MA 161 Exam 3 Solutions

1. If $f(1) = 3$ and $f'(1) = 5$, use a linear approximation to estimate $f(0.99)$.

A. $\boxed{2.95}$

B. 2.96

C. 2.97

D. 2.98

E. 2.99

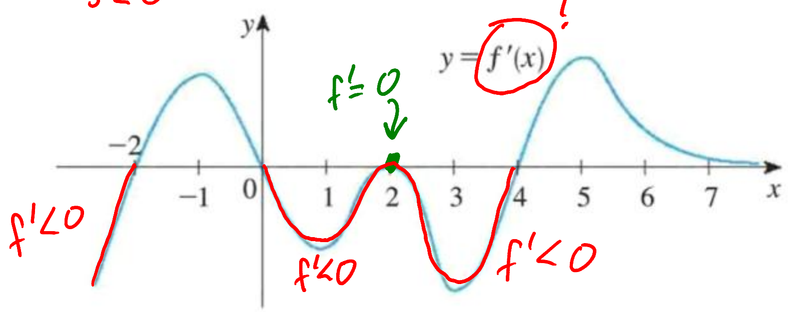
$L(x) = f(a) + f'(a)(x-a)$ $\quad a = 1$, $x = .99$

$L(.99) = 3 + 5(.99 - 1)$

$= 3 - .05 = 2.95$

2. The graph of the **first derivative** $f'(x)$ of a function $f(x)$ is shown. On what intervals is $f(x)$ **decreasing**?

$f' < 0$



f is dereasing when $f' < 0$

A. $(-1, 1) \cup (2, 3) \cup (5, \infty)$

B. $(-\infty, -1) \cup (1, 2) \cup (3, 5)$

C. $(-\infty, 0) \cup (1.5, 2.5) \cup (4.5, 5.5)$

D. $(-2, 0) \cup (4, \infty)$

E. $\boxed{(-\infty, -2) \cup (0, 2) \cup (2, 4)}$ ✓

3. Let $f(x) = -x^3 + 3x + 6$. Let M be the absolute maximum value of $f(x)$ and m the absolute minimum value of $f(x)$ on $[-2, 3]$. What is $M - m$?

A. 4

B. 28

C. $\boxed{20}$

D. 8

E. 16

$f'(x) = -3x^2 + 3 = -3(x^2 - 1) = 0$ when $x = \pm 1$

Critical numbers in $[-2, 3]$: $c = \pm 1$

x	$f(x)$
-2 (endpoints)	$8 - 6 + 6 = 8 \leftarrow M$
-1 (crt.#)	$1 - 3 + 6 = 4$
1	$-1 + 3 + 6 = 8 \leftarrow M$
3 (endpoints)	$-27 + 9 + 6 = -12 \leftarrow m$

$M - m = 8 - (-12) = 20$

4. What is the maximum value of $f(x) = \sqrt{3}\sin x + \cos x$ on $[0, \pi]$?

A. $\sqrt{3}$

B. $1/2$

C. 1

D. $\boxed{2}$

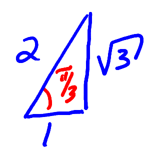
E. $\sqrt{3} + 1$

$f'(x) = \sqrt{3}\cos x - \sin x = 0$ when $\frac{\sin x}{\cos x} = \sqrt{3}$

$\tan x = \sqrt{3}$

Only one critical number in $[0, \pi]$:

$c = \frac{\pi}{3}$



x	$f(x)$
0 (endpoints)	$\sqrt{3} \cdot 0 + 1 = 1$
$c = \frac{\pi}{3}$	$\sqrt{3}\,\frac{\sqrt{3}}{2} + \frac{1}{2} = 2 \leftarrow$ Max
π (endpoints)	$\sqrt{3} \cdot 0 - 1 = -1$

5. A particle moves along the x axis with a position function $x(t)$ with $x(0) = 3$. What is the largest possible positive value for $x(5)$ if $x'(t) \leq 7$ for all $t \geq 0$?

A. 7

B. 10

C. 32

D. $\boxed{38}$

E. 5

MVT: $\dfrac{x(5)-x(0)}{5-0} = x'(c)$ for some c in $(0,5)$.

$x'(c) \leq 7$

So $\dfrac{x(5)-3}{5} \leq 7$

$x(5)-3 \leq 35$

$x(5) \leq 38$ ← happens if $x(t) = 3+7t$

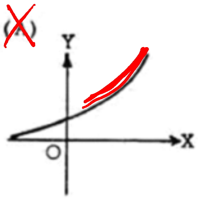
6. If y is a function of x such that $y' > 0$ for all x and $y'' < 0$ for all x, which of the following could be part of the graph of $y = f(x)$?

increasing

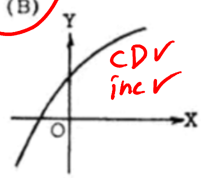
concave down

not C.D.

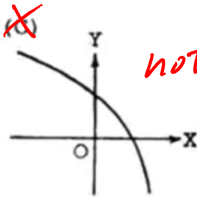
(A)



(B)

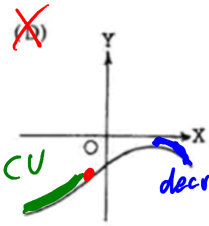


(C)

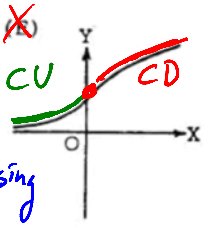


not increasing

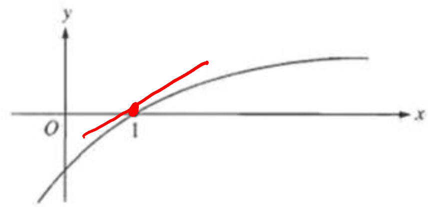
(D)



(E)



7. The graph of a twice-differentiable function f is show in the figure below. Which of the following is true?



concave down at 1

$f''(1) < 0.$

x-intercept : $f(1) = 0$

f increasing at 1: $f'(1) > 0$

A. $f(1) < f'(1) < f''(1)$

B. $f(1) < f''(1) < f'(1)$

C. $f'(1) < f(1) < f''(1)$

D. $\boxed{f''(1) < f(1) < f'(1)}$ ✓

E. $f''(1) < f'(1) < f(1)$

8. If $f(3) = 5$, $\underbrace{f'(3) = 0}_{\text{horiz tang}}$, and $\underbrace{f''(3) = -2}_{\text{concave down}}$, which of the following statements must be true about the point $(3, 5)$? You may assume that $f(x)$, $f'(x)$, and $f''(x)$ are continuous for all x.

I. f has a local maximum there ✓

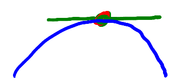
II. f has a local minimum there

III. The graph is concave upward in the neighborhood of $(3, 5)$

IV. The graph is concave downward in the neighborhood of $(3, 5)$ ✓

V. The tangent line at $(3, 5)$ is horizontal ✓

(3,5)



$f'(3) = 0$ (horizontal tangent)

A. II, III, and V

B. $\boxed{\text{I, IV, and V}}$

C. II, IV, and V

D. I and III

E. II and IV

$f''(3) = -2$: Concave down

Second Der. Test : (3,5) a local max

9. Consider the function $f(x) = xe^{2x}$. Which of the following statements are true?

(1) f is increasing on $(-\frac{1}{2}, \infty)$. ✓ $f' > 0$

(2) f is concave down on $(-\infty, -1)$. ✓

~~(3)~~ f has one local maximum on $(-\infty, \infty)$.

A. (1), (2), and (3)

B. (3) only

C. $\boxed{\text{(1) and (2)}}$

D. (1) only

E. (2) and (3)

$$f'(x) = 1 \cdot e^{2x} + x\,(2e^{2x})$$

$$= (1 + 2x)\,e^{2x}$$

First Der. Test: Local min

$$- - - - -\ \underset{-\frac{1}{2}}{0}\ + + + + + + + + \quad \text{Sign } f'$$

$f' > 0 \Rightarrow$ increasing

$$f''(x) = \frac{d}{dx}\left[e^{2x} + 2x\,e^{2x}\right] = 2e^{2x} + \left(2e^{2x} + 4xe^{2x}\right)$$

$$= 4e^{2x} + 4xe^{2x} = 4(1+x)\,e^{2x}$$

$$- - - - - - -\ \underset{-1}{0}\ + + + + + + + + + + + \quad \text{Sign } f''$$

$f'' < 0$

Concave down

10. Compute the limit $\lim_{x \to 0} \underbrace{(1 + 2x)^{3/x}}_{y}$.

A. 3

B. 6

C. $3e^2$

D. e^3

E. $\boxed{e^6}$

$$\ln y = \frac{3}{x} \ln(1+2x)$$

$$\lim_{x \to 0} \ln y = \lim_{x \to 0} \frac{3 \ln(1+2x)}{x} \underset{\left(\frac{0}{0}\right)}{\overset{L'H}{=}}$$

$$= \lim_{x \to 0} \frac{3 \cdot \frac{1}{(1+2x)} \cdot 2}{1} = 6$$

$$y = e^{\ln y} \longrightarrow e^6 \quad \text{as } x \to 0.$$

11. Compute the limit $\lim_{x \to 0+} \frac{\sin x}{e^x}$ $= \frac{\sin 0}{e^0} = \frac{0}{1} = 0$

A. $\boxed{0}$

B. 1

C. ∞

D. $\frac{1}{e}$

E. DNE

L'H out!

12. Find $\lim_{x \to 0+} x(\ln x)^2$. $= \lim_{x \to 0+} \frac{(\ln x)^2}{\left(\frac{1}{x}\right)} \underset{\left(\frac{\infty}{\infty}\right)}{\overset{L'H}{=}} \lim_{x \to 0+} \frac{2(\ln x) \cdot \frac{1}{x}}{\left(\frac{-1}{x^2}\right)}$

A. 1

B. -2

C. 2

D. ∞

E. $\boxed{0}$

$= \lim_{x \to 0+} \frac{2 \ln x}{\left(-\frac{1}{x}\right)} \underset{\left(\frac{-\infty}{-\infty}\right)}{\overset{L'H}{=}} \lim_{x \to 0+} \frac{2 \cdot \frac{1}{x}}{\left(\frac{1}{x^2}\right)}$

$= \lim_{x \to 0^+} 2x = 0$