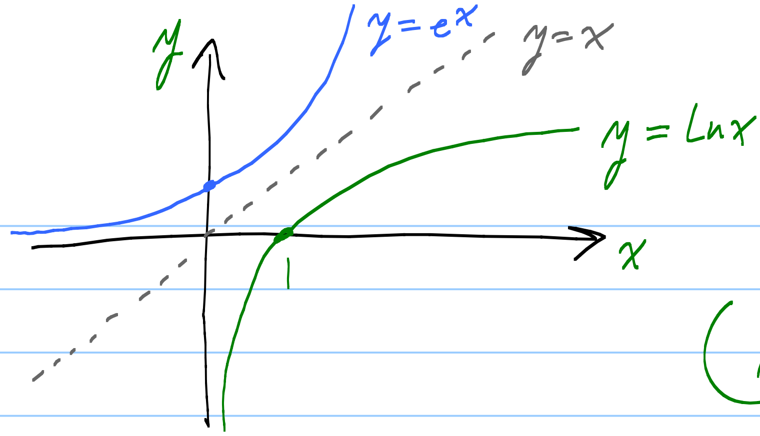
$$\lim_{x \to 3+} f(x) = \infty$$

$$\lim_{x \to 3-} f(x) = -\infty$$



Or: $h = \text{tiny} > 0$ $\quad \dfrac{(3+h)+1}{\underbrace{(3+h)}_{x}-3} = \dfrac{4+h}{h}$ ← Big! Positive if $h>0$ tiny,

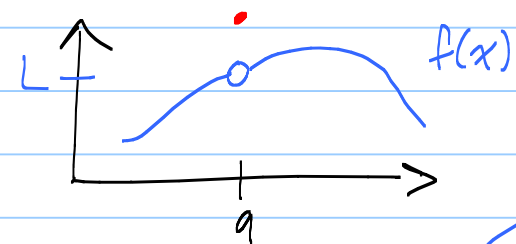
left side: $\dfrac{(3-h)+1}{\underbrace{(3-h)}_{x}-3} = \dfrac{4-h}{-h}$ ← Big! In negative direction



$$\lim_{x\to 0+} \ln x = -\infty$$

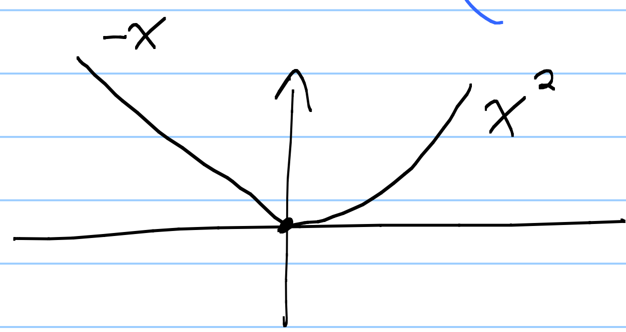
(Not even defined for $x<0$.)

Fact:

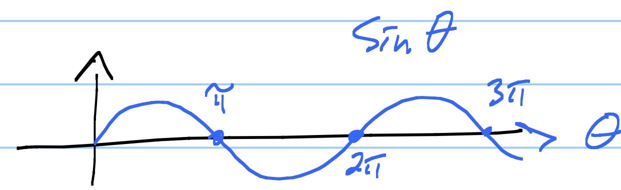


$$\lim_{x\to a} f(x) = L \iff \begin{cases} \lim_{x\to a-} f(x) = L & \underline{\text{and}} \\ \lim_{x\to a+} f(x) = L \end{cases}$$

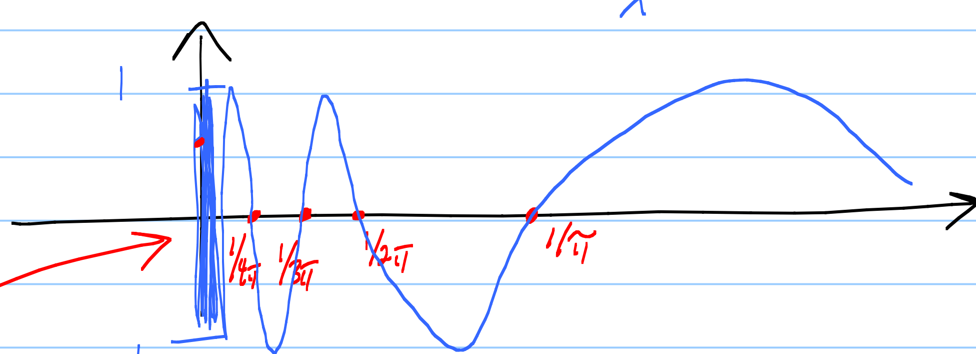
EX:



EX: $y = \sin \frac{1}{x}$

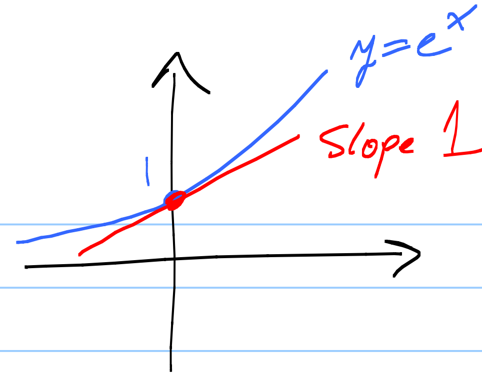


$$\frac{1}{x} = n\pi \qquad x = \frac{1}{n\pi}$$



No limit as $x \to 0+$.
DNE!

EX: $\lim_{x \to 0} \frac{e^x - 1}{x} = ?$



Hmmm. Aha! $\frac{e^x - e^0}{x - 0}$

$= DQ \to 1$ by special property of e^x!