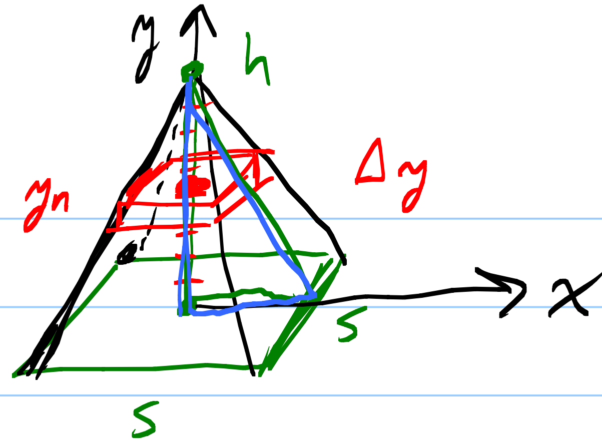
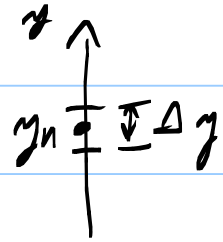
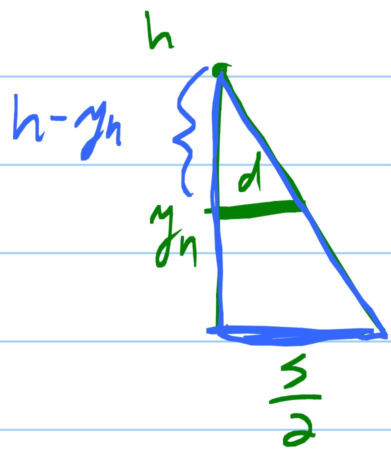
Volume of a pyramid:







$$\frac{h-y_n}{d} = \frac{h}{\left(\frac{s}{2}\right)}$$

$$d = \frac{h-y_n}{h} \cdot \frac{s}{2}$$

Side of red square $= 2d = s \cdot \frac{h-y_n}{h}$

$$V = \lim_{N\to\infty} \sum \left(s \cdot \frac{h-y_n}{h}\right)^2 \Delta y$$

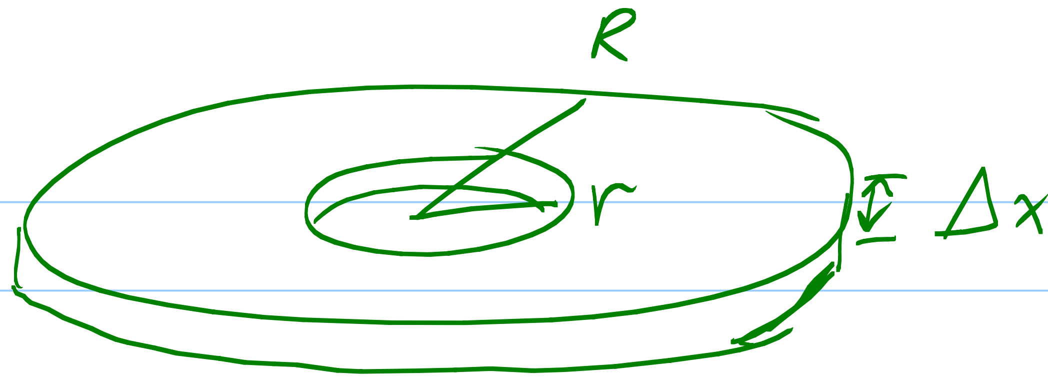
$$= \int_0^h s^2 \left(\frac{h-y}{h}\right)^2 dy = \frac{s^2}{h^2} \int_0^h (h-y)^2 \, dy$$

$$= \frac{s^2}{h^2} \int_0^h h^2 - 2hy + y^2 \; dy$$

$$= \frac{s^2}{h^2} \left[h^2 y - h y^2 + \frac{1}{3} y^3 \right]_0^h$$

$$= \frac{s^2}{h^2} \left(h^3 - h^3 + \frac{1}{3} h^3 - 0 \right) = \frac{1}{3} s^2 h$$

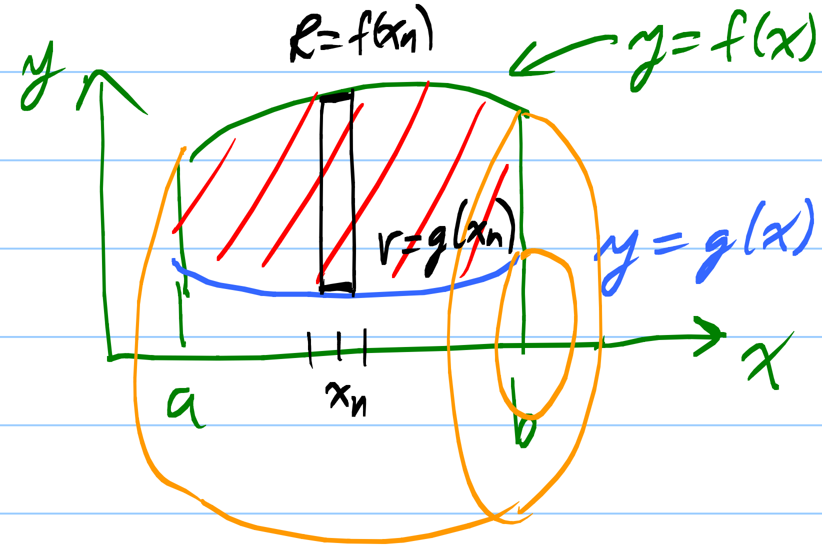
Washers



$$\Delta V = \pi R^2 \Delta x - \pi r^2 \Delta x = \pi \left[R^2 - r^2 \right] \Delta x$$

Volume of revolution:

Spin around x-axis

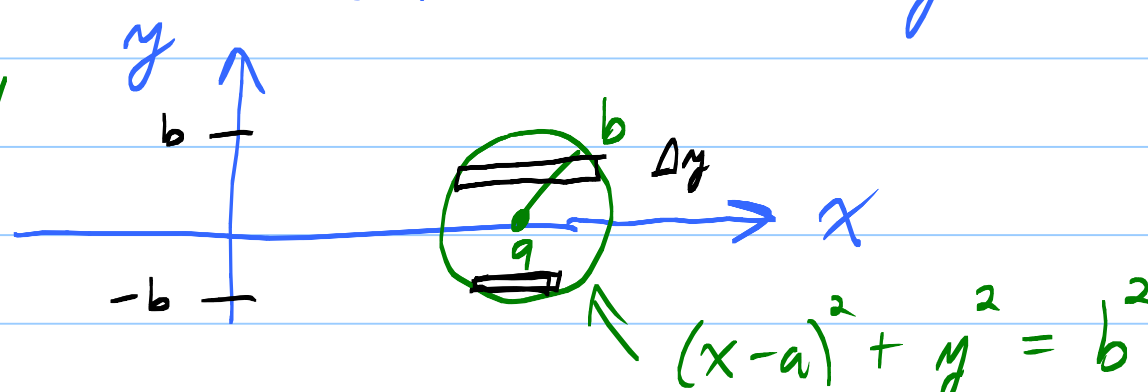


$$V = \lim_{N \to \infty} \sum \pi \left[f(x_n)^2 - g(x_n)^2 \right] \Delta x$$

$$= \int_a^b \pi \left[f(x)^2 - g(x)^2 \right] dx$$

Problem: Find the volume of a bagel.

Spin around y-axis



$$(x-a)^2 = b^2 - y^2$$

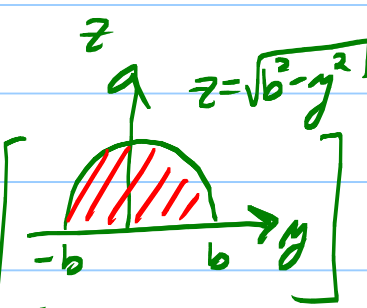
$$x - a = \pm\sqrt{b^2 - y^2}$$

$$x = a \pm \sqrt{b^2 - y^2} \leftarrow$$ + for outer radius, − for inner radius

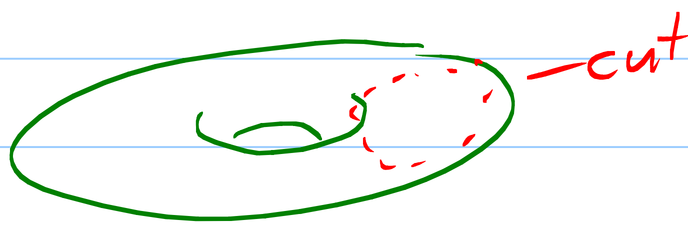
$$V = \lim_{N\to\infty} \sum \pi \left[\left(a + \sqrt{b^2 - y_n^2}\right)^2 - \left(a - \sqrt{b^2 - y_n^2}\right)^2 \right] \Delta y$$

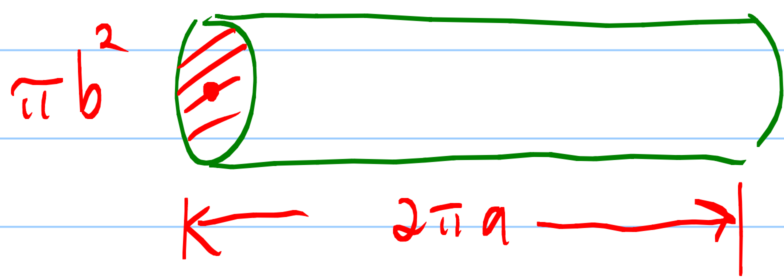
$$= \int_{-b}^{b} \pi \, 4a\sqrt{b^2 - y^2}\, dy$$

$$= 4\pi a \int_{-b}^{b} \sqrt{b^2 - y^2}\, dy = 4\pi a \left[\quad \right]$$



$$= 4\pi a \left[\tfrac{1}{2}\pi b^2\right] = (2\pi a)(\pi b^2)$$





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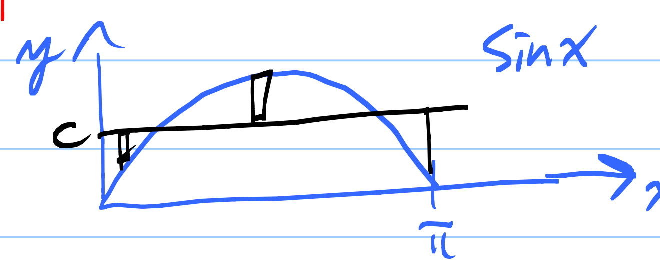
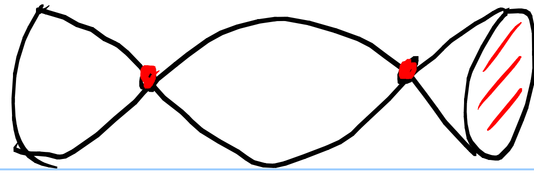
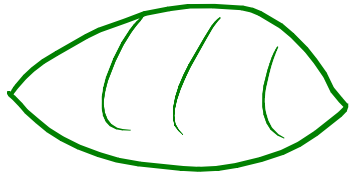






$c = 0$

$$dV = \pi \underbrace{\left[\sin x - c\right]^2}_{= \left[c - \sin x\right]^2} dx$$

Great! Don't have to cut up $\int$.

$$V = \int_0^{\pi} \pi \left(\sin x - c\right)^2 dx$$

$$= \pi \int_0^{\pi} \sin^2 x - 2c \sin x + c^2 \, dx$$

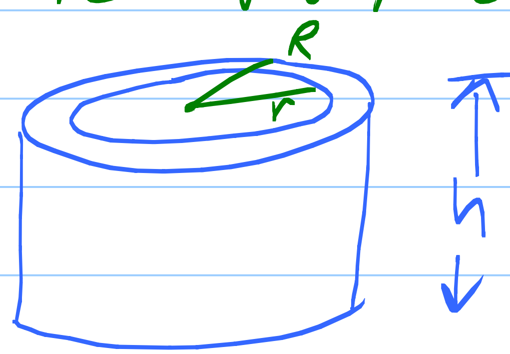
$\sin^2 x = \frac{1}{2}\left(1 - \cos 2x\right)$

$$= A - Bc + Dc^2 = V(c)$$

Check endpoints $c=0$, $c=1$.

Critical point where $V'(c) = 0$.

Cylindrical Shells:

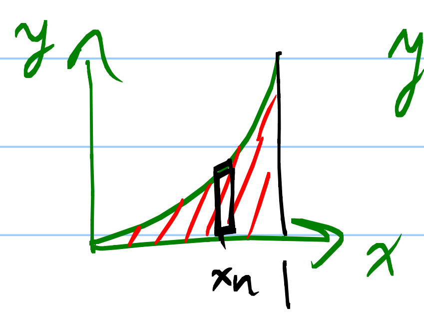


$$\Delta V = \pi R^2 h - \pi r^2 h = \pi (R^2 - r^2) h$$

$$= 2\pi \underbrace{\frac{(R+r)}{2}}\underbrace{(R-r)}_{\Delta r} h \qquad \frac{R+r}{2} = \bar{r} \leftarrow \text{ave or midpt.}$$

$$\boxed{\Delta V = (2\pi \bar{r}) h \, \Delta r}$$

EX: Spin about y-axis.



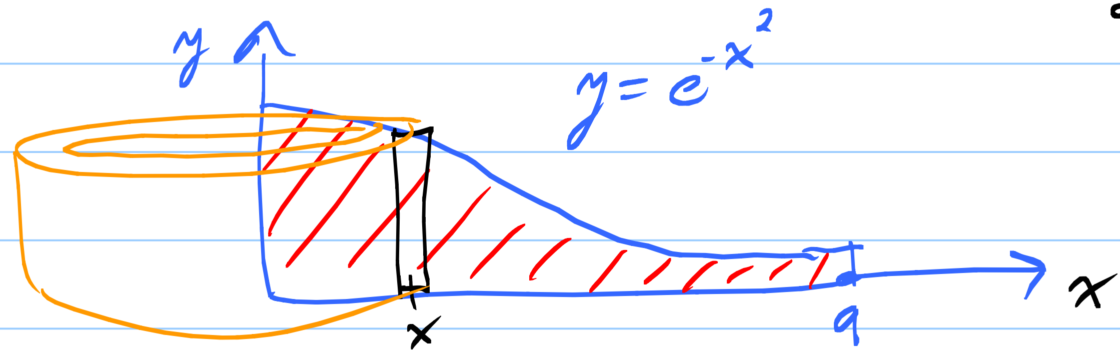
Take x_n = midpts.

$$V = \lim_{N \to \infty} \sum \underbrace{(2\pi x_n)}_{\bar{r}} \underbrace{[x_n^2]}_{h} \underbrace{\Delta x}_{\text{thickness}}$$

$$= \int_0^1 2\pi x \cdot x^2 \, dx = 2\pi \int_0^1 x^3 \, dx = 2\pi \tfrac{1}{4} x^4 \Big|_0^1$$

$$= \frac{\pi}{2}.$$

Problem:



Spin around y-axis.

$$V = \int_0^a (2\pi x)\, e^{-x^2}\, dx$$

$u = -x^2$

$du = -2x\, dx$

$\boxed{2x\, dx = -du}$

When $x=0$: $u = -0^2 = 0$
when $x=a$: $u = -a^2$

$$V = \int_{x=0}^{a} \pi\, e^{-x^2}\, (2x\, dx)$$

$$= \int_{u=0}^{-a^2} \pi\, e^{u}\, (-du)$$

$$= \pi \int_{-a^2}^{0} e^{u}\, du = \pi \left[e^{u}\right]_{-a^2}^{0}$$

$$= \pi\left[e^{0} - e^{-a^2}\right] = \pi\left(1 - e^{-a^2}\right).$$

Hmmm. Let $a \to \infty$. $V = \pi$ in limit.
A solid that holds π gallons of paint but that has infinite surface area and so cannot be painted!