HWK 6 on Home Page. Due Thurs. after Exam 1.
2 old Exam 1's on Home Page.

Population Growth: $\frac{dP}{dt} = kP$

$$\boxed{P(t) = P_0 e^{kt}} \quad \text{where} \quad P_0 = P(0).$$

Doubling time: When is $P(t) = 2P_0$? When is

$$2P_0 = P_0 e^{kt} \; ?$$

$$2 = e^{kt}$$

$$\ln 2 = \ln e^{kt} = kt$$

$$\boxed{t_D = \frac{\ln 2}{k}}$$

↑ Doubling time.

Radioactive Decay:

$$\frac{dR}{dt} = -kR \qquad R(0) = R_0.$$

$$R(t) = R_0 e^{-kt}$$

Half life:

$$\boxed{t_H = \frac{\ln 2}{k}}$$

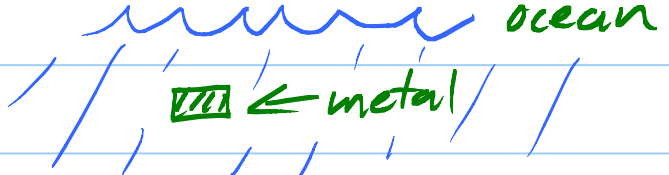
$$\tfrac{1}{2} R_0 = R_0 e^{-kt}$$

$$\ln \tfrac{1}{2} = \ln e^{-kt} = -kt$$

$\ln \tfrac{1}{2}$: $\ln 2^{-1} = -\ln 2$

Newton's Law of Cooling:

$T(t)$ = Temp of metal at time t



T_a = ambient temp of ocean.

$$\frac{dT}{dt} = -k(T - T_a)$$ Separable ODE!

Trick: Let $y = T - T_a$.

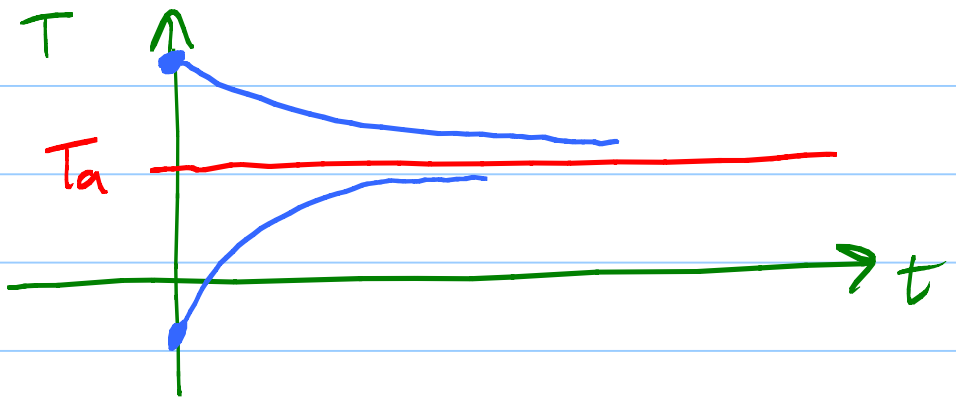
$$\frac{dy}{dt} = \frac{dT}{dt}.$$

$$\frac{dy}{dt} = -ky$$

$$y = y_0 e^{-kt}$$

$$(T - T_a) = (T_0 - T_a) e^{-kt}$$

Soln: $$T = T_a + (T_0 - T_a) e^{-kt}$$



Prob: A pan of warm water 46°C was put in a fridge. Ten minutes later, it was 39°C. 10 min after that, it was 33°C. What's the temp of the fridge?

Soln: $$T = T_a + (T_0 - T_a) e^{-kt}$$

Know: $T_0 = 46$, $T(10) = 39$, $T(20) = 33$

Want: T_a, and maybe k too.

$$39 = T(10) = T_a + (46 - T_a)\, e^{-10K} \quad (A)$$
$$33 = T(20) = T_a + (46 - T_a)\, e^{-20K} \quad (B)$$

MAPLE: solve({eqn1, eqn2}, {var1, var2});

Solve (A) for e^{-10K} : $\dfrac{39 - T_a}{46 - T_a} = e^{-10K}$

Solve (B) for e^{-20K} : $\dfrac{33 - T_a}{46 - T_a} = e^{-20K}$

But $e^{-20K} = (e^{-10K})^2$

$$\frac{33 - T_a}{46 - T_a} = \left(\frac{39 - T_a}{46 - T_a}\right)^2$$

$$(33 - T_a)(46 - T_a) = (39 - T_a)^2$$

$$1518 - 79T_a + T_a^2 = 1521 - 78T_a + T_a^2$$

$$\boxed{T_a = -3}$$

What is K?

$$e^{-10K} = \frac{39 - (-3)}{46 - (-3)} = \frac{42}{49}$$

$$-10K = \ln\frac{42}{49} = -\ln\frac{49}{42}$$

$$K = \frac{1}{10}\ln\frac{49}{42}.$$

7.3 Hyperbolic functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\operatorname{sech} = \frac{1}{\cosh} \qquad \operatorname{csch} = \frac{1}{\sinh}$$

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Derivatives:

$$\frac{d}{dx}\sinh x = \frac{d}{dx}\left(\frac{e^x - e^{-x}}{2}\right) = \frac{e^x + e^{-x}}{2}$$

$$= \cosh x$$

$$\frac{d}{dx}\cosh x = \frac{d}{dx}\left(\frac{e^x + e^{-x}}{2}\right) = \frac{e^x - e^{-x}}{2} = \sinh x$$

ODEs:

$$y'' - y = 0$$

$$y = A\sinh x + B\cosh x$$

$$y'' + y = 0$$

$$y = A\sin x + B\cos x$$

Identities:

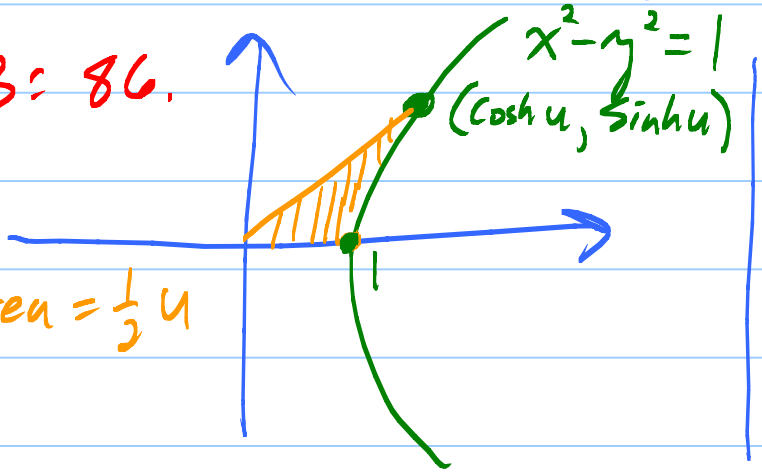
$$\cosh^2 x - \sinh^2 x = 1$$

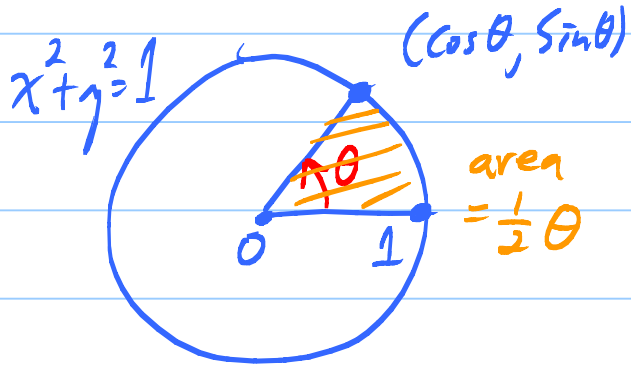
$$\left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2 = ?$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\frac{e^{2x}+2+e^{-2x}}{4} - \left[\frac{e^{2x}-2+e^{-2x}}{4}\right] = \frac{1}{2}+\frac{1}{2} = 1 \checkmark$$

7.3: 86.





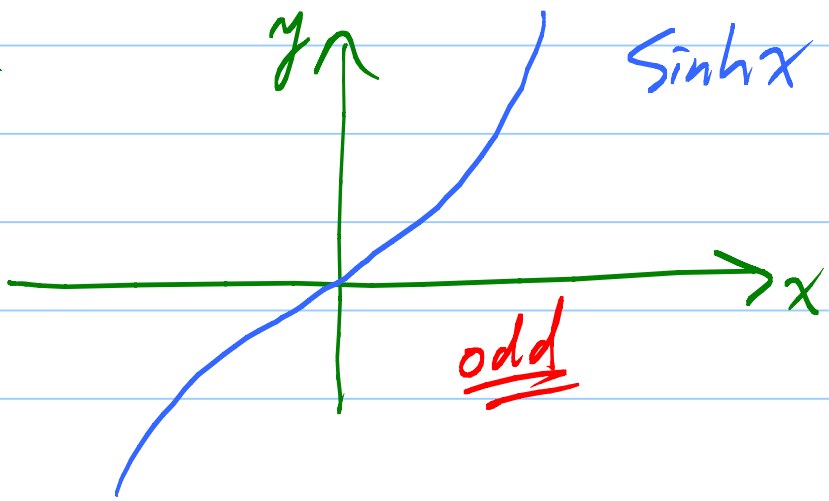
Fact: Analagous identities to trig identities.

$$\sinh x \cosh x = \left(\frac{e^x-e^{-x}}{2}\right)\left(\frac{e^x+e^{-x}}{2}\right) = \frac{e^{2x}-1+1+e^{-2x}}{4}$$

$$= \frac{1}{2}\left(\frac{e^{2x}-e^{-2x}}{2}\right) = \frac{1}{2}\sinh 2x$$

$$\boxed{\sinh 2x = 2\sinh x \cosh x}$$

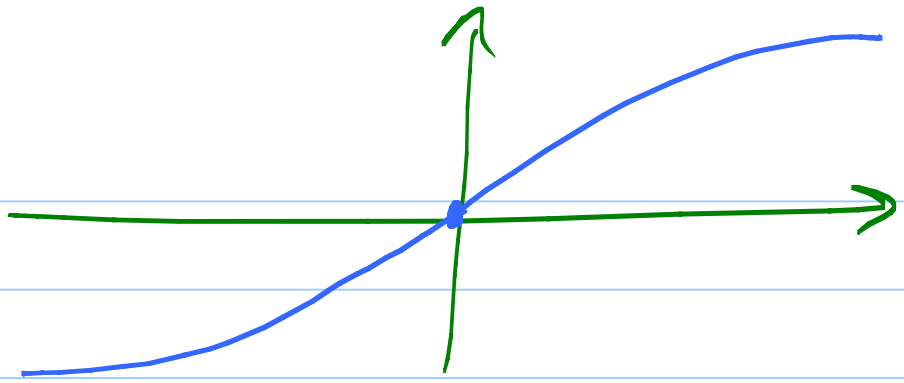
Graphs:

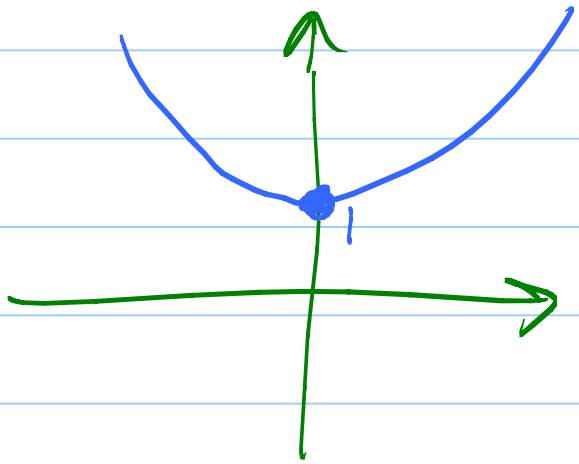


$\sinh 0 = 0$

$\lim_{x\to\infty} \sinh x = \infty$

$\lim_{x\to-\infty} \sinh x = -\infty$





$\cosh 0 = 1$

even