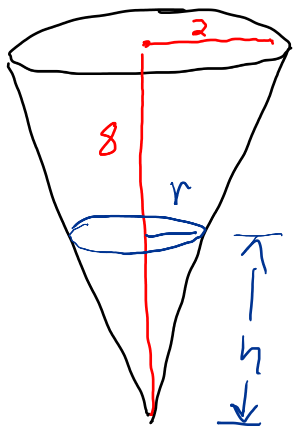
1) A tank has the shape of an inverted circular cone with radius 2 m and height 8 m. If water is poured into the tank at a rate of 4 m^3 per minute, find the rate at which the water level is rising (in m per minute) when the water is 4 m deep.



$\frac{r}{h} = \frac{2}{8}$ $\boxed{r = \frac{1}{4}h}$

Know: $\frac{dV}{dt} = 4$

Want: $\frac{dh}{dt}$ when $h = 4$

$V = \frac{\pi}{3} r^2 h = \frac{\pi}{3}\left(\frac{1}{4}h\right)^2 h$

$= \frac{\pi}{48} h^3$

$4 = \frac{dV}{dt} = \frac{\pi}{48} 3h^2 \frac{dh}{dt}$

$4 = \frac{\pi}{16} \cdot 4^2 \cdot \frac{dh}{dt}$ when $h = 4$

So $\frac{dh}{dt} = \frac{4}{\pi}$ m/min

A) $\frac{4}{\pi}$

B) $\frac{2}{\pi}$

C) $\frac{8}{3\pi}$

D) $\frac{3}{\pi}$

E) $\frac{4}{3\pi}$

2) Use a linear approximation to compute the approximate value of $\sqrt[3]{8.06}$.

$f(x) = x^{1/3}$ $a = 8$ $f(a) = 8^{1/3} = 2$

$f'(x) = \frac{1}{3}x^{-2/3}$ $f'(a) = \frac{1}{3}\frac{1}{(8)^{2/3}} = \frac{1}{12}$

A) 2.04

B) 2.02

C) 2.005

D) 2.01

E) 2.0025

$f(x) \approx f(a) + f'(a)(x-a)$

$x^{1/3} \approx 2 + \frac{1}{12}(x-8)$

$(8.06)^{1/3} \approx 2 + \frac{1}{12}(8.06-8) = 2 + \frac{1}{12} \cdot \frac{6}{100}$

$= 2 + \frac{.5}{100} = 2.005$

 $f'(x)=3x^2+1$

3) If $f(x) = x^3 + x - 1$ on the interval $[0, 2]$, find a number c that satisfies the Mean Value Theorem.

$$\frac{f(2)-f(0)}{2-0} = \frac{(2^3+2-1)-(0+0-1)}{2-0}$$

$$= \frac{10}{2} = 5 = f'(c)$$

$$5 = 3c^2+1$$

$$\frac{4}{3} = c^2$$

$$c = \pm\sqrt{\frac{4}{3}} \qquad c = \frac{2}{\sqrt{3}} \text{ is in } (0,2)$$

A) $\dfrac{2}{\sqrt{3}}$

B) $\sqrt{2}$

C) $\sqrt{\dfrac{5}{3}}$

D) $\dfrac{\sqrt{3}}{3}$

E) $\dfrac{4}{\sqrt{3}}$

4) If m_1 is the minimum of $f(x) = x^3 + 3x^2 - 9x$ on $[0, 2]$ and m_2 is the maximum, find $m_1 + m_2$.

$$f'(x) = 3x^2+6x-9 = 0$$

$$x^2+2x-3=0$$

$$(x+3)(x-1)=0 \qquad \text{crt. \#'s } x = 1, -3$$

out

A) 7

B) -3

C) 5

D) 2

E) -52

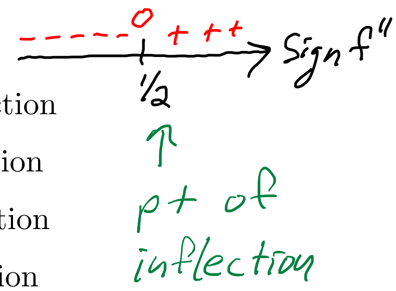
Closed Interval Theorem:

	x	$f(x)$
end pt. →	0	$0^3+3\cdot0^2-9\cdot0=0$
crt # →	1	$1^3+3\cdot1^2-9\cdot1=-5 \leftarrow m_1$
end pt. →	2	$2^3+3\cdot2^2-9\cdot2=8+12-18=2 \leftarrow m_2$

$$m_1+m_2 = -5+2=-3$$

5) Let $f(x) = 2x^3 - 3x^2$. f has

$f''(x) = 12x - 6 = 12(x - \frac{1}{2})$



$f'(x) = 6x^2 - 6x$

$= 6x(x-1)$

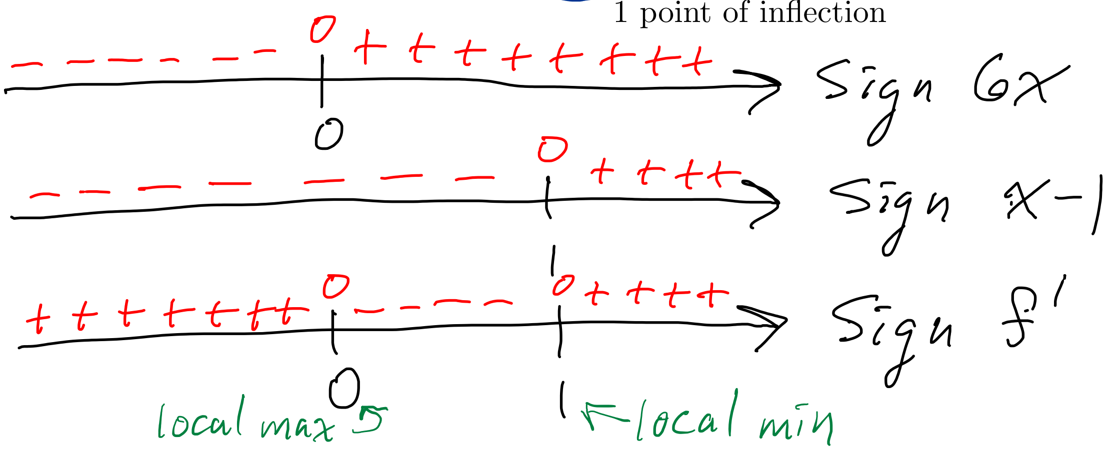
A) 1 local max and 2 points of inflection

B) 1 local max and 1 point of inflection

C) 1 local min and 2 points of inflection

D) 1 local min and 1 point of inflection

E) 1 local min, 1 local max and 1 point of inflection



6) If $f(t) = t^2 + 4\cos t$ on $(0, 2\pi)$ find the interval(s) where the graph of f is concave upward.

$f'(t) = 2t + 4(-\sin t)$

$f''(t) = 2 - 4\cos t > 0$ if Concave up

$\boxed{\cos t < \frac{1}{2}}$

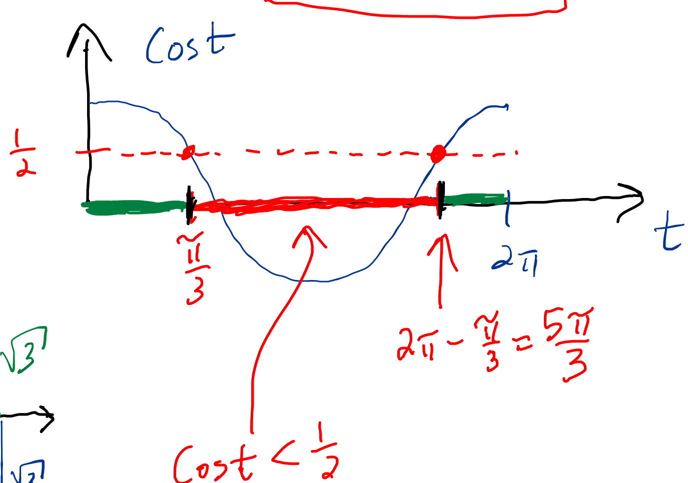
A) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{11\pi}{6}, 2\pi\right)$

B) $\left(\frac{\pi}{3}, \frac{5\pi}{3}\right)$

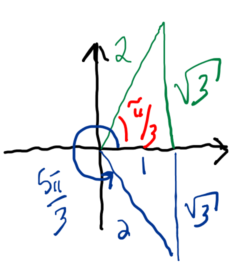
C) $\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$

D) $\left(\frac{\pi}{6}, \frac{11\pi}{6}\right)$

E) $\left(0, \frac{\pi}{3}\right) \cup \left(\frac{5\pi}{3}, 2\pi\right)$



Concave up on $\left(\frac{\pi}{3}, \frac{5\pi}{3}\right)$



7) $\lim_{x\to 0} \frac{\sin x - x}{\tan x - x} \overset{L'H}{=} \frac{0}{0}$ $\lim_{x\to 0} \frac{\cos x - 1}{\sec^2 x - 1} \overset{L'H}{=} \frac{0}{0}$

A) $-\frac{1}{2}$

B) -1

C) 0

D) $\frac{1}{2}$

E) 1

$= \lim_{x\to 0} \underbrace{\frac{-\sin x}{2\sec x(\sec x \tan x)}}$

Can do L'H again from here, or

$$\frac{-\sin x}{2\frac{1}{\cos x}\cdot\frac{1}{\cos x}\cdot\frac{\sin x}{\cos x}} = -\frac{1}{2}\cos^3 x$$

$$= \lim_{x\to 0} -\frac{1}{2}\cos^3 x = -\frac{1}{2}\cos^3 0 = -\frac{1}{2}$$

8) $\lim_{x\to 0^+} \underbrace{(1-3x)^{1/5x}}_{y} = 1^\infty$ $\quad \ln y = \frac{1}{5x}\ln(1-3x)$

A) 1

B) e^{-15}

C) $e^{-3/5}$

D) $e^{-5/3}$

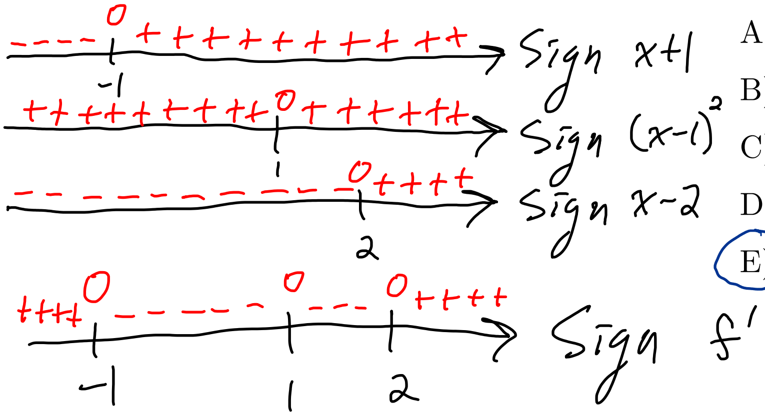
E) $e^{-1/15}$

$$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\ln(1-3x)}{5x} \overset{L'H}{=} \frac{0}{0}$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{1-3x}\cdot(-3)}{5} = \frac{\left(\frac{-3}{1-3\cdot 0}\right)}{5} = -\frac{3}{5}$$

So $y = e^{\ln y} \longrightarrow e^{-3/5}$ as $x \to 0+$

9) Let $f'(x) = (x+1)(x-1)^2(x-2)$. f has



Local max   not!   local min

A) no local maxima and 2 local minima

B) 2 local maxima and no local minima

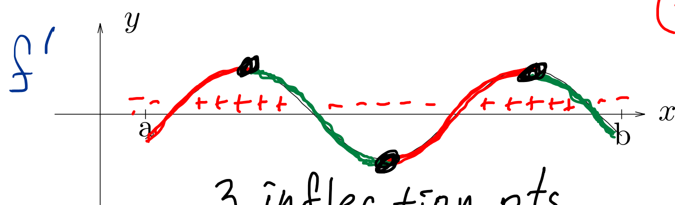
C) 1 local maximum and 2 local minima

D) 2 local maxima and 1 local minimum

E) 1 local maximum and 1 local minimum

10) The graph of f' is given below, $a \le x \le b$.

f' increasing $(f')' > 0$ $(f'' > 0)$ Concave up



3 inflection pts.

f' decreasing $(f')' < 0$ $(f'' < 0)$ conc. down

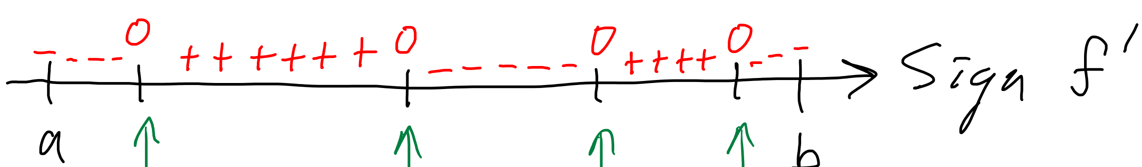
A) f has exactly 2 points of inflection and exactly 4 local extrema.

B) f has exactly 2 points of inflection and exactly 3 local extrema.

C) f has exactly 4 points of inflection and exactly 3 local extrema.

D) f has exactly 3 points of inflection and exactly 4 local extrema.

E) f has exactly 3 points of inflection and exactly 5 local extrema.



local min   local max   local min   local max