5.5: 36.

$$I=\int \frac{\cos\sqrt{\theta}}{\sqrt{\theta}\,\sin^2\sqrt{\theta}}\,d\theta$$

Two steps: $u=\sqrt{\theta}$ $\qquad du=\frac{1}{2}\theta^{-1/2}d\theta=\frac{1}{2\sqrt{\theta}}\,d\theta$

$$I=\int \frac{\cos\underbrace{\sqrt{\theta}}_{u}}{\sin^2\underbrace{\sqrt{\theta}}_{u}}\underbrace{\left(\frac{1}{\sqrt{\theta}}\,d\theta\right)}_{2\,du}$$

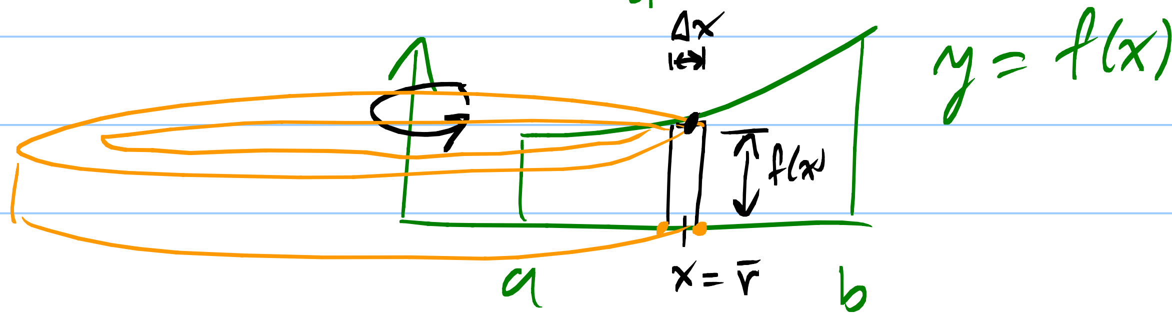
$$=\int \frac{\cos u}{\underbrace{\sin^2 u}_{v^2}}\,du \quad dv$$

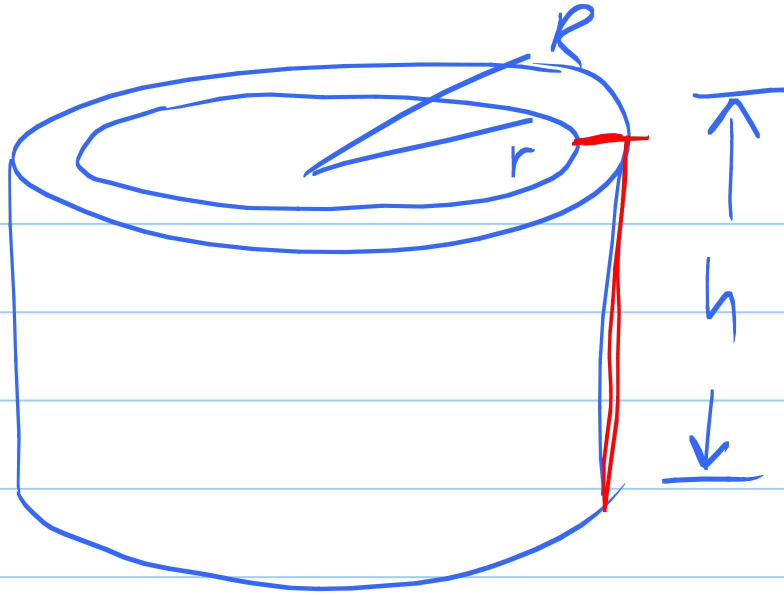
Next, Let $v=\sin u$.

One step: Let $u=\sin\sqrt{\theta}$.

<u>Cylindrical shells</u>: How to remember the formula:

$$V=\int_a^b (2\pi x)\,f(x)\,dx$$





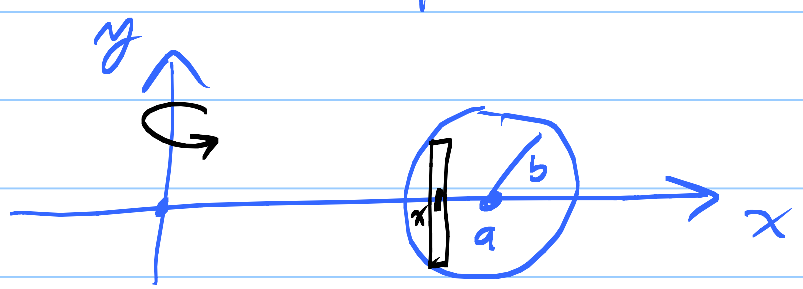
$h = f(x)$

$\Delta r = \Delta x$

$\bar{r} = x$

$$\Delta V = (2\pi \bar{r})\, h\, \Delta r$$

Bagel problem:



Think:

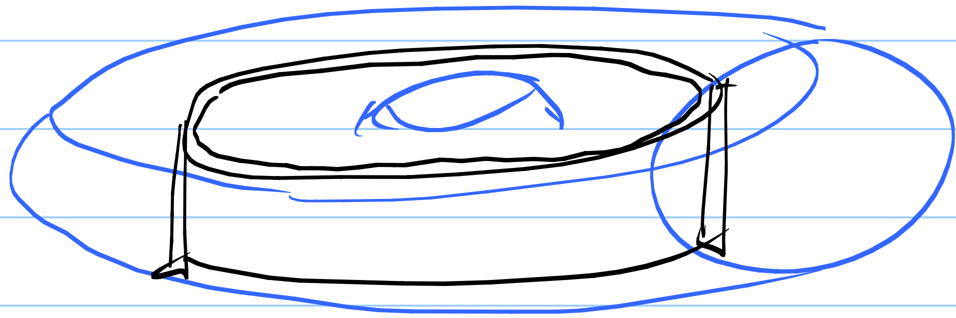
$$V = \int_{a-b}^{a+b} dV$$

$$= \int_{a-b}^{a+b} (2\pi x)\, 2\sqrt{b^2 - (x-a)^2}\, dx$$

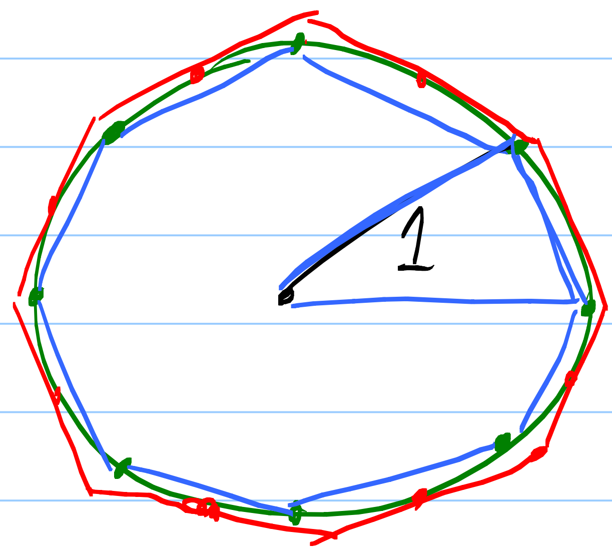
$$(x-a)^2 + y^2 = b^2$$

$$y = \pm\sqrt{b^2 - (x-a)^2}$$

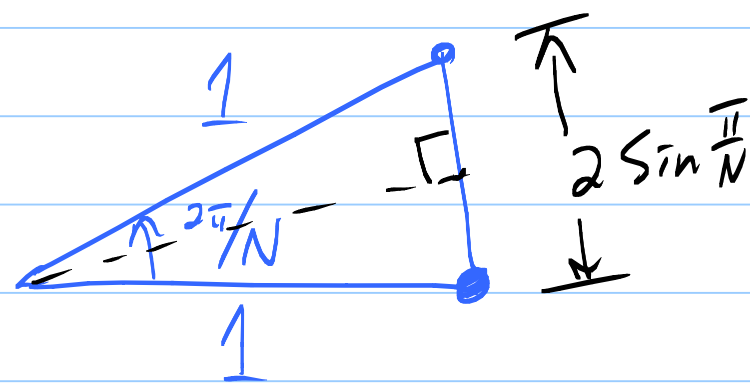
Ugh! This is harder than the washer method!



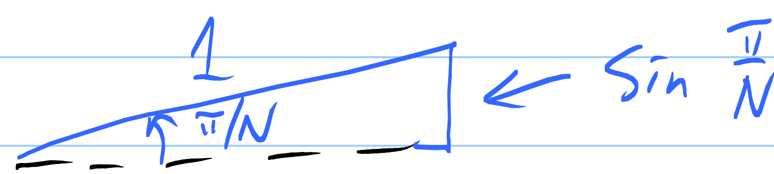
Arc Length:



Inner Δ:



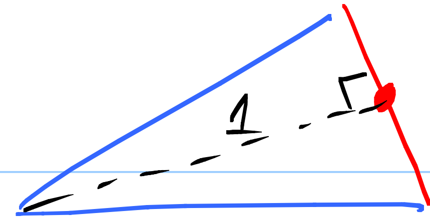
$\frac{1}{2}\Delta$:



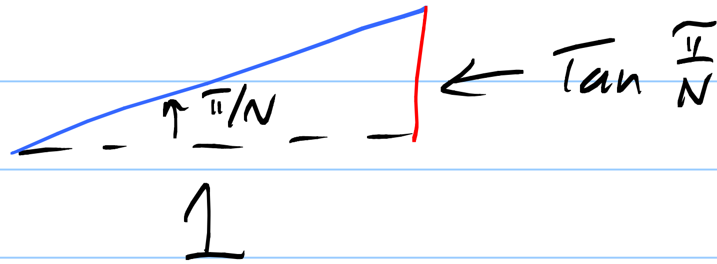
$$\text{Total length} = N\,2\sin\frac{\pi}{N} = 2\pi\,\frac{\sin\frac{\pi}{N}}{\frac{\pi}{N}}$$

$$\longrightarrow 2\pi\cdot 1 \text{ as } N\to\infty.$$

Outside Polygon:



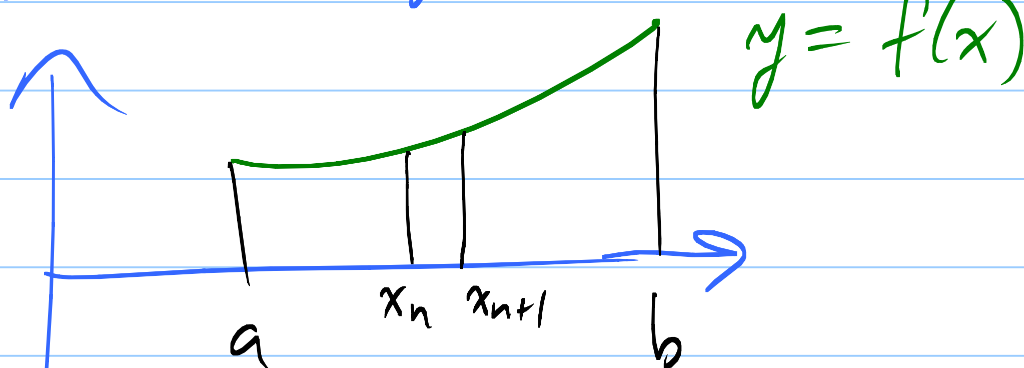
$\frac{1}{2}\Delta$:

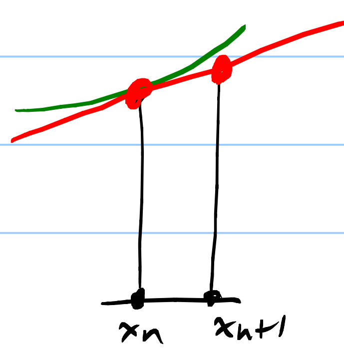


$$\text{Total length} = N\left(2\tan\frac{\pi}{N}\right) = \frac{2\tan\frac{\pi}{N}}{\left(\frac{1}{N}\right)} \quad \frac{0}{0}$$

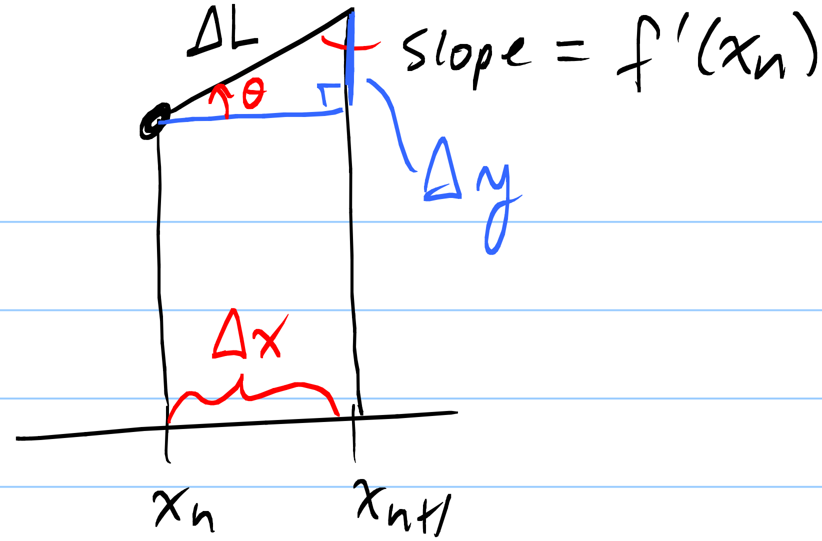
$$\xrightarrow{L'H} \lim_{N\to\infty} \frac{2\left(\sec^2\frac{\pi}{N}\right)\left(\frac{-\pi}{N^2}\right)}{\left(\frac{-1}{N^2}\right)} = 2\pi\sec^2 0 = 2\pi$$

Problem: Find the length of a curve described as the graph of $y = f(x)$.





Tangent vecter at $(x_n, f(x_n))$.



$$\text{Slope} = f'(x_n) = \operatorname{Tan}\theta = \frac{\Delta y}{\Delta x}$$

So $\boxed{\Delta y = f'(x_n)\,\Delta x}$

Now $\Delta L = \sqrt{\Delta x^2 + \Delta y^2}$

$$= \sqrt{(\Delta x)^2 + (f'(x_n)\,\Delta x)^2}$$

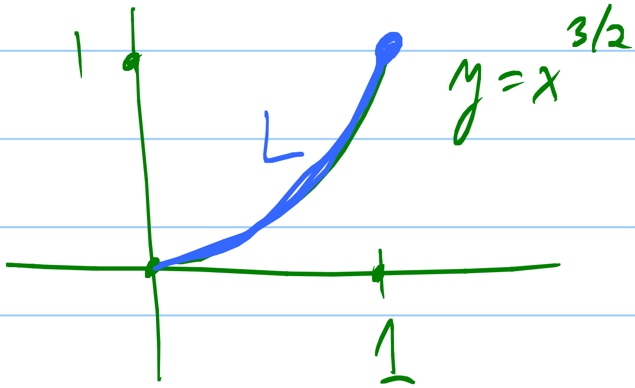
$$\Delta L = \sqrt{1 + f'(x_n)^2}\;\Delta x$$

Finally, $L = \lim\limits_{N \to \infty} \sum\limits_{n=1}^{N} \sqrt{1 + f'(x_n)^2}\;\Delta x$

$$= \int_a^b \sqrt{1 + f'(x)^2}\; dx$$

or $$= \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Ex: $y = x^{3/2}$. $\quad \frac{dy}{dx} = \frac{3}{2}x^{1/2}$



$$L = \int_0^1 \sqrt{1 + \left(\tfrac{3}{2}x^{1/2}\right)^2}\, dx$$

$$= \int_0^1 \sqrt{\underbrace{1 + \tfrac{9}{4}x}_{u}}\ \underbrace{dx}_{\frac{4}{9}du}$$

$u = 1 + \frac{9}{4}x$

$du = \frac{9}{4}dx$

When $x = 0$: $\quad u = 1 + \frac{9}{4}\cdot 0 = 1$

When $x = 1$: $\quad u = 1 + \frac{9}{4}\cdot 1 = \frac{13}{4}$.

So $$L = \int_{u=1}^{13/4} \sqrt{u}\ \frac{4}{9}\, du = \frac{4}{9}\int_1^{13/4} u^{1/2}\, du$$

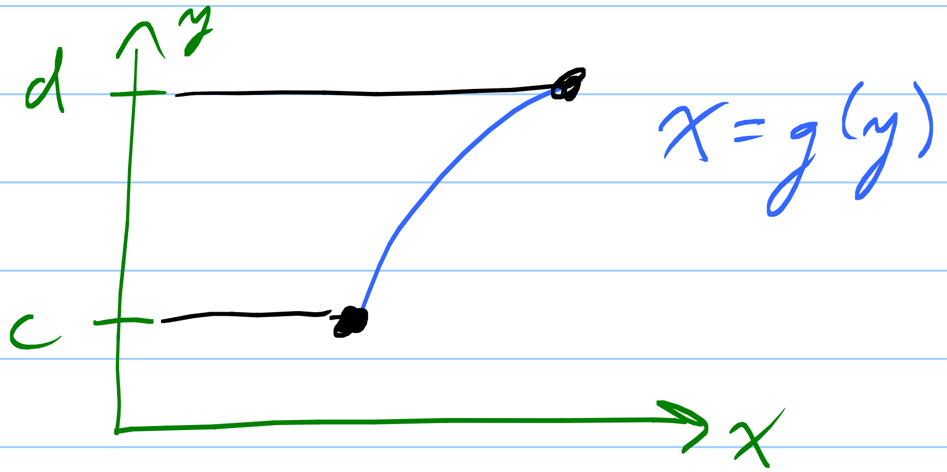
$$= \frac{4}{9}\left[\frac{2}{3}u^{3/2}\right]_1^{13/4} = \frac{4}{9}\cdot\frac{2}{3}\left[\left(\frac{13}{4}\right)^{3/2} - 1\right]$$

EX: Try this for $y = x^{2/3}$.

$\frac{dy}{dx} = \frac{2}{3} x^{-1/3} \leftarrow$ Ouch! Vertical tangent at $x = 0$.

But $x = y^{3/2}$ is the same curve sideways as the one above.

Fact:



$$L = \int_c^d \sqrt{1 + g'(y)^2}\, dy$$

That's how to do $y = x^{2/3}$. $x = y^{3/2}$.