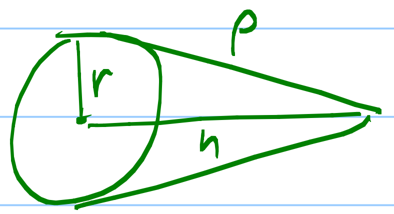
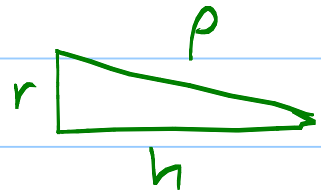
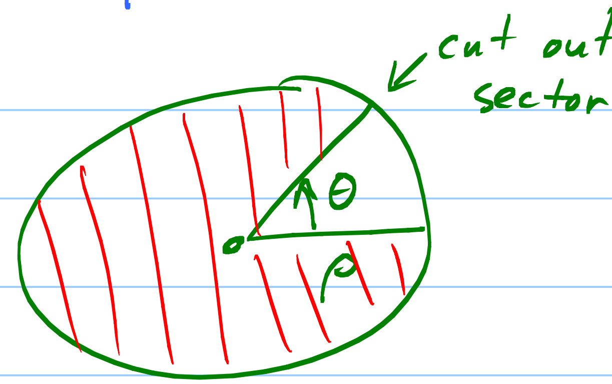
Exam 1 in class Tuesday, Oct 4

Surface Area: Cone



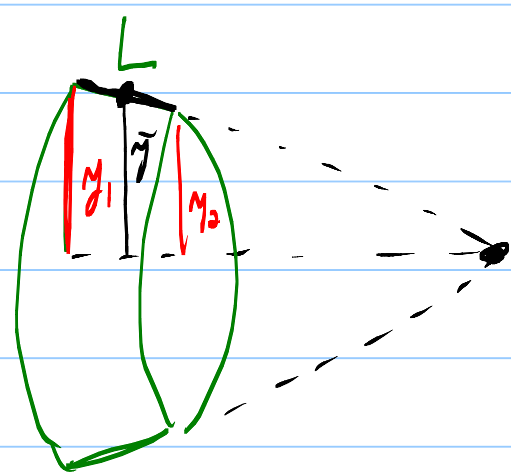




$$\text{Area} = \pi\rho^2 - \tfrac{1}{2}\theta\rho^2$$

$$2\pi r = (2\pi - \theta)\rho$$

Frustrum of a cone

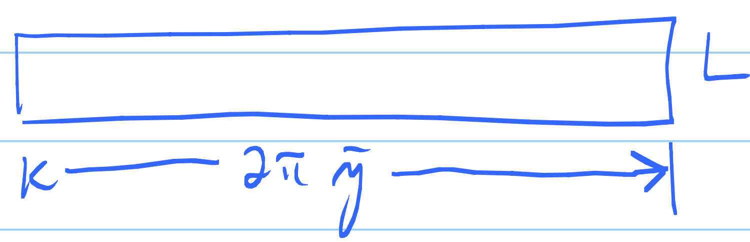


Surface area of the band:

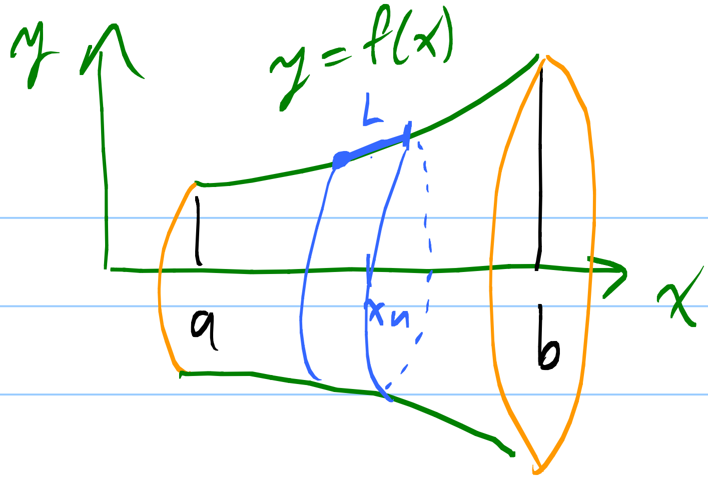
$$= 2\pi\underbrace{\left(\frac{y_1+y_2}{2}\right)}_{\bar{y}} L$$

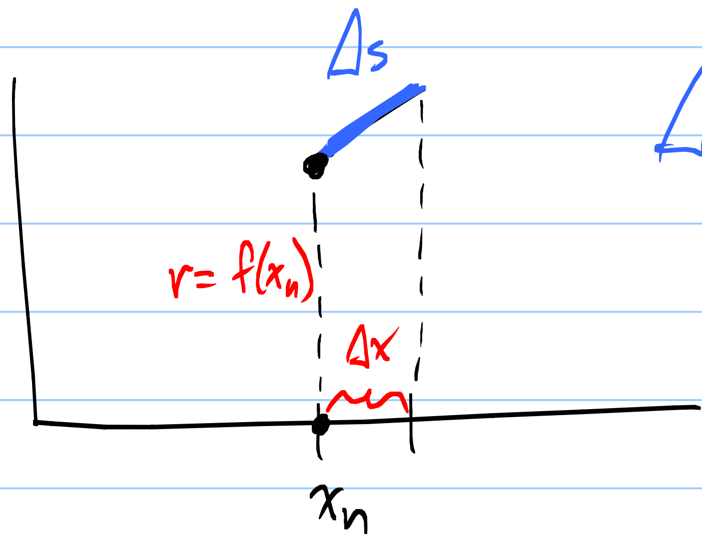
$$= 2\pi\,\bar{y}\,L$$

How to remember: Cut and unroll:



Problem: Find the surface area of a surface of revolution.





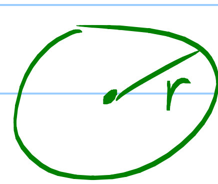
$$\Delta s = \sqrt{1 + f'(x_n)^2}\,\Delta x$$

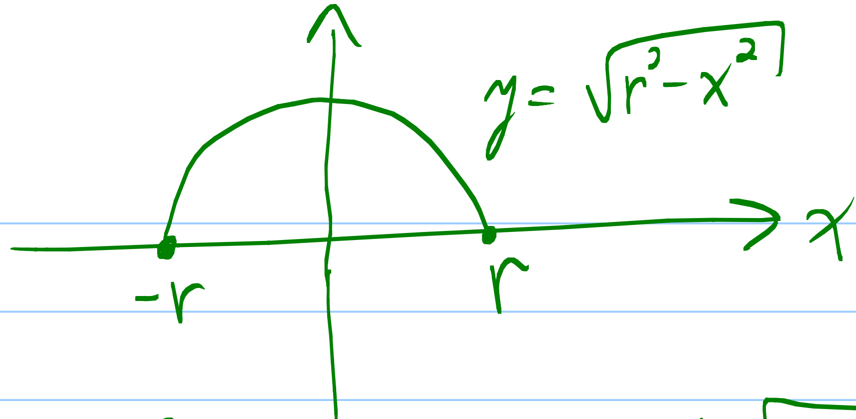
$$\Delta A = \text{area of ribbon} = \left(2\pi f(x_n)\right)\cdot\sqrt{1 + f'(x_n)^2}\,\Delta x$$

$$A = \lim_{N\to\infty} \sum_{n=1}^{N} \left(2\pi f(x_n)\right)\cdot\sqrt{1 + f'(x_n)^2}\,\Delta x$$

$$= \int_a^b 2\pi \underset{\substack{\uparrow \\ y}}{f(x)} \sqrt{1 + \underset{\substack{\uparrow \\ dy/dx}}{f'(x)}^2}\,dx$$

<u>Problem</u>: Find the surface area of a sphere.



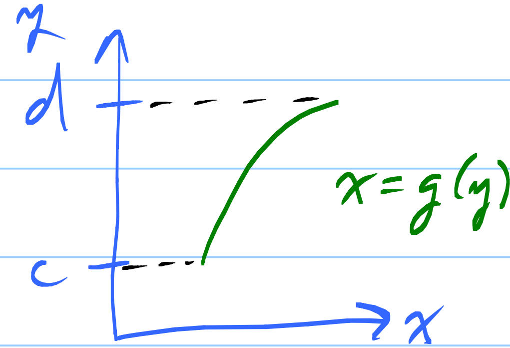


$$A = \int_{-r}^{r} \left(2\pi \sqrt{r^2 - x^2}\right) \sqrt{1 + \left[\tfrac{1}{2}(r^2-x^2)^{-1/2}(-2x)\right]^2}\, dx$$

$$= 2\pi \int_{-r}^{r} \underbrace{\sqrt{r^2-x^2} \sqrt{1 + \frac{x^2}{r^2-x^2}}}_{\sqrt{(r^2-x^2)+x^2} = r} \, dx$$

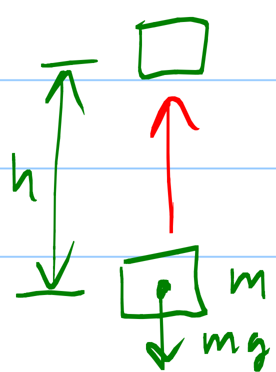
$$= 2\pi \int_{-r}^{r} r \, dx = 2\pi r \left[x\right]_{-r}^{r} = 4\pi r^2$$

Can revolve around y-axis:



$$A = \int_{c}^{d} 2\pi\, g(y) \sqrt{1 + g'(y)^2}\, dy$$

Work:



$$\text{Work} = Fd = mgh$$

Newton's 2nd Law: $F = ma$

Integrate with respect to distance x.

Use chain rule: $a = \dfrac{dv}{dt} = \dfrac{dv}{dx} \cdot \underbrace{\dfrac{dx}{dt}}_{v} = v\dfrac{dv}{dx}$

$$\int F\,dx = \int m a\,dx = \int m v \frac{dv}{dx}\,dx$$

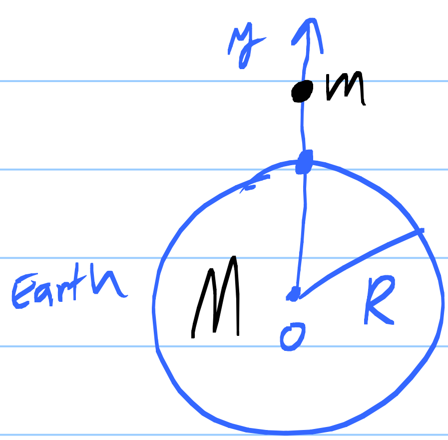
$$= \int m v\,dv$$

$$= \tfrac{1}{2} m v^2 + C$$

$$\underbrace{\int_{x_1}^{x_2} F\,dx}_{\text{Work}} = \underbrace{\tfrac{1}{2} m v^2 \Big|_{x_1}^{x_2}}_{\Delta K.E.}$$

Δ K.E. ← Increase in Kinetic Energy

<u>Problem</u>: Blast a cannonball straight up from the North Pole.



$$F = \frac{-GMm}{y^2}$$

Blast velocity V_0

At top y_{max}, $V_{Top} = 0$.

$$\int_R^{y_{max}} -\frac{GMm}{y^2}\,dy = \frac{1}{2}m\left(\underbrace{v_{Top}^2}_{0} - v_0^2\right)$$

$$\left[\frac{GMm}{y}\right]_R^{y_{max}} = -\frac{1}{2}mv_0^2$$

$$\frac{GMm}{y_{max}} - \frac{GMm}{R} = -\frac{1}{2}mv_0^2$$

$$\frac{1}{y_{max}} - \frac{1}{R} = -\frac{v_0^2}{2GM}$$

$$\frac{1}{y_{max}} = \frac{1}{R} - \frac{v_0^2}{2GM} = \frac{2GM - Rv_0^2}{2RGM}$$

$$y_{max} = \frac{2RGM}{2GM - Rv_0^2}$$

What? If $Rv_0^2 = 2GM$

$$v_0 = \sqrt{\frac{2GM}{R}}$$

the ball never stops. $y_{max} \to \infty$ as $v_0 \to \sqrt{\frac{2GM}{R}}$.

Problem: Find $y(t)$ if $v_0 = \sqrt{\frac{2GM}{R}}$.

Work-Energy Integral

$$\frac{1}{y} - \frac{1}{R} = \frac{1}{2} m v^2 - \frac{1}{2} m \underbrace{\left(\frac{2GM}{R}\right)}_{v_0^2}$$

Solve for $\underbrace{v}_{\frac{dy}{dt}} =$ function of $y = g(y)$

Separable ODE: $\frac{dy}{dt} = g(y)$

$$\frac{1}{g(y)}\, dy = dt$$

$$\int \frac{1}{g(y)}\, dy = t + C$$

Get C from $v = v_0$ at $t = t_0$, $y(0) = R$.

Solve for y.