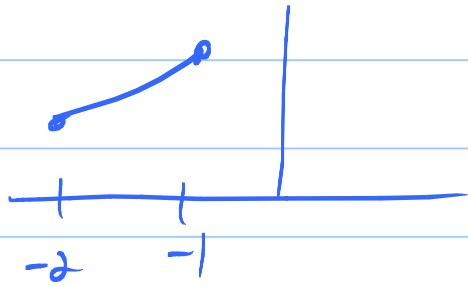
386: 10. $y=\int_{-2}^{x}\sqrt{3t^4-1}\,dt \qquad -2\le x\le -1$



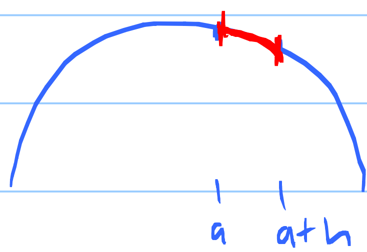
$$L=\int_{-2}^{-1}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x).$$

6.3: 24. $y=\sqrt{r^2-x^2}$

$$L = 2\int_{-r}^{r}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

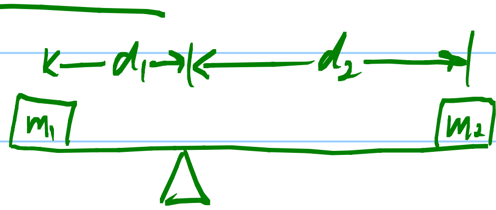
6.4: 28.



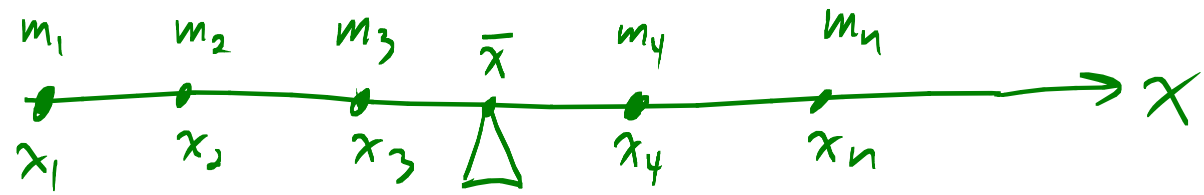
$$S=\int_a^{a+h}2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$y=\sqrt{r^2-x^2}$

Centers of Mass:



Balanced if $m_1 d_1 = m_2 d_2$



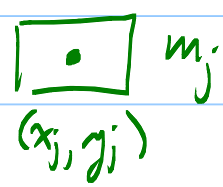
Sum of moments = 0 at balance point.

$$\sum_{j=1}^{n} m_j (x_j - \bar{x}) = 0$$

$$\sum_{j=1}^{n} m_j x_j - \sum m_j \bar{x} = 0$$

$$\bar{x} = \frac{\sum m_j x_j}{\sum m_j}$$ ← Moment about origin ← Total mass.

Repeat for masses distributed in a plane.

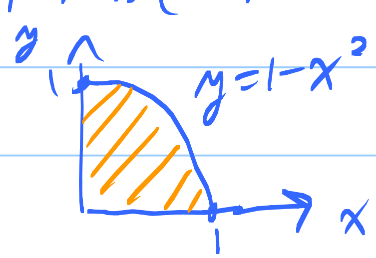


← bunch of these all over the place.

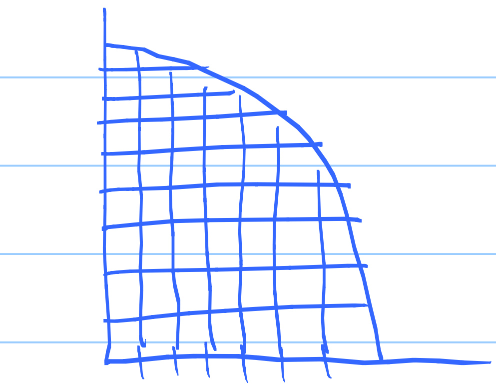
$$\bar{x} = \frac{\sum m_j x_j}{\sum m_j} = \frac{M_y}{M}$$ ← Moment about y-axis ← Total mass.

$$\bar{y} = \frac{\sum m_j y_j}{\sum m_j} = \frac{M_x}{M}$$ ← Moment about x-axis.

Problem: Find the center of mass of a uniform plate

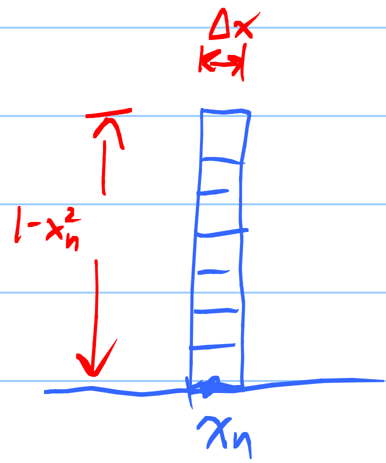


Cut into tiny rectangles.



$$\bar{x} = \frac{\sum m_i x_i}{\sum m_i}$$

First sum vertical stacks, then add the stacks.



$$m_1 x_n + m_2 x_n + \cdots + m_m x_n$$

$$= \underbrace{(m_1 + \cdots + m_m)}_{\text{mass of stack}} x_n$$

mass of stack = density × area

$$= \rho\,(1 - x_n^2)\,\Delta x$$

For the stack: $\sum m_j x_n = \rho(1-x_n^2)\,\Delta x \cdot x_n$

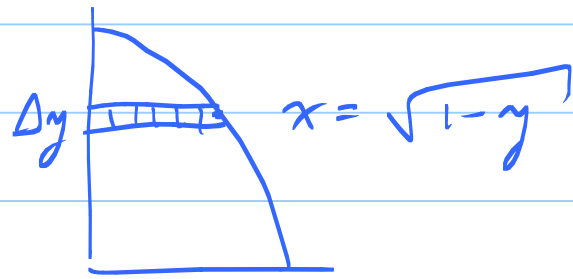
Add up stacks: $\displaystyle\sum_{n=1}^{N} \rho\, x_n (1 - x_n^2)\,\Delta x$

$$\longrightarrow \rho \int_0^1 x(1-x^2)\,dx$$

Total mass $= \rho \cdot \text{Area} = \rho \int_0^1 (1-x^2)\,dx$

So $\displaystyle \bar{x} = \frac{\rho \int_0^1 x(1-x^2)\,dx}{\rho \int_0^1 (1-x^2)\,dx} = \frac{\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1}{\left[x - \frac{x^3}{3}\right]_0^1} = \frac{3}{8}$

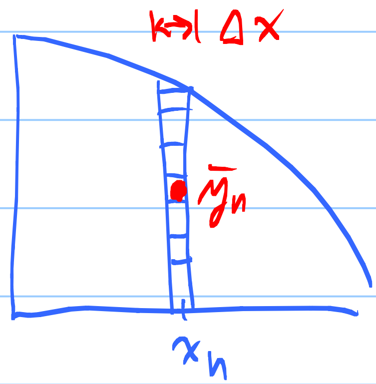
Next, get $\bar{y}$. Could start over.



$$y = 1 - x^2$$
$$x^2 = 1 - y$$

First add horizontally.
Then add stacks.

Easier way that keeps dx integral. (like book)



For vertical stack:

$$m_1 y_1 + m_2 y_2 + \cdots + m_k y_k$$

Multiply and divide by the mass $\underbrace{m_1 + \cdots + m_k}_{M}$ of stack:

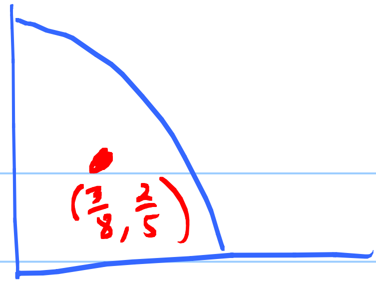
$$M \cdot \frac{m_1 y_1 + \cdots + m_k y_k}{M} = M \bar{y}_n = \underset{\uparrow \, \rho (1 - x_n^2) \Delta x}{M} \cdot \frac{1 - x_n^2}{2}$$

Now add up stacks:

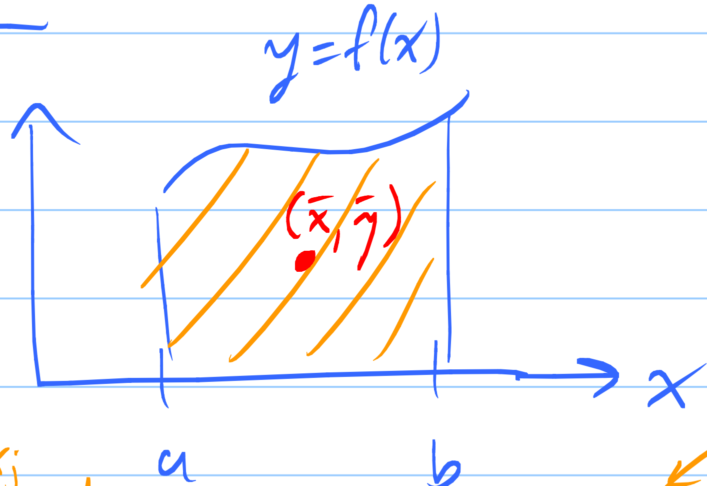
$$\sum m_j y_j = \sum \rho (1 - x_n^2) \Delta x \cdot \frac{1 - x_n^2}{2}$$

$$\to \rho \int_0^1 \frac{1 - x^2}{2} \cdot (1 - x^2) \, dx = \rho \frac{4}{15}$$

Finally, $\bar{y} = \dfrac{\sum m_j y_j}{M} = \dfrac{\rho \frac{4}{15}}{\rho \cdot \frac{2}{3}} = \dfrac{2}{5}$

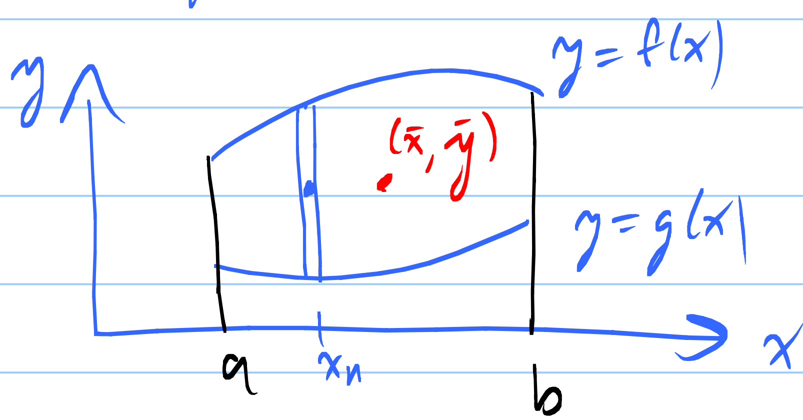


Summary:



$$\bar{x} = \frac{\cancel{\rho}\int_a^b \overset{x_j}{x}\, \overbrace{f(x)\,dx}^{dm_j}}{\cancel{\rho}\int_a^b f(x)\,dx} \qquad \bar{y} = \frac{\int_a^b \overset{\bar{y}_j}{\frac{f(x)}{2}} \cdot \overbrace{f(x)\,dx}^{dm_j}}{\int_a^b f(x)\,dx}$$

When $\rho = \text{const.}$, this is also called the Centroid.

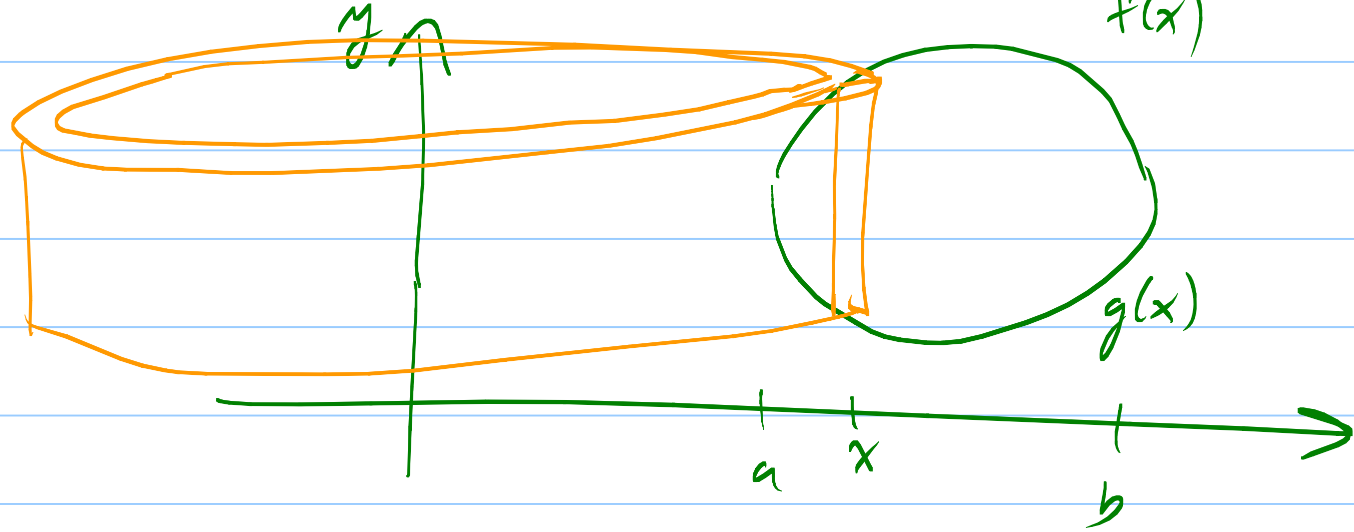


$\bar{y}_n$

y coord of centroid of strip

$$\bar{x} = \frac{\int_a^b \underset{x_j}{x}\, \underbrace{[f(x)-g(x)]\,dx}_{dm_j}}{\text{Area}} \qquad \bar{y} = \frac{\int_a^b \frac{f(x)+g(x)}{2} \cdot \underbrace{[f(x)-g(x)]\,dx}_{dm_j}}{\text{Area}}$$

Theorem of Pappus:

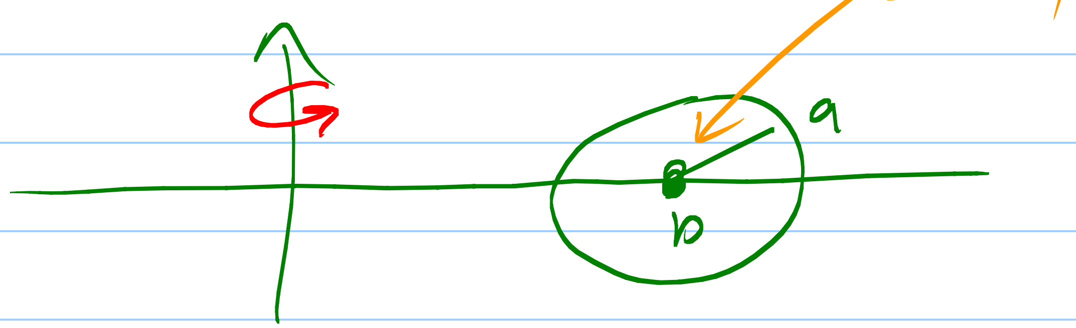


$$V = \int_a^b 2\pi x \, [f(x) - g(x)] \, dx$$

$$= 2\pi A \underbrace{\frac{\int_a^b x \, [f(x) - g(x)] \, dx}{A}}_{\bar{x}}$$

$$V = (2\pi \bar{x}) A$$

Saw this for a bagel.



$$V = (2\pi b)(\pi a^2)$$