Review for Final Exam 1

Final Exam, Tues. Dec. 13 LAMB F101,
10:30-12:30

1. $\lim_{x\to1}\frac{x^2-1}{x^2-x} =$ A. -1 B. 0 C. 1 D. 2 E. Does not exist

$$\frac{x^2-1}{x^2-x} = \frac{\cancel{(x-1)}(x+1)}{x\cancel{(x-1)}} = \frac{x+1}{x} \to \frac{1+1}{1} = 2$$

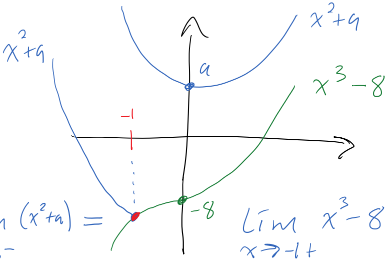
or L'H $\frac{0}{0}$ $= \lim_{x\to1}\frac{2x}{2x-1} = \frac{2\cdot1}{2\cdot1-1} = 2$ ✓

2. If $y = \underbrace{(x^2+1)}_{u}\underbrace{\tan x}_{v}$, then $\frac{dy}{dx} =$ A. $2x\tan x + (x^2+1)\sec^2 x$ B. $2x\sec^2 x$ C. $2x\tan x + (x^2+1)\tan x$ [✓ D.] $2x\tan x + 2x\sec^2 x$ E. $2x\tan x$

$$y' = u'v + uv' = (2x)\tan x + (x^2+1)\underline{\underline{\sec^2 x}}$$

3. If $h(x) = \begin{cases} x^2+a, & \text{for } x < -1 \\ x^3-8 & \text{for } x \geq -1 \end{cases}$ determine all values of a so that h is continuous for all values of x.
A. $a=-1$ B. $a=-8$ C. $a=-9$ [✓ D.] $a=-10$ E. There are no values of a.

Continuous: $\lim_{x\to a} f(x) = f(a)$



$\lim_{x\to-1^-}(x^2+a) = \lim_{x\to-1^+} x^3-8$

To make them hook up at $x=-1$, need

$(-1)^2+a = (-1)^3-8 = -9$

$1+a=-9$

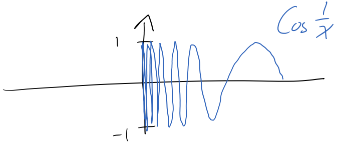
$\boxed{a=-10}$

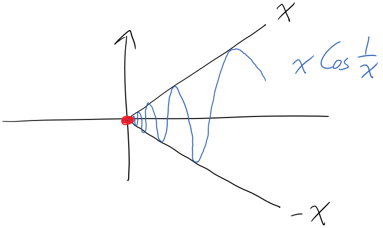
4. Evaluate $\lim_{x\to0^+} x\cos(\frac{1}{x})$. (Hint: $-1 \leq \cos(\frac{1}{x}) \leq 1$ for all $x \neq 0$.) [✓ A. 0] B. 1 C. -1 D. $\frac{\pi}{2}$ E. Does not exist

$$\frac{\cos\frac{1}{x}}{\left(\frac{1}{x}\right)} \overset{L'H?}{=} \left(\frac{DNE}{\infty}\right)$$

No!

Squeeze Theorem!





5. If $f(x)=\frac{1}{x+3}$, then $\lim_{x\to1}\frac{f(x)-f(1)}{x-1}=$ A. $\frac{1}{4}$ [✓ B. $\frac{1}{16}$] C. $-\frac{1}{16}$ D. $-\frac{1}{4}$ E. Does not exist

$$\frac{\frac{1}{x+3}-\frac{1}{1+3}}{x-1}$$

Algebra.

Or $= f'(1)$

$$f(x) = (x+3)^{-1}$$

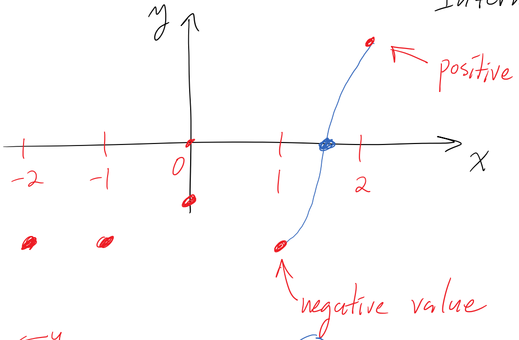
$$f'(x) = (-1)\cdot(x+3)^{-2}\,(1)$$

$$= -(x+3)^{-2}$$

$$f'(1) = \frac{-1}{(1+3)^2} = \frac{-1}{16}$$

6. The equation $x^3-x-5=0$ has one root for x between -2 and 2. The root is in the interval: A. $(-2,-1)$ B. $(-1,0)$ C. $(0,1)$ [✓ D. $(1,2)$] E. $(-1,1)$

Intermediate Value theorem!



7. If $f(x)=\frac{\overset{u}{1-x}}{\underset{v}{1+x}}$, then $f'(1)=$ A. -1 [✓ B. $-\frac{1}{2}$] C. 0 D. $\frac{1}{2}$ E. 1

$$f' = \frac{u'v-v'u}{v^2} = \frac{(-1)\,(1+x)-(1)\cdot(1-x)}{(1+x)^2} = \frac{-2}{(1+x)^2}$$

$$f'(1) = \frac{-2}{(1+1)^2} = -\frac{2}{4} = -\frac{1}{2}$$

8. If $y = \ln(1-x^2) + \sin^2 x$, then $\frac{dy}{dx} =$ A. $\frac{1}{1-x^2} + \cos^2 x$ B. $\frac{1}{1-x^2} + 2\sin x \cos x$ C. $\frac{1}{1-x^2} + 2\sin x$ D. $\frac{-2x}{1-x^2} + \cos^2 x$ [✓ E. $\frac{-2x}{1-x^2} + 2\sin x \cos x$]

$$y = \ln u + (\underbrace{\sin x}_{v})^2 \quad \text{where } u = 1-x^2$$

$$\boxed{\frac{d}{du}\ln u = \frac{1}{u}}$$

$$\frac{dy}{dx} = \frac{1}{u}\cdot\frac{du}{dx} + 2v\cdot\frac{dv}{dx}$$

$$= \frac{1}{(1-x^2)}\cdot[-2x] + 2(\sin x)[\cos x]$$

9. Find $f''(x)$ if $f(x) = \frac{1-x}{1+x}$ [✓ A. $\frac{4}{(1+x)^3}$] B. $\frac{-4}{(1+x)^3}$ C. $-\frac{4x}{(1+x)^3} + \frac{2}{(1+x)^2}$ D. $\frac{2(1+x)^2 - 2x(1+x)}{(1+x)^4}$ E. -1

$$f'(x) = -\frac{2}{(1+x)^2} \quad \text{from \#7 above.}$$

$$= -2(1+x)^{-2}$$

$$f''(x) = \frac{d}{dx}\left[-2v^{-2}\right] \quad \text{where } v = 1+x$$

$$= (-2)(-2v^{-3})\underbrace{\frac{dv}{dx}}_{1} = \frac{4}{v^3} = \frac{4}{(1+x)^3}$$

10. Assume that y is defined implicitly as a differentiable function of x by the equation $\underline{xy^2 - x^2 + y + 5 = 0}$. Find $\frac{dy}{dx}$ at $(-2, 1)$ A. 9 B. $\frac{-5}{3}$ C. 1 D. 2 [✓ E. $\frac{5}{3}$]

$x=-2,\ y=1$

$$\frac{d}{dx}\left[\underbrace{x}_{u}\,\underbrace{y^2}_{v=y^2} - x^2 + y + 5\right] = \frac{d}{dx}[0] = 0$$

$$\left(\underline{(1)(y^2)} + x\left[2y\frac{dy}{dx}\right]\right) \underline{-2x} + \frac{dy}{dx} + 0 = 0$$

$$\frac{dy}{dx}(2xy+1) = 2x - y^2 \qquad \boxed{\frac{dy}{dx} = \frac{2x-y^2}{2xy+1}} = \frac{2(-2)-1^2}{2(-2)\cdot 1+1} = \frac{-5}{-3} = \frac{5}{3}$$

11. Find the maximum and minimum values of the function $f(x) = 3x^2+6x-10$ on the interval $-2 \le x \le 2$.
A. max is 14, min is -10. B. max is -10, min is -13 [✓ C.] max is 14, min is -13 D. no max, min is -10
E. max is 14, no min.

Closed Interval Theorem: Find crt #'s : $f'(x)=0$

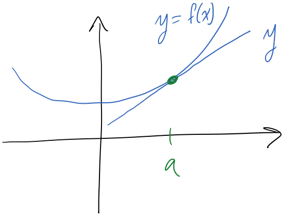
$f'(x) = 6x+6 = 0$

$\boxed{x=-1}$

x	$f(x)$
-2	$3(-2)^2+6(-2)-10 = -10$
-1	$3(-1)^2+6(-1)-10 = -13 \leftarrow$ Min
2	$3(2)^2+6\cdot 2-10 = 14 \leftarrow$ Max

12. For a differentiable function $f(x)$ it is known that $f(3) = 5$ and $f'(3) = -2$. Use a linear approximation to get the approximate value of $f(3.02)$. A. 6.02 B. 5.02 C. 5.04 D. 3 [✓ E.] 4.96.

$a=3$



$y = f(a) + f'(a)(x-a)$

$$\frac{y-f(a)}{x-a} = f'(a)$$

$$f(x) \approx f(a) + f'(a)(x-a)$$

$$f(3.02) \approx 5 + (-2)(3.02-3)$$

$$5 - 2(.02) = 5 - .04$$

$$= 4.96$$

$\sqrt[3]{8.02}$

$f(x) = x^{1/3}$

$a = 8$

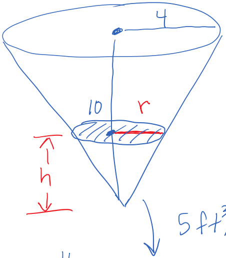
$\sqrt[3]{8} = 2$

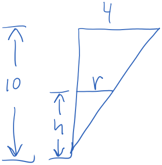
13. Water is withdrawn from a conical reservoir, 8 feet in diameter and 10 feet deep (vertex down) at the constant rate of 5 ft^3/min. How fast is the water level falling when the depth of the water in the reservoir is 5 ft? ($V = \frac{1}{3}\pi r^2 h$). A. $\frac{15}{16\pi}$ ft/min B. $\sqrt{\frac{3}{\pi}}$ ft/min C. $\frac{2}{\pi}$ ft/min D. $5\sqrt[3]{3/4\pi}$ ft/min [✓ E.] $\frac{5}{4\pi}$ ft/min.

$$\boxed{V = \text{Volume} = \frac{1}{3}\pi r^2 \cdot h}$$

Given $\frac{dV}{dt} = -5$

Want $\left|\frac{dh}{dt}\right|$ when $h = 5$.





$$\frac{4}{10} = \frac{r}{h}$$

$$\boxed{r = \frac{2}{5}h}$$

$$V = \frac{1}{3}\pi\left(\frac{2}{5}h\right)^2 \cdot h$$

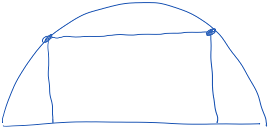
$$V = \frac{4}{75}\pi h^3$$

$$\frac{dV}{dt} = \frac{4}{75}\pi\left(3 \cdot h^2\right)\frac{dh}{dt}$$

$$(-5) = \frac{4}{25}\pi(5)^2\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{-5}{4\pi} \text{ ft/m}$$

14.



Fall 2015

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