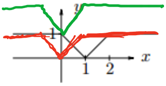
Review for Exam 1 Thursday, 9/22 8:00-9:00 pm

Closed book, no calculators, no notes. #2 pencil, eraser

MA 16100 Fall 2008

1. The graph of $y = f(x)$ is shown below.

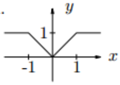


up ↑

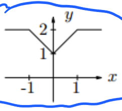
left ←

Which is the graph for $y = 1 + f(x+1)$. ← shift left 1 unit, up 1 unit.

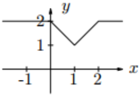
A.



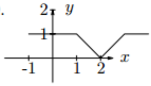
[✓ B.]



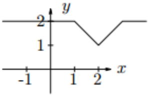
C.



D.



E.



6. If $f(x) = \frac{\sqrt{x}}{x^2-1}$, find the domain of f.

$\sqrt{x}$ ← $x \geq 0$

← $x^2 - 1 \neq 0$

$x \neq \pm 1$

A. $(-\infty,-1) \cup (-1,1) \cup (1,\infty)$
B. $[0,\infty)$
[✓ C.] $[0,1) \cup (1,\infty)$
D. $(1,\infty)$
E. $(-1,1) \cup (1,\infty)$

$[0,1) \cup (1,\infty)$

9. The inverse of the function $f(x) = \sqrt{4 + e^{3x}}$ is $f^{-1}(x) =$

$y = \sqrt{4 + e^{3x}}$

$y^2 = 4 + e^{3x}$

$y^2 - 4 = e^{3x}$

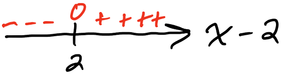
$3x = \ln(y^2 - 4)$

$x = \frac{1}{3}\ln(y^2 - 4)$

$x = f^{-1}(y)$ ← change y to x

A. $\frac{1}{3}\ln(x^2+4)$
B. $3\ln(x^2-4)$
[✓ C.] $\frac{1}{3}\ln(x^2-4)$
D. $3\ln(x^2+4)$
E. $\frac{1}{3}\ln(4-x^2)$

10. $\lim_{x\to 2^-} \frac{x+1}{x-2} =$ At $x = 2$: $x+1 = 2+1 = 3$



$\frac{x+1}{x-2} = \frac{(3\text{-ish})}{(\text{small} < 0)}$ left of 2.

→ $-\infty$ as $x \to 2^-$

A. ∞
[✓ B.] $-\infty$
C. 0
D. 3
E. 1

11. $\lim_{x\to 2} \frac{2x-x^2}{x^2-x-2} =$ $\frac{x(2-x)}{(x-2)(x+1)} = \frac{-x}{x+1}$

$\frac{0}{0} \nearrow$

slides into $\frac{-2}{2+1} = -\frac{2}{3}$ as

$x \to 2.$

A. $-\frac{2}{3}$

B. $\frac{2}{3}$

C. 0

D. ∞

E. $-\infty$

$1-\frac{1}{4}=3/4$ $\quad$ $1/4-(-\frac{1}{2})=\frac{3}{4}$ ✓

12. If $1-(x+1)^2 \le H(x) \le \frac{1}{4} - x$ for $-1 < x < 0$, then $\lim_{x\to -\frac{1}{2}} H(x) =$

Squeeze Thm! Important to check bottom fuction = top function at $x=-\frac{1}{2}$ to get the squeeze.

If not, $\longrightarrow$

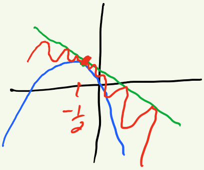
A. $\frac{4}{3}$

[✓ B. $\frac{3}{4}$]

C. $-\frac{3}{4}$

D. $-\frac{4}{3}$

E. cannot be determined



15. $\lim_{x\to -\infty} \frac{x^3-x^2}{1-x^2} =$ $\frac{x^3(1-\frac{1}{x})}{x^2(\frac{1}{x^2}-1)} =$

$\swarrow$ boss $\quad$ $\nwarrow$ boss

$x \cdot \left[\frac{1-\frac{1}{x}}{-1+1/x^2}\right] = \left(\begin{matrix}\text{Big}\\ <0\end{matrix}\right) \cdot \left[\begin{matrix}\text{close}\\ \text{to } \frac{1}{-1}=-1\end{matrix}\right]$

$\to +\infty$ as $x \to -\infty.$

[✓ A. ∞]

B. $-\infty$

C. 1

D. -1

E. 0

6. The domain of $\ln\left(\frac{4x^2}{x+1}\right)$ is

Lnx only for $x > 0.$

$+ + + + + + \; 0 \; + + + + +$ $\longrightarrow$ $4x^2$ (0)

$- - - \; 0 \; + + + + + + +$ $\longrightarrow$ $x+1$ (-1)

A. $(1,\infty)\cup(-\infty,-1)$

B. $(0,\infty)\cup(-\infty,-1)$

[✓ C. $(-1,0)\cup(0,\infty)$]

D. $(-1,0]$

Good region: $(-1,0)\cup(0,\infty)$

E. All real numbers except $x=0$ and $x=-1$

9. Compute $\lim_{t\to 0} \frac{\sqrt{2+t}-\sqrt{2-t}}{t}$ $\cdot$ $\frac{\sqrt{2+t}+\sqrt{2-t}}{\sqrt{2+t}+\sqrt{2-t}}$

A. 2

B. $\frac{1}{2\sqrt{2}}$

C. $\frac{1}{2}$

[✓ D.] $\frac{1}{\sqrt{2}}$

E. $\sqrt{2}$

Algebra, t's cancel, limit slides into value

$$\frac{2}{\sqrt{2+t}+\sqrt{2-t}} \xrightarrow[t\to 0]{} \frac{2}{\sqrt{2}+\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$1-0=1$ at 0. ← jump at $x=0$

10. Let $G(x) = \begin{cases} 1-x & \text{if } x<0 \\ x+x^2 & \text{if } 0 \le x < 1 \\ 2-x & \text{if } x \ge 1 \end{cases}$. Then G is discontinuous

$0+0^2=0$

$1+1^2=2$

$2-1=1$

← jump at $x=1$.

A. Only at 0

B. Only at 1

[✓ C.] Only at 0 and 1

D. Only at $-1, 0$, and 1

E. The function is continuous everywhere

12. Compute $\lim\limits_{x\to\infty} \frac{2x-5x^2}{\sqrt{4x^2+9}}$

A. $-\frac{5}{2}$

B. 1

C. $-\frac{5}{4}$

D. $\frac{1}{2}$

[✓ E.] $-\infty$

$$\frac{x^2\left[\frac{2}{x}-5\right]}{\underbrace{\sqrt{x^2}}\sqrt{4+9/x^2}}$$

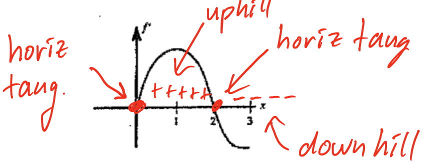
$= x$ if $x\to\infty$

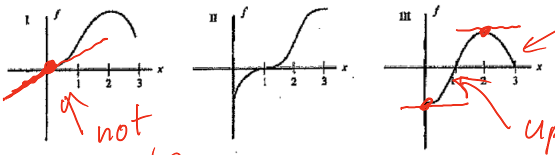
$$= x\cdot\left[\frac{-5+\frac{2}{x}}{\sqrt{4+9/x^2}}\right] = (\text{Big} > 0)\left[\text{close to } -5/\sqrt{4}\right]$$

$\longrightarrow -\infty$ as $x\to\infty$

(But $\sqrt{x^2}=|x|=-x$ if $x\to-\infty$)

2. Given f' as graphed, which could be a graph of f?





downhill

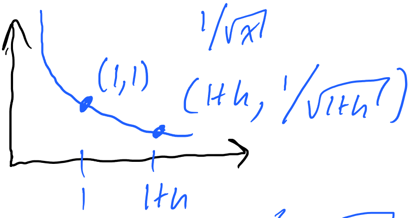
not horiz.

uphill ✓

- A. I only ← allowed because of sloppy graph in 2014
- B. II only
- [✓ C. III only]
- D. I and III
- E. II and III

6. The secant line to the curve $y = x^{-1/2}$ connecting the points on the graph over $x = 1$ and $x = 1 + h$ has slope

- [✓ A.] $\frac{1-\sqrt{1+h}}{h\sqrt{1+h}}$
- B. $\frac{1}{1+\sqrt{1+h}}$
- C. $\frac{1}{\sqrt{1+h}} - 1$
- D. $\frac{\sqrt{1+h}-1}{h} - 1$
- E. $-\frac{1}{2} + h$



$$\text{Slope} = \frac{\frac{1}{\sqrt{1+h}} - 1/1}{h} = \frac{\left(\frac{1-\sqrt{1+h}}{\sqrt{1+h}}\right)}{h}$$

10. $\lim\limits_{h\to0} \frac{(5+h)^2 - 25}{h} =$

- A. 25
- B. Does not exist
- C. 0
- [✓ D. 10]
- E. 5

Aha! $\frac{f(a+h)-f(a)}{h}$

where $f(x) = x^2$, $a = 5$.

Ans $= f'(5) = 2\cdot5 = 10$

$f'(x) = 2x$

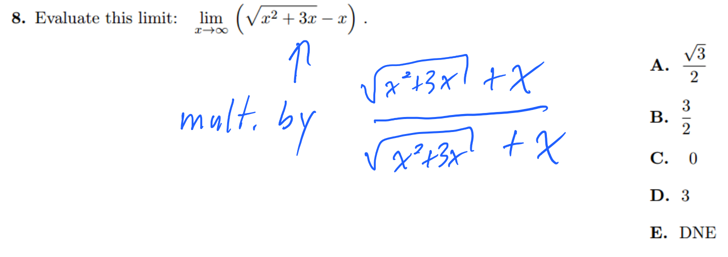
6. $\lim\limits_{x\to1}\left(\frac{1-x}{1-\sqrt{x}}\right)^2 =$

$$\frac{1-x}{1-\sqrt{x}}\cdot\frac{1+\sqrt{x}}{1+\sqrt{x}} = 1+\sqrt{x} \xrightarrow{x\to1} 1+\sqrt{1} = 2$$

$(\ )^2$ continuous. So

$$\left(\frac{1-x}{1-\sqrt{x}}\right)^2 \longrightarrow (2)^2 = 4 \text{ as } x\to1$$

- [✓ **A.** 4]
- **B.** 1
- **C.** $\frac{1}{2}$
- **D.** $\frac{1}{\sqrt{2}}$
- **E.** DNE



8. Evaluate this limit: $\lim_{x\to\infty}\left(\sqrt{x^2+3x}-x\right)$.

mult. by $\dfrac{\sqrt{x^2+3x}+x}{\sqrt{x^2+3x}+x}$

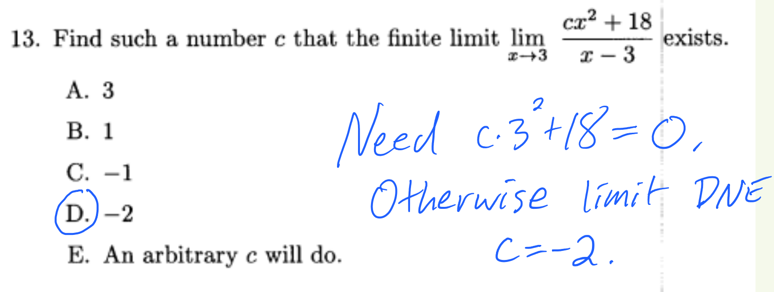
A. $\frac{\sqrt{3}}{2}$

B. $\frac{3}{2}$

C. 0

D. 3

E. DNE



13. Find such a number c that the finite limit $\lim_{x\to 3}\dfrac{cx^2+18}{x-3}$ exists.

A. 3

B. 1

C. −1

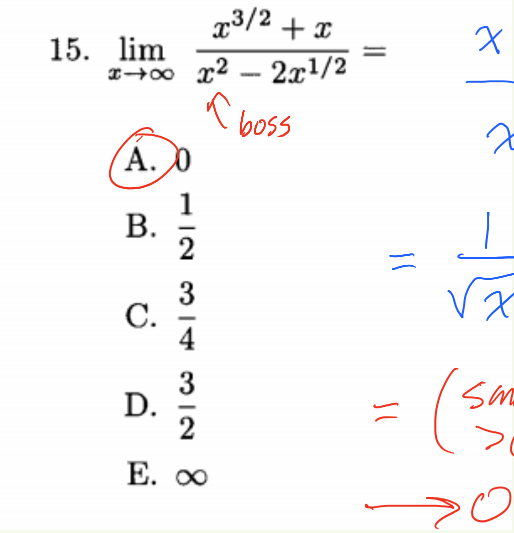
[✓ D.] −2

E. An arbitrary c will do.

Need $c\cdot 3^2+18=0$,

Otherwise limit DNE

$c=-2$.



boss

15. $\lim_{x\to\infty}\dfrac{x^{3/2}+x}{x^2-2x^{1/2}} =$

boss

[✓ A.] 0

B. $\frac{1}{2}$

C. $\frac{3}{4}$

D. $\frac{3}{2}$

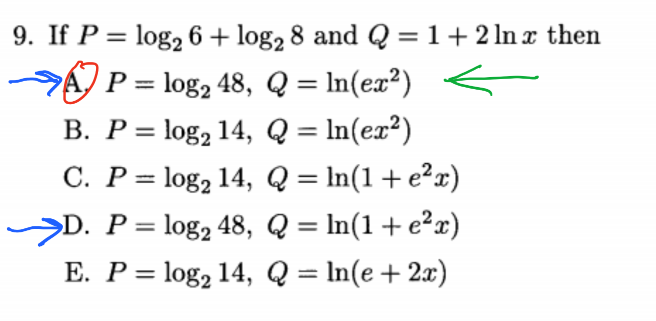
E. ∞

$$\frac{x^{3/2}\left[1+\frac{1}{x^{1/2}}\right]}{x^2\left[1-\frac{2}{x^{3/2}}\right]}$$

$$=\frac{1}{\sqrt{x}}\left[\frac{1+1/x^{1/2}}{1-2/x^{3/2}}\right]$$

$$=\begin{pmatrix}\text{small}\\ >0\end{pmatrix}\cdot\begin{pmatrix}\text{close}\\ \text{to } 1\end{pmatrix}$$

$\longrightarrow 0$ as $x\to\infty$.



9. If $P=\log_2 6+\log_2 8$ and $Q=1+2\ln x$ then

→ [✓ A.] $P=\log_2 48,\ Q=\ln(ex^2)$ ←

B. $P=\log_2 14,\ Q=\ln(ex^2)$

C. $P=\log_2 14,\ Q=\ln(1+e^2x)$

→ D. $P=\log_2 48,\ Q=\ln(1+e^2x)$

E. $P=\log_2 14,\ Q=\ln(e+2x)$

$P=\log_2 6+\log_2 8=\log_2 6\cdot 8$

$Q=1+2\ln x=\ln e+\ln x^2$

$1=\ln e$

$=\ln(ex^2)$