Review for Final Exam 3

Final Exam, Tuesday, Dec. 13
10:30 am - 12:30 pm
Lambert Fieldhouse (LAMB F101)

Off Hrs : Fri 2-3:30, Mon 2-3:30

21. $\int_3^4 x\sqrt{\underbrace{25-x^2}_{u}}\,dx =$ A. 0 B. -37 C. $\frac{37}{3}$ D. $-\frac{74}{3}$ E. $\frac{7}{12}$

$u = 25 - x^2$ $\quad \frac{du}{dx} = -2x \quad du = -2x\,dx \quad \boxed{x\,dx = -\frac{1}{2}du}$

When $x = 3$: $u = 25 - 3^2 = 16$
When $x = 4$: $u = 25 - 4^2 = 9$

$$\int_{x=3}^{4} \underbrace{(25-x^2)^{1/2}}_{u^{1/2}} \underbrace{x\,dx}_{-\frac{1}{2}du} = \int_{u=16}^{9} u^{1/2}\left(-\frac{1}{2}du\right)$$

$$\left(\int_a^b = -\int_b^a\right) \qquad = \frac{1}{2}\int_9^{16} u^{1/2}\,du = \frac{1}{2}\left[\frac{1}{\frac{1}{2}+1}u^{\frac{1}{2}+1}\right]_9^{16}$$

$$= \frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_9^{16} = \frac{1}{3}\left(16^{3/2} - 9^{3/2}\right) = \frac{1}{3}\left(4^3 - 3^3\right) = \frac{37}{3}$$

22. $\lim_{x\to\infty} \frac{x^2+2x}{3x^2+4} \overset{\text{L'H}}{=}$ $\left(\frac{\infty}{\infty}\right)$ A. 1 B. 3/7 C. 1/4 D. 0 E. 1/3.

boss

$$\frac{x^2 + 2x}{3x^2 + 4} = \frac{\cancel{x^2}\left(1 + \frac{2}{x}\right)}{\cancel{x^2}\left(3 + \frac{4}{x^2}\right)} \xrightarrow[\substack{x\to\infty \\ \text{or} \\ x\to-\infty}]{} \frac{1+0}{3+0} = \frac{1}{3}$$

boss

$y = \frac{1}{3}$ is horiz. asymp.

boss $x\to\infty$

boss $x\to-\infty$

$$\frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\frac{x^2 + 2x}{3x^2+4} \xrightarrow[x\to 0]{\text{as}} \frac{0^2 + 2\cdot 0}{3\cdot 0^2 + 4} = \frac{0}{4} = 0$$

23. $\lim\limits_{x\to 0}\dfrac{2x-\sin^{-1}x}{2x+\tan^{-1}x} = \dfrac{0}{0}$ L'H A. 1/2 B. 2 C. 1/3 D. 1 E. 0.

$$\lim_{x\to 0}\frac{2-\frac{1}{\sqrt{1-x^2}}}{2+\frac{1}{1+x^2}} = \frac{2-\frac{1}{\sqrt{1}}}{2+\frac{1}{1+0^2}} = \frac{1}{3}$$

24. Suppose that a function f has the following properties: decreasing. (Down hill)

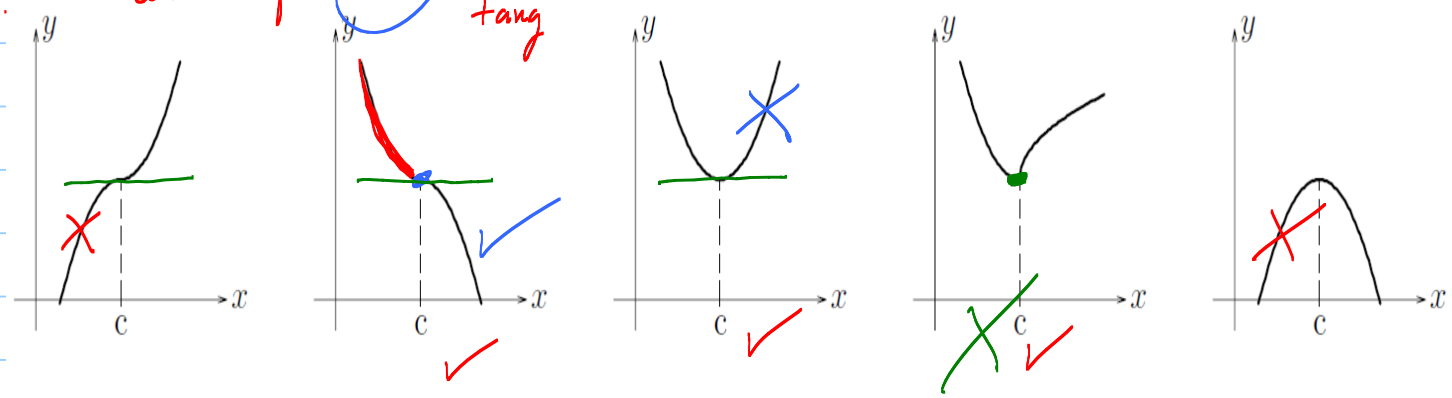
$f''(x) > 0$ for $x < c$, $f'(c) = 0$, and $f'(x) < 0$ for $x > c$.

Which of the following could be the graph of f?

concave up

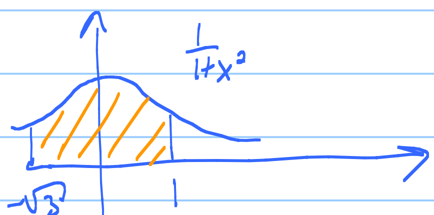
horiz tang

A. B. C. D. E.

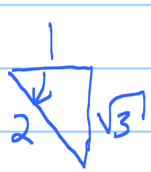


26. Find the area of the region between the graph of $y = \dfrac{1}{1+x^2}$ and the x-axis, from $x = -\sqrt{3}$ to $x = 1$.

A. $\frac{\pi}{2}$ B. $\frac{3\pi}{4}$ C. $\frac{15\pi}{12}$ D. $\frac{\pi}{3}$ E. $\frac{7\pi}{12}$



$$= \int_{-\sqrt{3}}^{1}\frac{1}{1+x^2}\,dx = \Big[\tan^{-1}x\Big]_{-\sqrt{3}}^{1}$$



$\theta = -\frac{\pi}{3}$

$$= \frac{\pi}{4} - \left(-\frac{\pi}{3}\right) = \frac{7\pi}{12}$$

27. $\frac{d}{dx}\left(e^{2x}\ln\sqrt{1+x}\right) =$ A. $e^{2x}\ln(1+x)+\frac{e^{2x}}{2(1+x)}$ B. $\frac{e^{2x}}{\sqrt{1+x}}+2e^{2x}\ln\sqrt{1+x}$ C. $\frac{1}{2}e^{2x}\ln(1+x)+\frac{e^{2x}}{2(1+x)}$ D. $\frac{2e^{2x}}{\sqrt{1+x}}$ E. $\frac{e^{2x}}{1+x}$

$$= (2e^{2x})\underbrace{\ln(1+x)^{1/2}}_{\frac{1}{2}\ln(1+x)} + e^{2x}\cdot\frac{1}{(1+x)^{1/2}}\cdot\frac{1}{2}(1+x)^{-1/2}\cdot 1$$

$$= 2e^{2x}\cdot\frac{1}{2}\ln(1+x) + \frac{e^{2x}}{2}\;\frac{1}{[(1+x)^{1/2}]^2}$$

$$= e^{2x}\ln(1+x) + \frac{e^{2x}}{2(1+x)}$$

28. $\frac{d}{dx}x^{\sin x} =$ A. $(\cos x)x^{\sin x}$ B. $(\sin x)x^{\sin x-1}$ C. $x^{\cos x}$ D. $x^{\sin x}\left[\frac{\sin x}{x}+(\cos x)\ln x\right]$ E. $(\ln x)x^{\sin x}$

$x^{\sin x} = e^{\ln(x^{\sin x})} = e^{\sin x \ln x}$ ← one way

$y = x^{\sin x}$

$\ln y = \ln x^{\sin x} = \sin x \ln x$ ← Log Diff'tion

$$\frac{1}{y}\cdot\frac{dy}{dx} = \cos x \ln x + \sin x\left(\frac{1}{x}\right)$$

$$\frac{dy}{dx} = y\cdot\left[\;\cdots\;\right] = x^{\sin x}\left[\cos x \ln x + \frac{\sin x}{x}\right]$$

29. $\frac{d}{dx}\tan^{-1}(\underbrace{e^{3x}}_{u}) =$ A. $\frac{1}{1+e^{3x}}$ B. $\frac{e^{3x}}{1+e^{3x}}$ C. $\frac{3e^{3x}}{1+e^{6x}}$ D. $\frac{3e^{3x}}{1+e^{9x^2}}$ E. $\frac{3e^{3x}}{\sqrt{1-e^{6x}}}$

$$\frac{d}{dx}\tan^{-1}u = \frac{1}{1+u^2}\cdot\frac{du}{dx} = \frac{1}{1+\underbrace{(e^{3x})^2}_{e^{2\cdot 3x}}}\left[e^{3x}\cdot 3\right]$$

30. $\int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}dx =$ A. $\frac{\pi}{2}$ B. $\frac{\pi}{6}$ C. $\sin^{-1}\sqrt{3}$ D. $\frac{\pi}{3}$ E. 1

Know $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C$

$$\sqrt{4-x^2} = \sqrt{4\left(1-\frac{x^2}{4}\right)} = 2\sqrt{1-\left(\frac{x}{2}\right)^2}$$

Aha! $u=\frac{x}{2}$ $du=\frac{1}{2}dx$

$$\int \frac{1}{2\sqrt{1-\left(\frac{x}{2}\right)^2}}\,dx = \frac{1}{2}\int\frac{1}{\sqrt{1-u^2}}(2du) = \sin^{-1}u + C$$

$\boxed{dx = 2du}$

$$= \sin^{-1}\frac{x}{2} + C$$

$$\int_0^{\sqrt{3}}\frac{1}{\sqrt{4-x^2}}\,dx = \sin^{-1}\frac{x}{2}\Bigg]_0^{\sqrt{3}} = \sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}0 = \frac{\pi}{3} - 0$$

$$= \pi/3$$

Or

When $x=0$: $u=\frac{0}{2}=0$

When $x=\sqrt{3}$: $u=\frac{\sqrt{3}}{2}$

$$\int_{x=0}^{\sqrt{3}} \frac{1}{2\sqrt{1-\left(\frac{x}{2}\right)^2}}(2du) = \int_{u=0}^{\sqrt{3}/2}\frac{1}{\sqrt{1-u^2}}\,du$$

$$= \sin^{-1}u\Big|_0^{\sqrt{3}/2} = \sin^{-1}u - \sin^{-1}u$$

$$= \sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}0 = \frac{\pi}{3} - 0$$

31. $\int_0^4 \frac{x}{\sqrt{1+2x}}\,dx =$ A. $\frac{7}{2}$ B. $\frac{10}{3}$ C. $\frac{11}{4}\tan^{-1}3$ D. 3 E. 4

$x \leftarrow \frac{1}{2}(u-1)$, $\quad \sqrt{1+2x}$: u, $\quad dx$: $\frac{1}{2}du$

$u = 1+2x$ ← solve for x: $u-1 = 2x$ $\quad x = \frac{1}{2}(u-1)$

$du = 2\,dx$

$\boxed{dx = \frac{1}{2}du}$

When $x=0$: $u = 1+2\cdot 0 = 1$
when $x=4$: $u = 1+2\cdot 4 = 9$

$$\int_{u=1}^{9} \frac{\frac{1}{2}(u-1)}{\sqrt{u}}\left(\frac{1}{2}du\right) = \frac{1}{4}\int_1^9 \frac{u-1}{u^{1/2}}\,du = \frac{1}{4}\int_1^9 u^{1/2} - u^{-1/2}\,du$$

$$= \frac{1}{4}\left[\frac{2}{3}u^{3/2} - 2u^{1/2}\right]_1^9$$

$$= \frac{1}{4}\left[\left(\frac{2}{3}9^{3/2} - 2\,9^{1/2}\right) - \left(\frac{2}{3}\cdot 1^{3/2} - 2\cdot 1^{1/2}\right)\right]$$

$$= \frac{1}{4}\left[\left(\frac{2}{3}\cdot 27 - 6\right) - \left(\frac{2}{3} - 2\right)\right] = \frac{1}{4}\cdot\frac{40}{3} = \frac{10}{3}$$

32. $\int_0^1 \frac{e^x}{1+e^x}\,dx =$ A. $\ln\frac{1+e}{2}$ B. $\ln(1+e)$ C. $\frac{1}{2}$ D. $1-\ln 2$ E. e

$1+e^x$: u

$u = 1+e^x$

$\frac{du}{dx} = 0 + e^x$

$\boxed{du = e^x\,dx}$

When $x=0$: $u = 1+e^0 = 2$
When $x=1$: $u = 1+e^1 = 1+e$

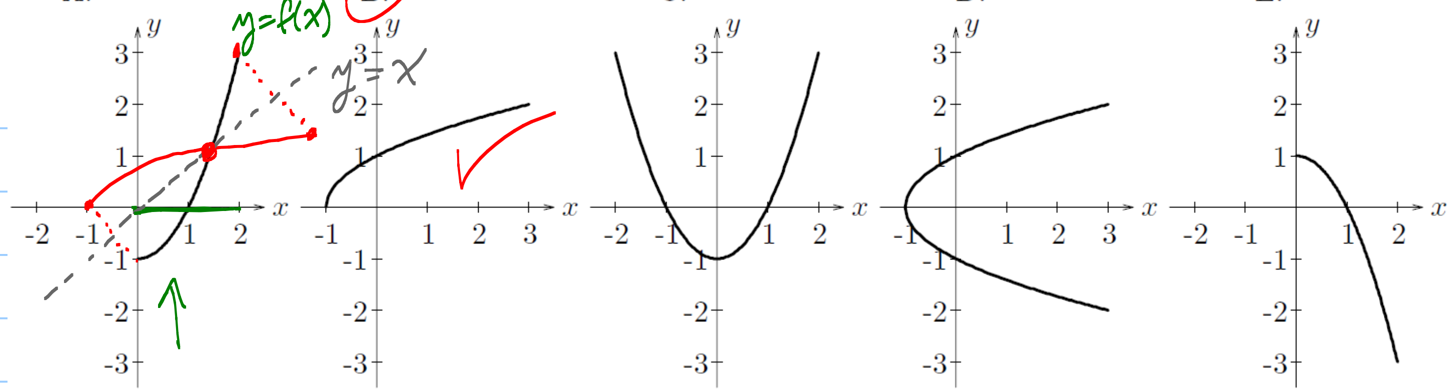
$$\int_{x=0}^{1} \frac{e^x\,dx}{(1+e^x)} = \int_{u=2}^{1+e} \frac{du}{u}$$

$$= \ln|u|\Big]_2^{1+e} = \ln(1+e) - \ln 2$$

$$= \ln\frac{1+e}{2}$$

33. If $f(x) = x^2 - 1$, $0 \le x \le 2$, then the graph of $y = f^{-1}(x)$ is

A. B. C. D. E.



$y = x^2 - 1$

$y + 1 = x^2$

$x = \pm\sqrt{y+1}$

Answers: 1.D, 2.A, 3.D, 4.A, 5.C, 6.D, 7.B, 8.E, 9.A, 10.E, 11.C, 12.E, 13.E, 14.E, 15.A, 16.B, 17.C, 18.C, 19.A, 20.A, 21.C, 22.E, 23.C, 24.B, 25.A, 26.E, 27.A, 28.D, 29.C, 30.D, 31.B, 32.A, 33.B