

MATH 181

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Two in class exams: 100 pts each

Final Exam: 200 pts.

Homework & quizzes 100 pts.

$$y = e^x \quad \frac{dy}{dx} = e^x \quad \text{Wow!}$$

$$y(0) = e^0 = 1.$$

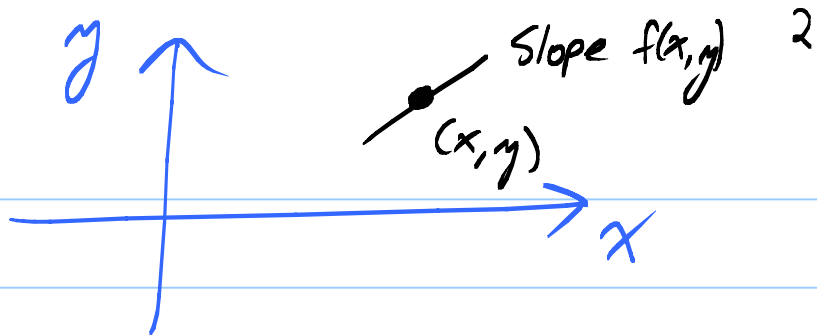
Fact: e^x is the one and only function $f(x)$ such that

- A) $f'(x) = f(x)$
- B) $f(0) = 1.$

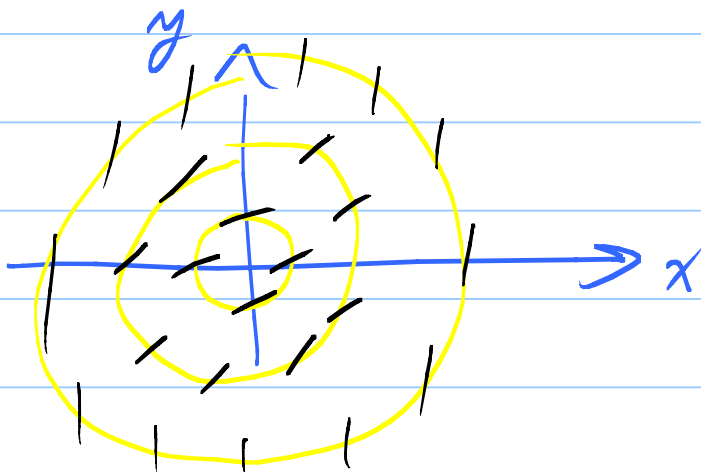
Famous Diff. Egn: $\frac{dy}{dx} = ky$

General First order Ordinary Diff. Egn

$$\frac{dy}{dx} = f(x, y)$$



$$\frac{dy}{dx} = x^2 + y^2$$



EX: $\frac{d}{dx}(e^x) = e^x$

$$\left[\begin{array}{l} y = e^{7x} : \quad y = e^u \text{ where } u = 7x \\ \frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = e^u (7) = 7e^{7x} \end{array} \right.$$

$$\frac{d}{dx}(e^{-7x}) = -7e^{-7x}$$

Fact: If $f'(x) = k f(x)$, then $f(x)$ has to be $f(x) = C e^{kx}$ where $C = f(0)$.

Why: Notice that $f'(x) - k f(x) \equiv 0$

Multiply by e^{-kx} :

$$[uv]' = u'v + uv'$$

$$e^{-kx} f'(x) - k e^{-kx} f(x) \equiv 0 \quad 3$$

$$\frac{d}{dx} [e^{-kx} f(x)] \equiv 0$$

Conclude that

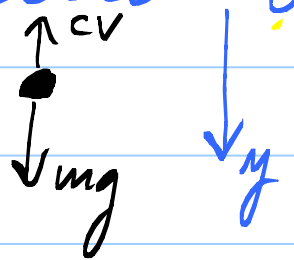
$$e^{-kx} f(x) \equiv C, \text{ a const.}$$

Plug in $x=0$: $e^0 f(0) = C$. So $C = f(0)$.

Solve for $f(x)$: $f(x) = \frac{C}{e^{-kx}} = C e^{kx}$ ✓

EX: Drop a heavy ball in ocean.

Newton's Law : $F = ma$



$$(mg - cv) = m \frac{dv}{dt}$$

$$-c \left(v - \frac{mg}{c} \right) = m \frac{dv}{dt}$$

$$\frac{dv}{dt} = -\frac{c}{m} \underbrace{\left(v - \frac{mg}{c} \right)}_u$$

$$\frac{du}{dt} = \frac{dv}{dt} !$$

$$\boxed{\frac{du}{dt} = -\frac{c}{m} u}$$

$$\text{so } u = \underbrace{u(0)}_{= -\frac{mg}{c}} e^{-\frac{c}{m} t}$$

$$u = v - \frac{mg}{c}$$

$$u(0) = \underbrace{v(0)}_0 - \frac{mg}{c} = -\frac{mg}{c}$$

Now

$$u = -\frac{mg}{c} e^{-\frac{c}{m}t}$$

$$\left(v - \frac{mg}{c}\right) = -\frac{mg}{c} e^{-\frac{c}{m}t}$$

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Finally $v = \frac{mg}{c} (1 - e^{-\frac{c}{m}t})$

Terminal velocity : $\frac{mg}{c}$.

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