Quiz: $\dfrac{d}{dx}\Big(\underbrace{\int_0^{x^4} h(t)\,dt}_{G(x)}\Big)^5$ $\qquad h(1)=3$, $\int_0^1 h(t)\,dt = 2$

$$y = \int_0^{x^4} h(t)\,dt = \int_0^u h(t)\,dt \text{ where } u = x^4.$$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = h(u)\,4x^3 = h(x^4)\cdot 4x^3$$

$$\frac{d}{dx}(y^5) = 5y^4\frac{dy}{dx}$$

$$= 5\Big(\int_0^{x^4} h(t)\,dt\Big)^4 h(x^4)\cdot 4x^3$$

Finally, $G'(1) = 5\Big(\int_0^1 h(t)\,dt\Big)^4 h(1)\cdot 4\cdot 1^3$

$$= 5\cdot 2^4\cdot 3\cdot 4\cdot 1^3$$

EX: $\displaystyle\int_0^1 x^3\sqrt{\underbrace{x^4+3}_{u}}\,dx$

Let $u = x^4+3$

$du = \dfrac{du}{dx}\cdot dx$

$= 4x^3\,dx$

$\boxed{x^3\,dx = \tfrac{1}{4}\,du}$

When $x=0$: $u = 0^4+3 = \underline{\underline{3}}$

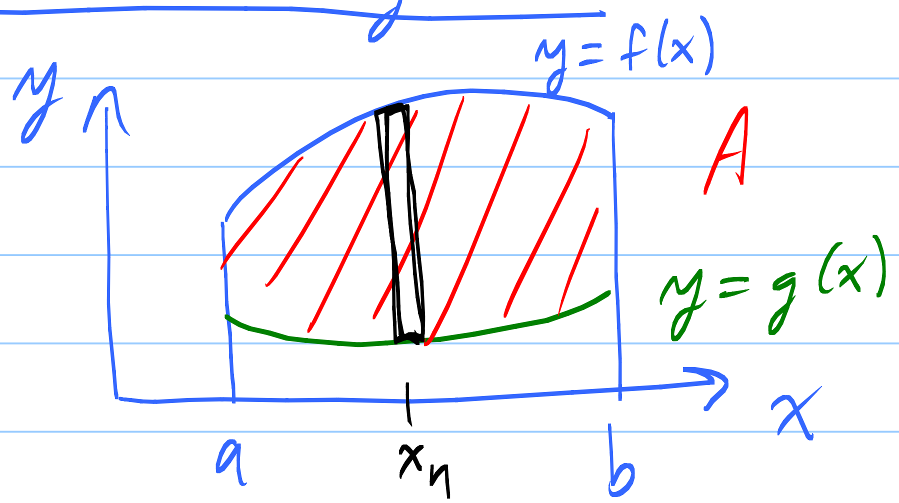
When $x=1$: $u = 1^4+3 = \underline{\underline{4}}$

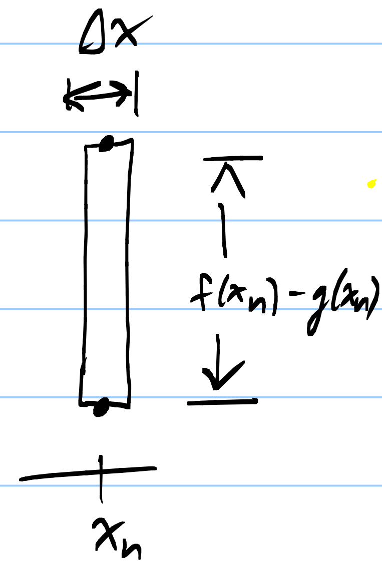
$$\int_{x=0}^{1} \sqrt{x^4+3}\,\left(x^3\,dx\right) = \int_{u=3}^{4} \sqrt{u}\,\left(\tfrac{1}{4}\,du\right)$$

$$= \frac{1}{4}\int_3^4 u^{1/2}\,du = \frac{1}{4}\left[\frac{1}{\frac{1}{2}+1}\,u^{\frac{1}{2}+1}\right]_3^4$$

$$= \frac{1}{6}\,u^{3/2}\Big|_3^4 = \frac{1}{6}\left[(4)^{3/2} - (3)^{3/2}\right].$$

Applications of integration:

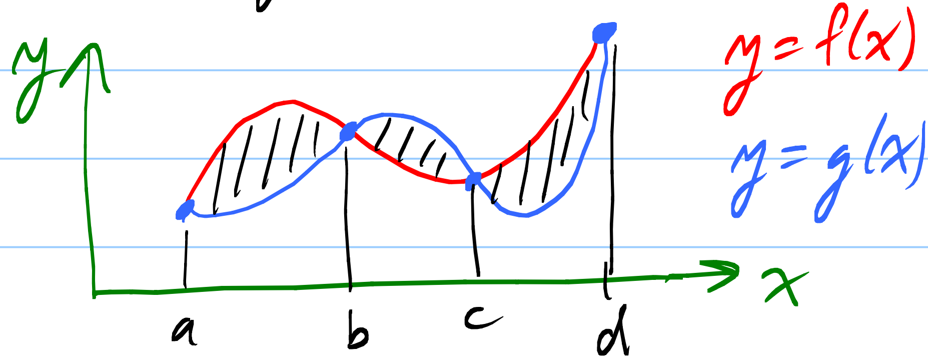




$$A = \int_a^b \Big[\underset{\text{top}}{f(x)} - \underset{\text{bottom}}{g(x)}\Big]\,dx$$

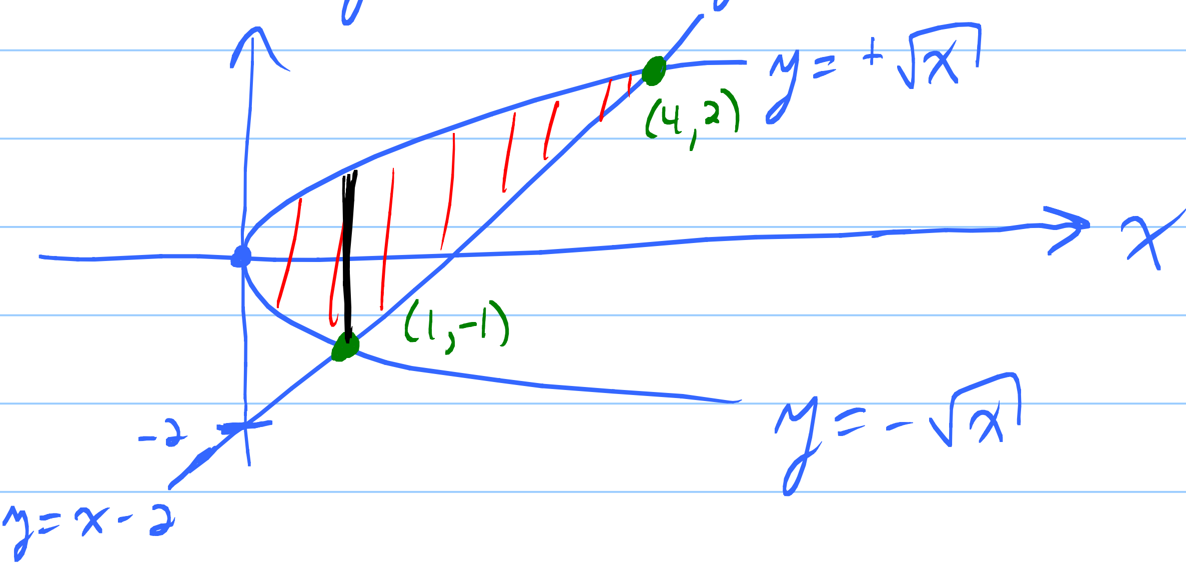
$$= \lim_{N\to\infty} \sum \left[f(x_n) - g(x_n)\right]\Delta x$$

Careful:



$$A = \int_a^b [f(x) - g(x)]\,dx + \int_b^c [g(x) - f(x)]\,dx + \int_c^d [f(x) - g(x)]\,dx$$

EX: Find the area enclosed by the curves $x = y^2$ and $y = x - 2$.



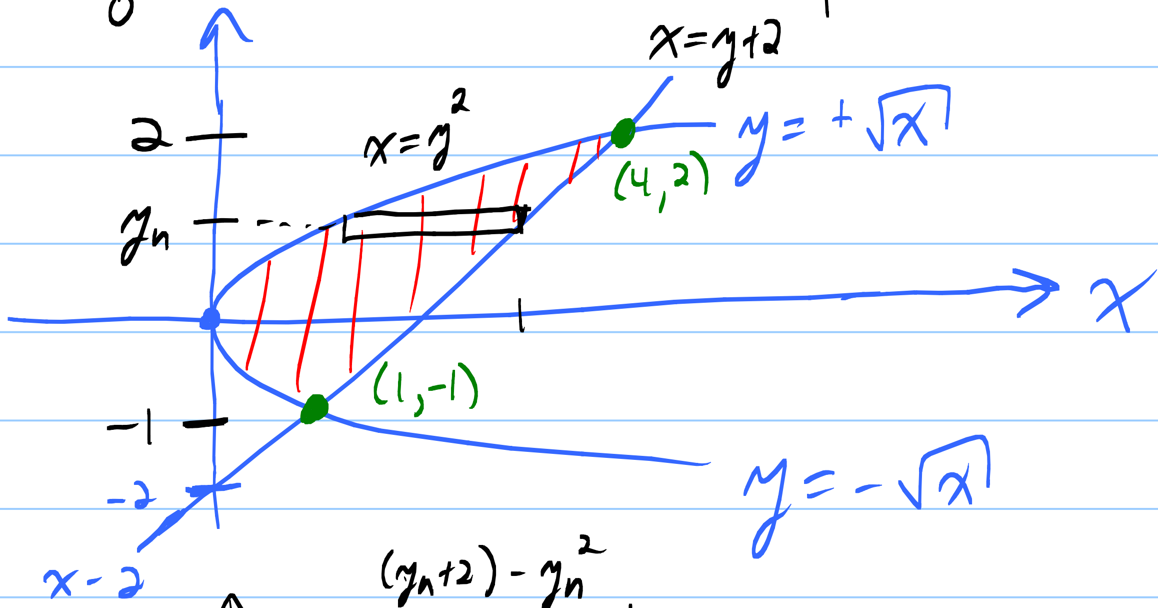
To find intersection points, plug $y = x-2$ into $x = y^2$:

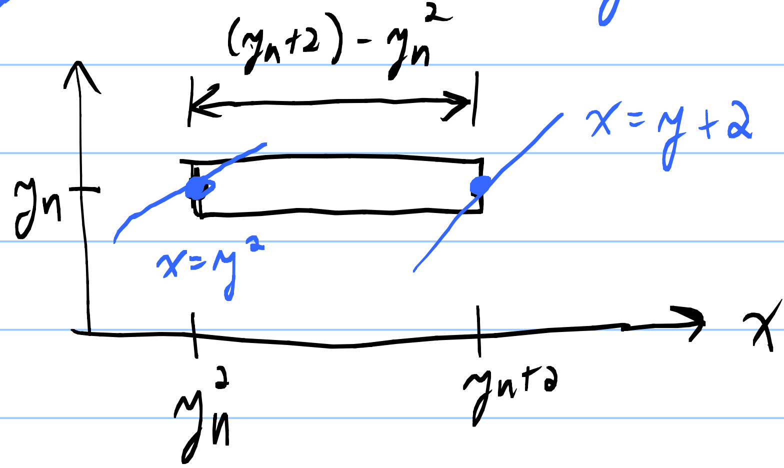
$$x = (x-2)^2 = x^2 - 4x + 4$$
$$x^2 - 5x + 4 = 0$$
$$(x-1)(x-4) = 0$$

Roots $x = 1, 4$.

When $x = 1$, $y = 1 - 2 = -1$.
When $x = 4$, $y = 4 - 2 = 2$

$$A = \int_0^1 \left[\sqrt{x} - (-\sqrt{x})\right] dx + \int_1^4 \left[\sqrt{x} - (x-2)\right] dx$$

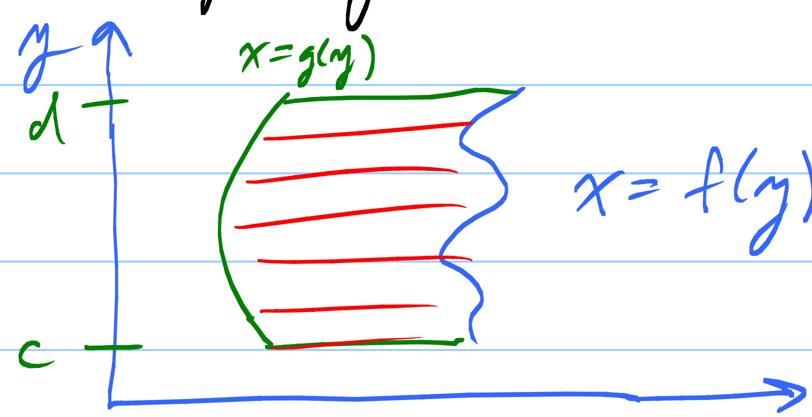




$$A = \lim_{N\to\infty} \sum \left[(y_n+2) - y_n^2\right] \Delta y$$

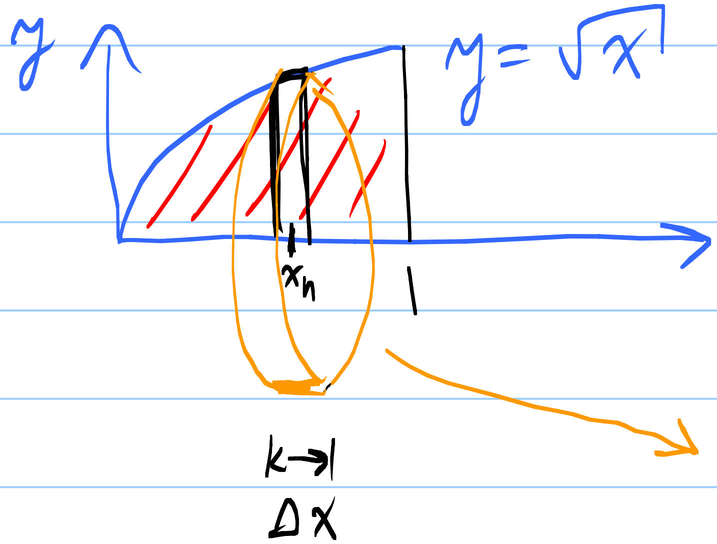
$$= \int_{-1}^{2} (y+2) - y^2 \, dy$$

In general

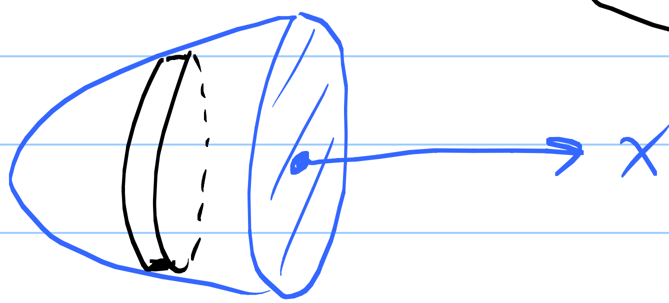


$$A = \int_c^d \left[\underset{\text{right}}{f(y)} - \underset{\text{left}}{g(y)} \right] dy$$

Problem: Find the volume of a gum drop.



Spin around x-axis.



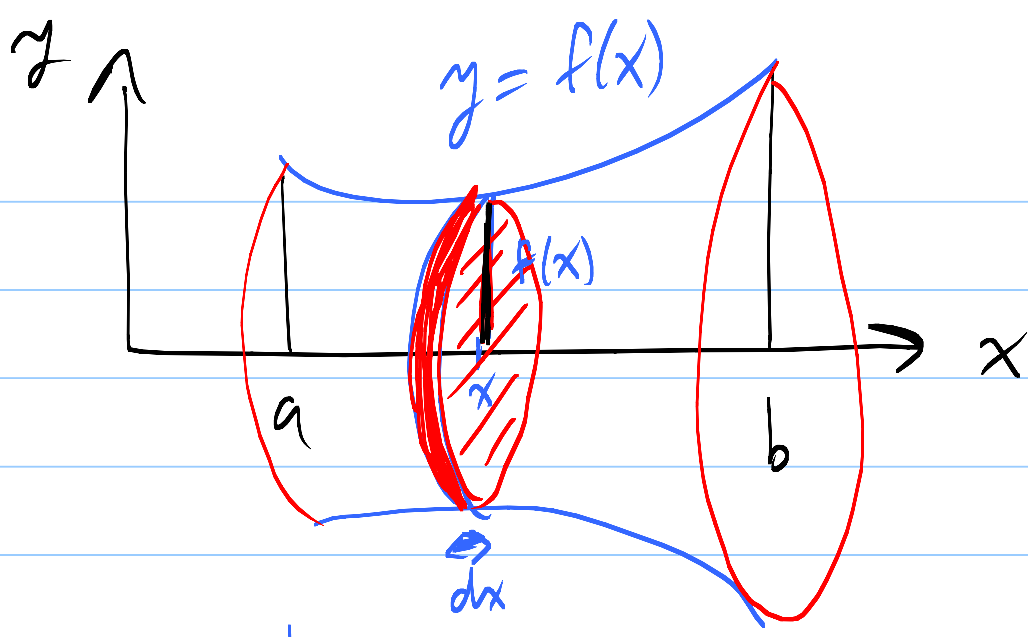
Volume of slab

$$\pi \left(\sqrt{x_n}\right)^2 \Delta x$$

$$\text{Total Volume} = \lim_{N \to \infty} \sum \pi \left(\sqrt{x_n}\right)^2 \Delta x$$

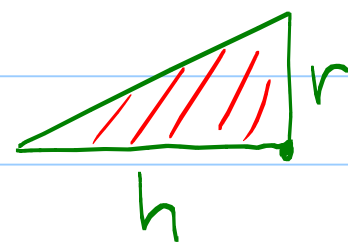
$$= \int_0^1 \pi x \, dx = \pi \left[\tfrac{1}{2} x^2 \right]_0^1 = \frac{\pi}{2}$$

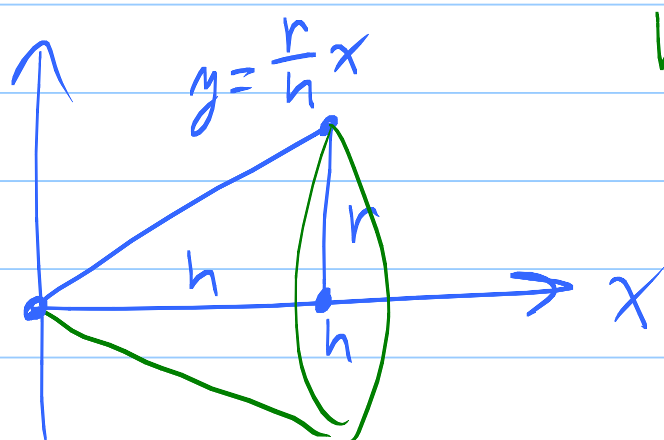
In general:



$$V = \int_a^b \pi f(x)^2 \, dx$$

Volume of a cone:



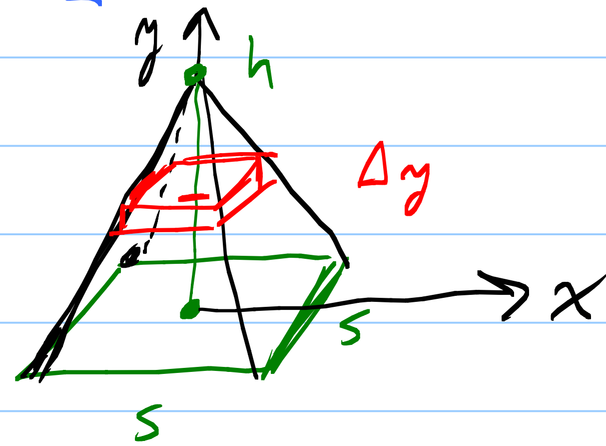


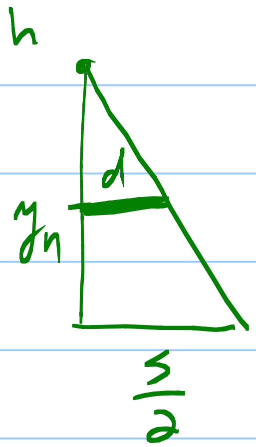
$$V = \int_0^h \pi \left(\frac{r}{h} x\right)^2 dx$$

$$= \frac{\pi r^2}{h^2} \int_0^h x^2 \, dx = \frac{\pi r^2}{h^2} \left[\frac{1}{3} x^3\right]_0^h$$

$$= \frac{\pi r^2}{h^2}\left[\frac{1}{3}h^3 - 0\right] = \frac{1}{3}\pi r^2 h$$

Volume of a pyramid:





Play: Square slice has area $= (2d)^2$

write in terms of y_n

$$V = \lim_{N\to\infty} \sum (2d)^2 \, \Delta y$$

$= f(y_n)$

$$= \int_0^h f(y)\, dy$$