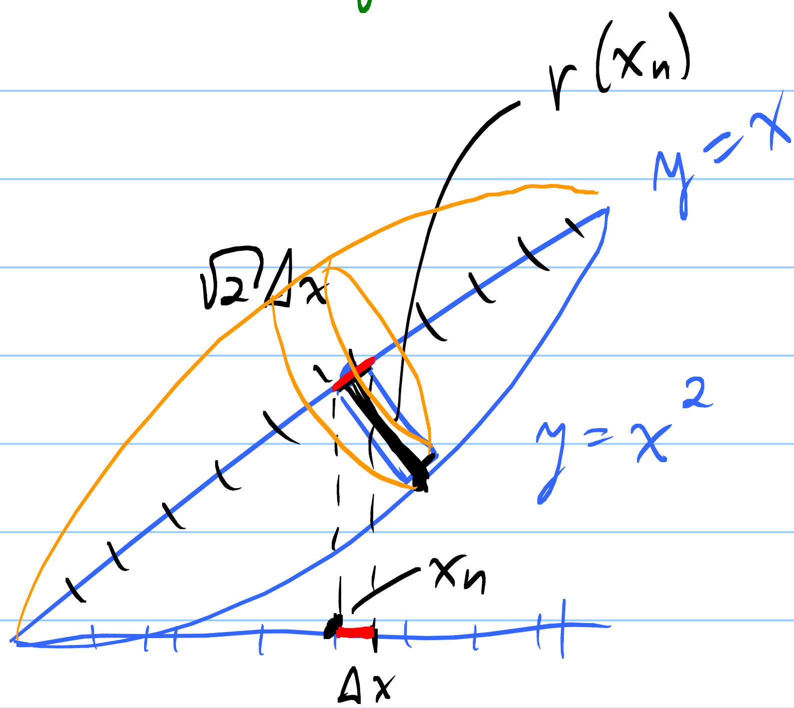
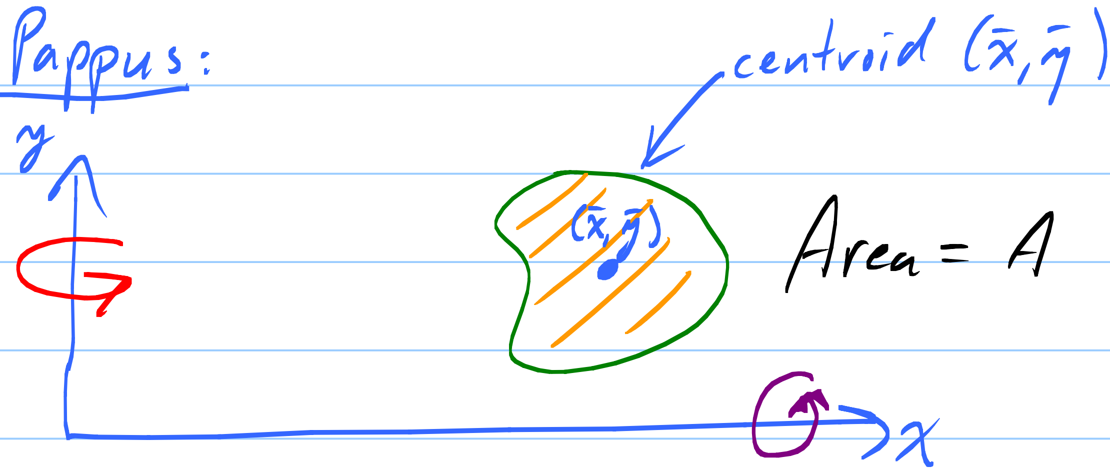
How to rotate about $y=x$?



$$\text{Volume} = \int_0^1 \pi \left[r(x)^2 \right] \cdot \sqrt{2}\, dx$$

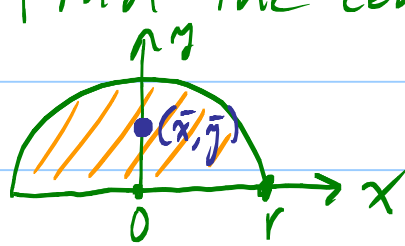
Thms of Pappus:

$2\pi\bar{x}A =$ volume



$2\pi\bar{y}\,A = \text{Volume}$

Problem: Find the centroid of a half circle.



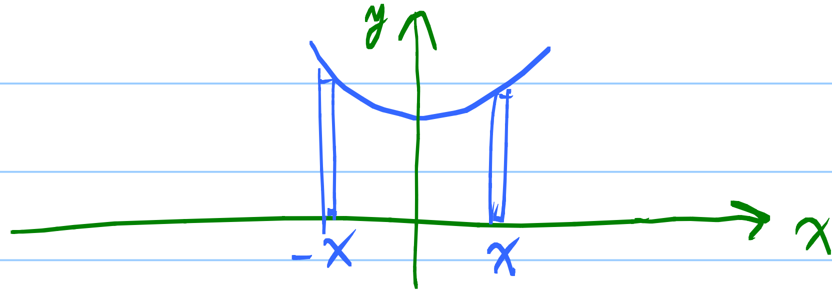
Volume of Sphere $= 2\pi\bar{y}\,(\text{Area of }\frown)$

$$\frac{4}{3}\pi r^3 = 2\pi\bar{y}\left(\frac{1}{2}\pi r^2\right)$$

$$\bar{y} = \frac{4r}{3\pi}$$

Clear that $\bar{x} = 0$.

<u>Fact</u>: If a figure is symmetric about a line, the centroid falls on the line.

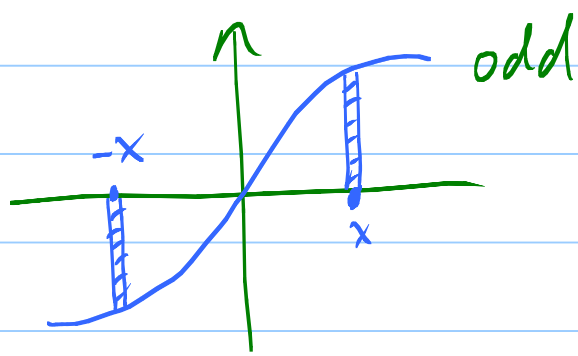


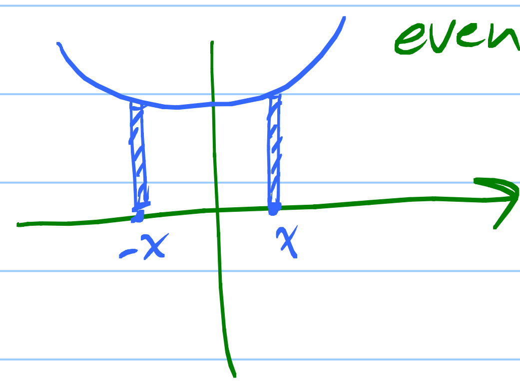
$$-x\, f(-x)\, dx \quad \text{cancels} \quad x\, f(x)\, dx$$

<u>Fact</u>: f odd $\quad f(-x) = -f(x) \quad \leftarrow \sin x$

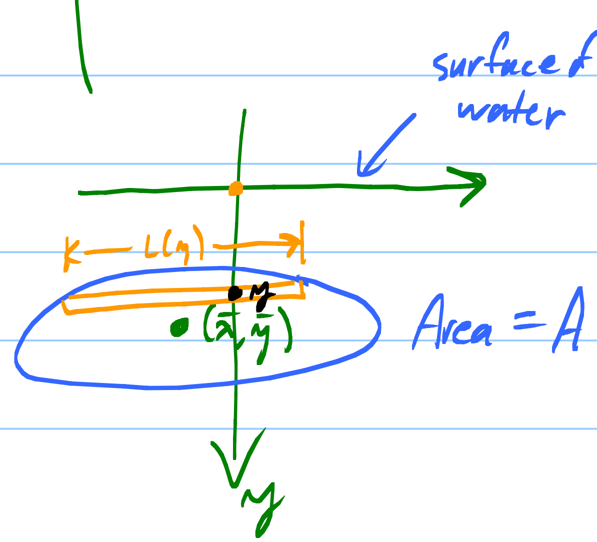
f even $\quad f(-x) = f(x) \quad \leftarrow \cos x$

$$\int_{-a}^{a} f(x)\, dx = \begin{cases} 0 & f \text{ odd} \\ 2\int_0^a f(x)\, dx & f \text{ even} \end{cases}$$





<u>Fluid forces and centroids:</u>



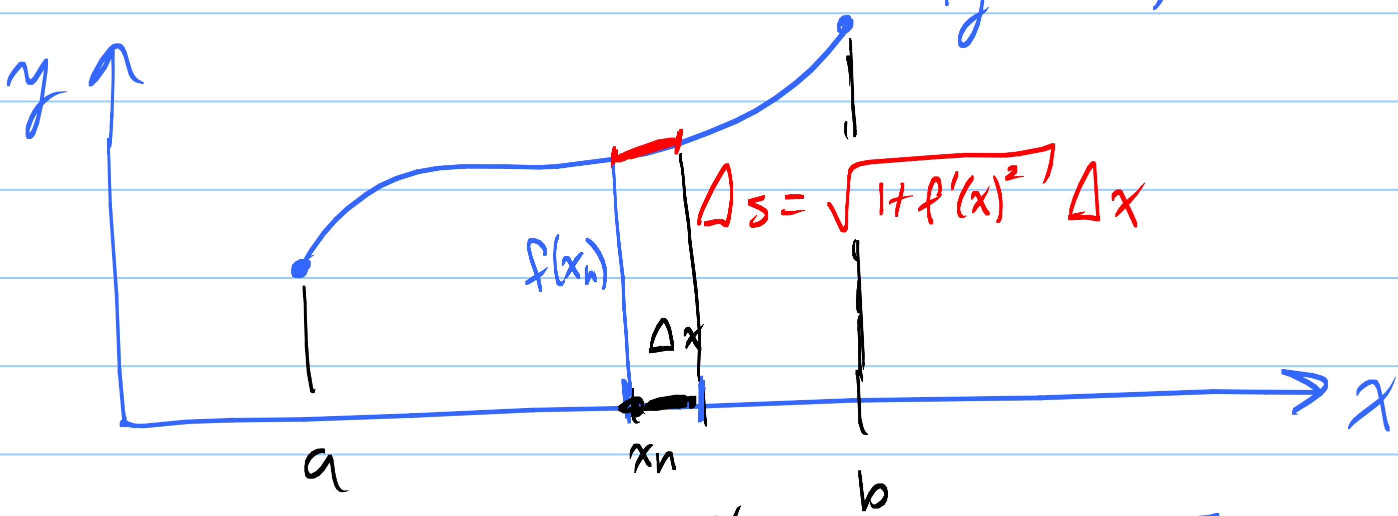
Fluid force on area $\approx \sum \underset{\uparrow \text{ wt. density}}{w} \; \underset{\uparrow \text{ depth of rectangle}}{y} \; \underbrace{L(y)\,\Delta y}_{\text{area of rect.}}$

$$\to \int_a^b w\,y\,L(y)\,dy \cdot A \qquad A = \int_a^b L(y)\,dy$$

$$\underbrace{\frac{\int_a^b w\,y\,L(y)\,dy}{A} \cdot A}_{w\,\bar{y}}$$

$$\boxed{F = w\,\bar{y}\,A}$$

Center of mass of a wire:



$$\bar{x} \approx \frac{\sum m_n x_n}{\sum m_n} \approx \frac{\sum_{n=1}^{N} x_n \left(\rho\sqrt{1+f'(x_n)^2}\,\Delta x\right)}{\sum_{n=1}^{N} \rho\sqrt{1+f'(x_n)^2}\,\Delta x}$$

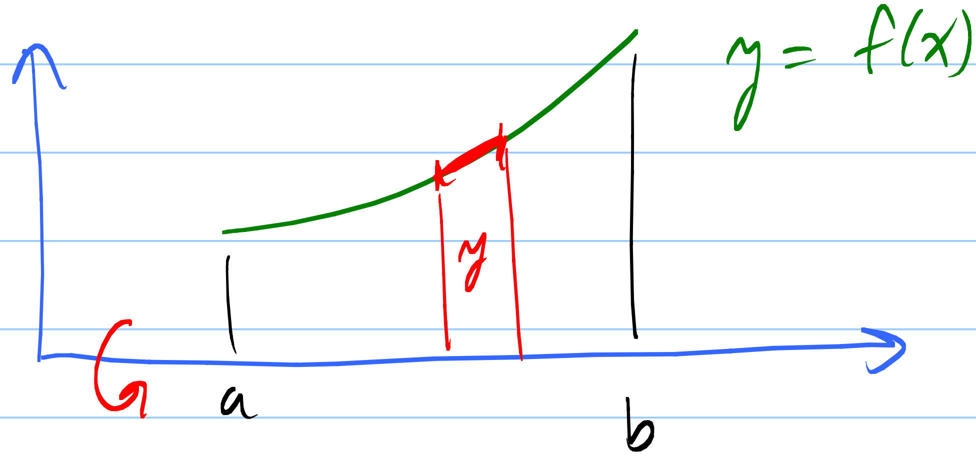
$$\bar{x} = \frac{\rho \int_a^b x\sqrt{1+f'(x)^2}\,dx}{\rho\, L}$$

where $L = \int_a^b \sqrt{1 + f'(x)^2}\, dx$

$$\bar{y} = \frac{\rho \int_a^b f(x) \sqrt{1 + f'(x)^2}\, dx}{\rho L}$$

Note, when the density is constant, it cancels out.

The second Theorem of Pappus:

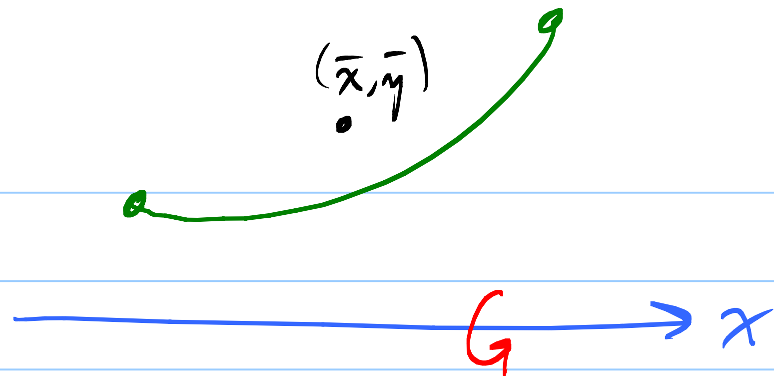


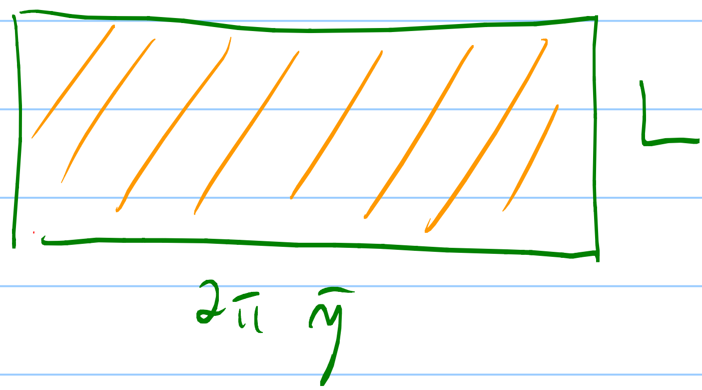
$$2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \text{Area of ribbon.}$$

$$\text{Surface Area} = S = \int_a^b 2\pi f(x) \sqrt{1 + f'(x)^2}\, dx$$

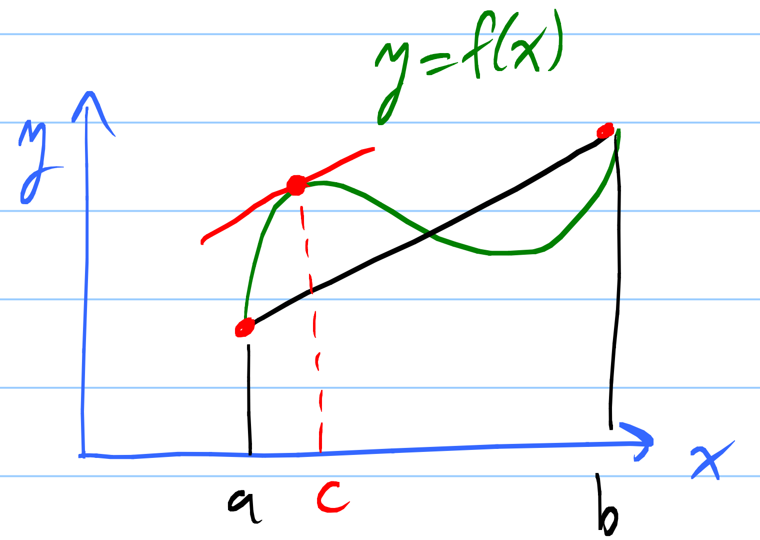
$$= 2\pi \underbrace{\frac{\int_a^b f(x) \sqrt{1 + f'(x)^2}\, dx}{L}}_{\bar{y}} \cdot L$$

$$S = 2\pi \bar{y} \cdot L$$





Mean Value Theorem I:



If f is continuous on $a \leq x \leq b$ and differentiable on $a < x < b$, then there is a value c with $a < c < b$ where

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Mean Value Theorem II: If f is continuous on $a \leq x \leq b$, then there is a c with $a < c < b$

such that $\int_a^b f(x)\,dx = f(c)(b-a)$.

Error estimate for the Rectangle Rule:

$$I = \int_a^b f(x)\,dx$$

$$RS = \sum_{n=1}^{N} f(x_n)\,\Delta x$$

$$I = \int_a^b f(x)\,dx = \int_{a=x_0}^{x_1} f(x)\,dx + \int_{x_1}^{x_2} f(x)\,dx + \cdots + \int_{x_{N-1}}^{x_N} f(x)\,dx$$

MVT II

$$= f(c_1)\,\Delta x + f(c_2)\,\Delta x + \cdots + f(c_N)\,\Delta x$$

Now $\displaystyle I - RS = \sum_{n=1}^{N} \left[f(c_n) - f(x_n) \right] \Delta x$

MVT I : $\displaystyle \frac{f(c_n) - f(x_n)}{c_n - x_n} = f'(e_n)$

where e_n between c_n and x_n.

Now $\left| f(c_n) - f(x_n) \right| = \left| f'(e_n)\,(c_n - x_n) \right|$

$$\leq \max_{[a,b]} |f'| \cdot \Delta x \leftarrow \frac{b-a}{N}$$

Finally, $|I - Rs| \leq \sum_{n=1}^{N} |f(c_n) - f(x_n)| \cdot \Delta x$

$$\leq \sum_{n=1}^{N} \left(M \cdot \frac{b-a}{N} \right) \cdot \Delta x$$

$$M \frac{b-a}{N} \underbrace{\left(\sum_{n=1}^{N} \Delta x \right)}_{b-a}$$

$$|I - Rs| \leq \frac{M(b-a)^2}{N} \quad \text{where } M = \underset{[a,b]}{\mathrm{Max}} |f'|$$