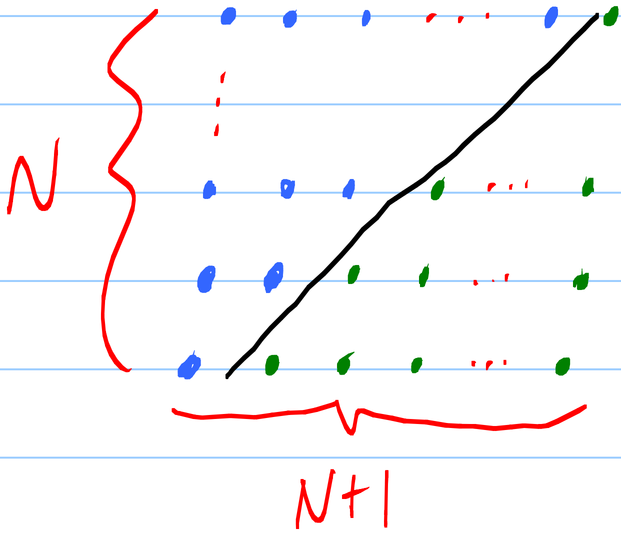
$$1 + 2 + 3 + \cdots + N = \frac{N(N+1)}{2}$$



$$2(1+2+\cdots+N) = N(N+1)$$

<u>Fact</u>: $1 + 2^2 + 3^2 + \cdots + N^2 = \frac{N(N+1)(2N+1)}{6}$

$N=1$. True ✓. Mathematical Induction.
Suppose the formula holds for N. Then

$$(1 + 2^2 + \cdots + N^2) + (N+1)^2 = \frac{N(N+1)(2N+1)}{6} + (N+1)^2$$

$$\stackrel{?}{=} \frac{[N+1]([N+1]+1)(2[N+1]+1)}{6}$$

Yes! So true for $N+1$ too. Use MAPLE to check:

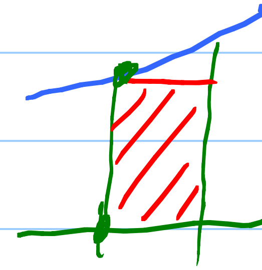
```
expand(N*(N+1)*(2*N+1)/6 + (N+1)^2);
expand((N+1)(N+2)(2N+3)/6);
```

Get same thing. Or use
simplify(eeee - mmm);

Def^n: $$\int_a^b f(x)\,dx = \lim_{N\to\infty} \sum_{n=1}^{N} f(x_n)\,\Delta x$$

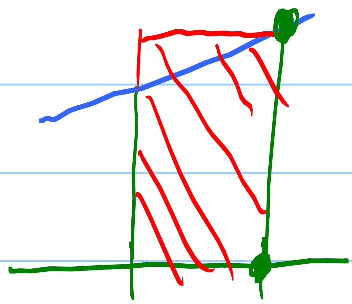
where $\Delta x = \frac{b-a}{N}$ and x_n any point from n-th interval.

Typical choices for x_n: Left



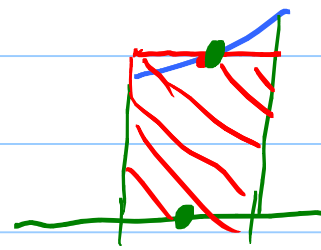
x_n

Right



x_n

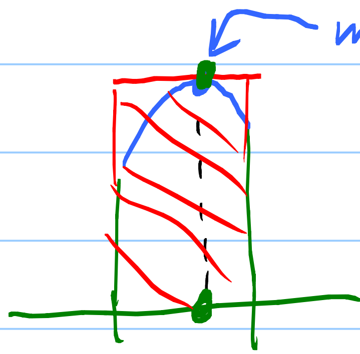
Midpoint



x_n

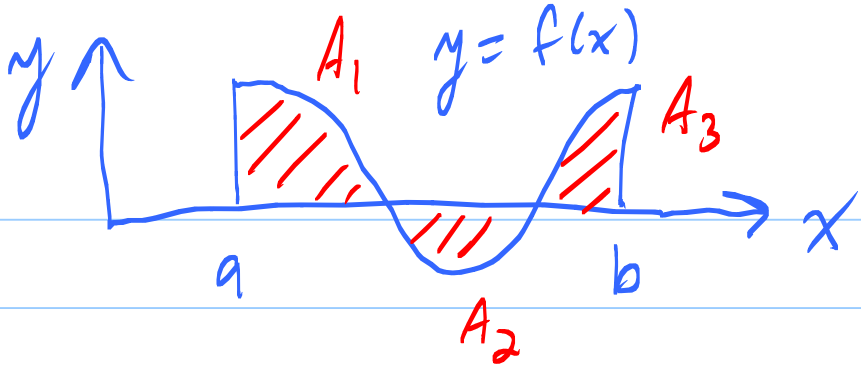
Special

max



x_n

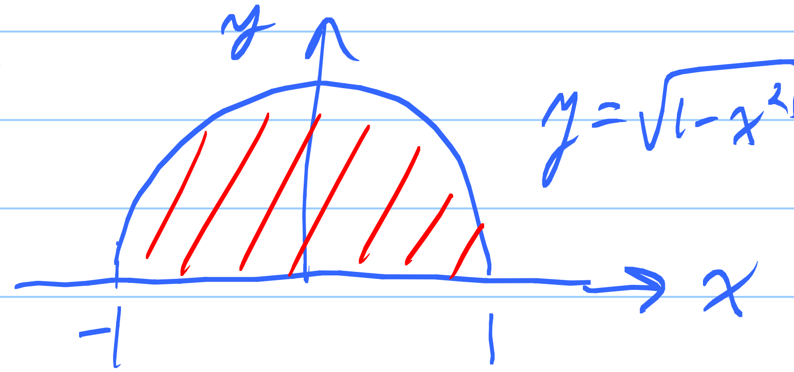
Fact: The limit exists if f is continuous. Limit does not depend of choice of x_n's.



$$\int_a^b f(x)\,dx = A_1 - A_2 + A_3$$

= the "signed area" under the graph of $y = f(x)$.

<u>Problem:</u> Calculate π



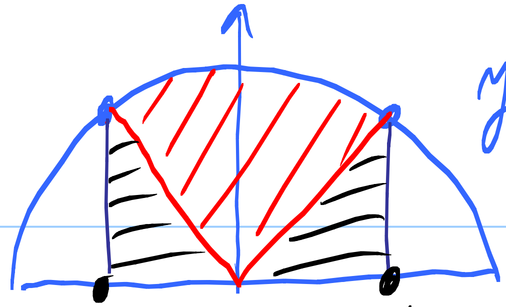
$$\Delta x = \frac{1-(-1)}{1000} = \frac{1}{500}$$

$$\frac{\pi}{2} \cong \sum_{n=1}^{\infty} \underbrace{\sqrt{1-\left(-1+\frac{n}{500}\right)^2}}_{f(x_n)} \underbrace{\left(\frac{1}{500}\right)}_{\Delta x}$$

```
= (1/500)*sum(sqrt(1-(-1+n/500)^2),n=1..1000);

evalf(%);
```

Not very accurate because of vertical tangents at $x = \pm 1$. Better way:



$y = \sqrt{1-x^2}$

$$\int_{-1/\sqrt{2}}^{1/\sqrt{2}} \sqrt{1-x^2}\, dx =$$

$$\frac{\pi}{4} + 2\left[\frac{1}{2} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}\right]$$