Solutions to old exams posted on web now.

Exam 1 in class tomorrow. Closed book, closed notes, no calculators.

1.b. $\int x^5 \sqrt{\underbrace{x^3+1}_{u}}\, dx$

$u = x^3+1 \Leftarrow x^3 = u-1$

$du = 3x^2\, dx$

$x^2 dx = \frac{1}{3} du$

$$\int \underset{\overset{\uparrow}{u-1}}{x^3} \sqrt{\underbrace{x^3+1}_{u}} \underbrace{(x^2\, dx)}_{\frac{1}{3}du}$$

$$= \int (u-1)\, u^{1/2}\, \frac{1}{3} du = \frac{1}{3} \int u^{3/2} - u^{1/2}\, du$$

$$= \frac{1}{3} \left(\frac{1}{\frac{3}{2}+1} u^{3/2+1} - \frac{1}{\frac{1}{2}+1} u^{1/2+1} \right) + C$$

$$= \frac{1}{3} \left(\frac{2}{5} (x^3+1)^{5/2} - \frac{2}{3} (x^3+1)^{3/2} \right) + C$$

$\int_{x=0}^{1} x^5 \sqrt{x^3+1}\, dx$ when $x=0$, $u = 0^3+1 = 1$

$x = 1$, $u = 1^3+1 = 2$

$$\int_{u=1}^{2} (u-1) \sqrt{u} \left(\frac{1}{3} du\right)$$

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2. $F(x) = \int_0^{\sqrt{x}} e^{t^2}\, dt$.

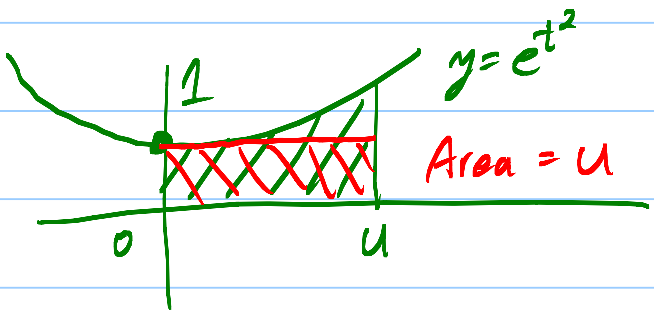
$$= \int_0^u e^{t^2}\,dt \quad \text{where } u = \sqrt{x}.$$

$$F'(x) = \underbrace{\frac{d}{du}\left(\int_0^u e^{t^2}\,dt\right)}_{\substack{\text{Fund. Thm. Calc.} \\ \shortparallel \\ e^{u^2}}} \cdot \underbrace{\frac{du}{dx}}_{\frac{1}{2x^{1/2}}}$$

$$= e^{(\sqrt{x})^2}\,\frac{1}{2x^{1/2}} = \frac{e^x}{2x^{1/2}}$$

What is $\displaystyle\lim_{x\to\infty} \frac{x}{F(x)}$?

Hmmm. Does $F(x) \to \infty$ as $x \to \infty$?



$$\int_0^u e^{t^2}\,dt > 1\cdot u$$

$$F(x) = \int_0^{\sqrt{x}} e^{t^2}\,dt \geq \sqrt{x}$$

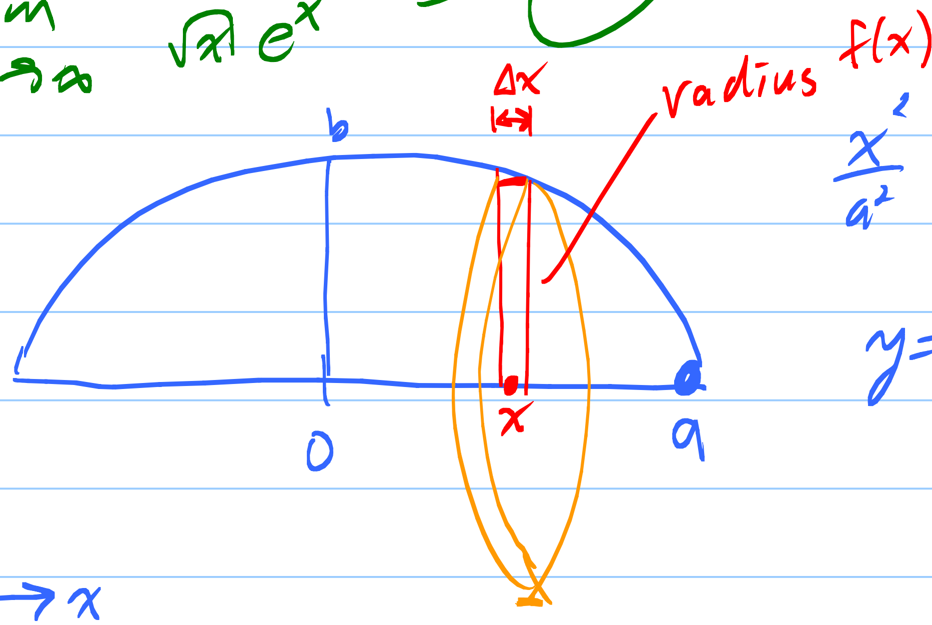
So, yes, $F(x) \to \infty$ as $x \to \infty$.

$$\lim_{x\to\infty} \frac{x}{F(x)} \overset{\left(\frac{\infty}{\infty}\right)}{\underset{L'H}{=}} \lim_{x\to\infty} \frac{1}{F'(x)} = \lim_{x\to\infty} \left(\frac{1}{\frac{e^x}{2x^{1/2}}}\right)$$

$$= \lim_{x\to\infty} \frac{2x^{1/2}}{e^x} \overset{\left(\frac{\infty}{\infty}\right)}{\underset{L'H}{=}} \lim_{x\to\infty} \frac{x^{-1/2}}{e^x}$$

$$= \lim_{x\to\infty} \frac{1}{\sqrt{x}\, e^x} = 0$$

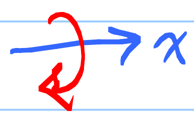
5.



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

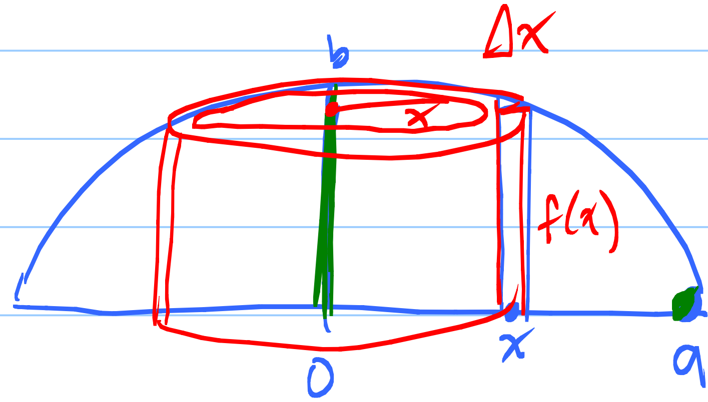
$$y = \frac{b}{a}\sqrt{a^2 - x^2}$$

a)



$$V = \int_{-a}^{a} \pi f(x)^2\, dx = \int_{-a}^{a} \pi \left[\frac{b}{a}\sqrt{a^2 - x^2}\right]^2 dx$$

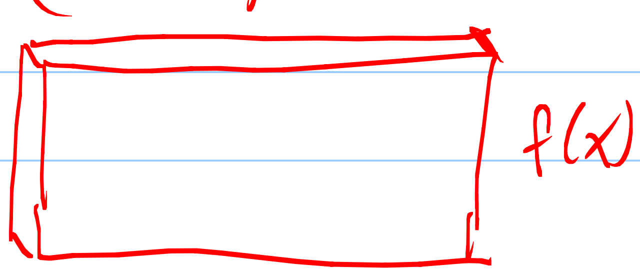
b)



$$V = \int \quad dx$$

Must use cylindrical shells.

$$\Delta V = (2\pi x)\, f(x)\, \Delta x$$

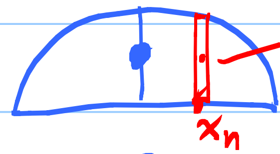


Thickness Δx

$|\leftarrow 2\pi x \rightarrow|$

$$V = \int_0^a 2\pi x \left(\frac{b}{a}\sqrt{a^2 - x^2} \right) dx$$

c) $\bar{y}$ of



$\bar{x} = 0$

$$\tilde{y}_n = \frac{f(x_n)}{2}$$

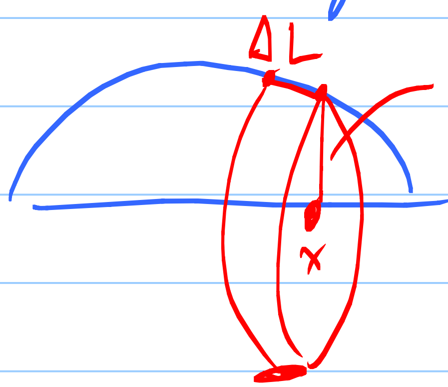
$$\bar{y} = \lim \frac{\sum m_i \tilde{y}_i}{\sum m_i} = \frac{\int_{-a}^{a} \underbrace{\frac{f(x)}{2}}_{\tilde{y}} \cdot \underbrace{\left(f(x)\,dx\right)}_{dm}}{\int_{-a}^{a} f(x)\,dx}$$

Plug in $f(x) = \frac{b}{a}\sqrt{x^2 - a^2}$.

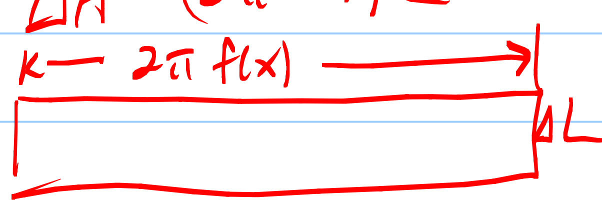
d) Skip (d). $L = \int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$x = a\cos t$, $y = b\sin t$.

e)



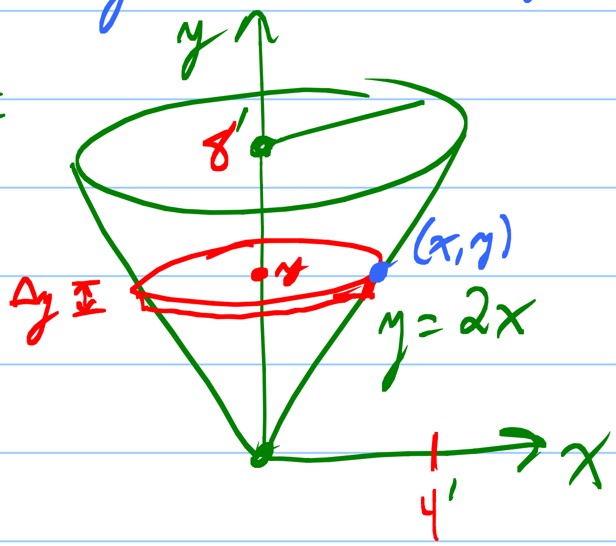
$$\Delta A = (2\pi f(x))L$$



$$S = \int_{-a}^{a} 2\pi f(x) \underbrace{\sqrt{1+f'(x)^2}\, dx}_{\Delta L}$$

Plug in $f(x) = \frac{b}{a}\sqrt{a^2-x^2}$.

Work:



Work to pump out full tank. $w = 64.2$ lbs/ft^3

$\Delta W = (\text{weight})(\text{heigth lifted})$

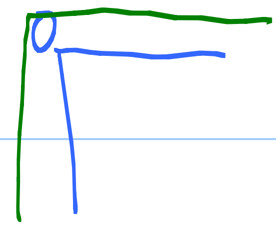
$= (\pi r^2 \Delta y) w (8 - y)$

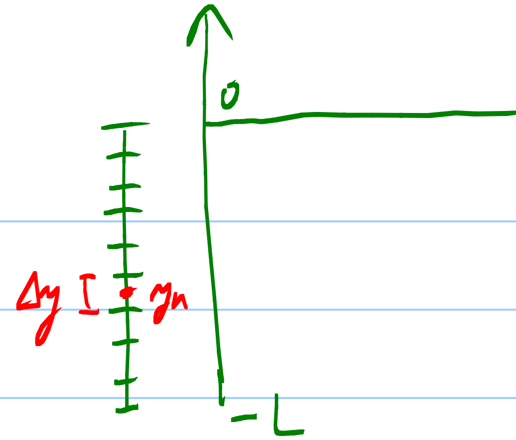
need r in terms of y

$r = x = \frac{y}{2}$

$$W = \int_0^8 \pi \left(\frac{y}{2}\right)^2 w\,(8-y)\,dy$$

Rope:

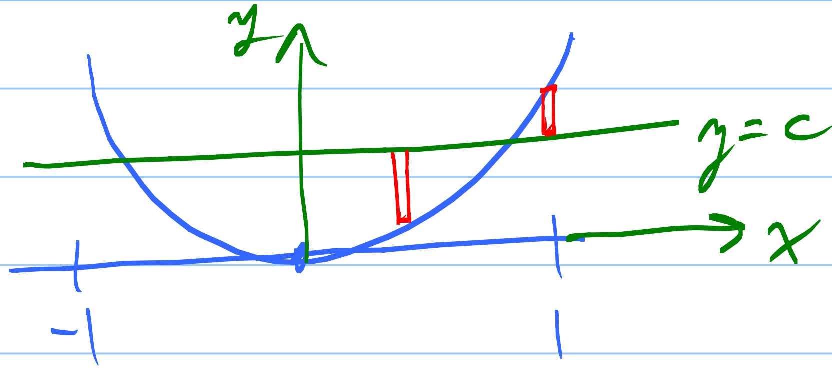


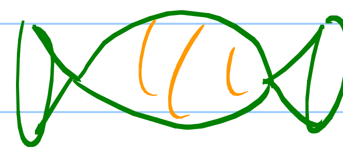


$$\Delta W = (\rho \Delta y)(0 - y_n)$$

$$W = \int_{-L}^{0} \rho\,(-y)\,dy \qquad \rho = \text{lbs}/\text{ft}$$

6.





$$V = \int_{-1}^{1} \pi \left(c - x^2\right)^2 dx$$

$$= \pi \int_{-1}^{1} c^2 - 2cx^2 + x^4 \, dx$$

$$V(c) = \pi \left[2c^2 - \frac{4}{3}c + \frac{2}{5}\right] \quad \text{parabola.}$$

$V(0) = \pi \frac{2}{5} = \frac{6}{15}\pi$

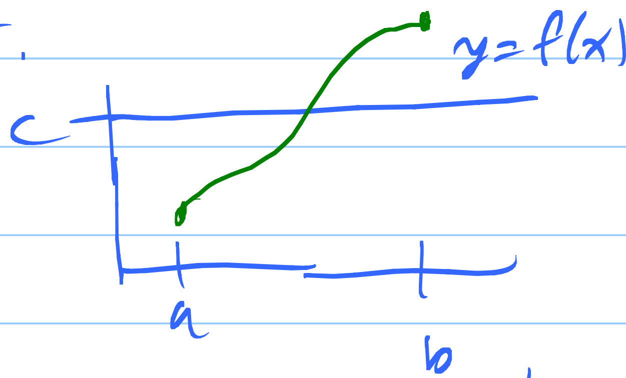
$V(1) = \pi\left(2 - \frac{4}{3} + \frac{2}{5}\right) = \frac{16}{15}\pi$

Critical point: $V'(c) = \pi\left(4c - \frac{4}{3}\right) = 0$

$c = 1/3$

$V\left(\frac{1}{3}\right) =$ do it.

Interesting thing.



$c_{min} = \text{Ave of } f = \frac{1}{b-a}\int_a^b f(x)\,dx.$

3. $f'' = \sin 3t$.

$f'(t) = -\frac{1}{3}\cos 3t + C$

$f(t) = -\frac{1}{9}\sin 3t + Ct + B$

$f(0) = 5$, $B = 5$.