100
100
100
100
98
98
98
97
95
95
95
95
94
92
90
90 A

86
82
80
79
76 B

75
75
75
75
70
70 B−

63 C+

Ave = 87

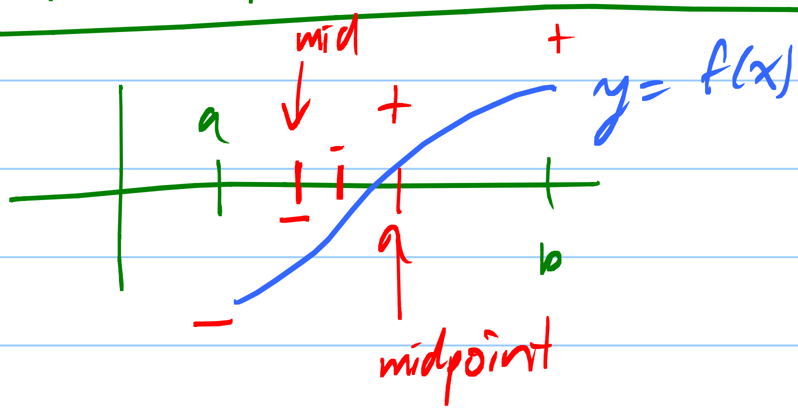
Δx

$$\pi(R^2 - r^2)\,\Delta x$$

$$\int_0^1 \pi\left((x^3)^2 - (x^4)^2\right)dx$$

$$\int_3^7 e^{(2x+1)^2}\,dx = c\int_a^b e^{u^2}\,du = \frac{1}{2}\int_7^{15} e^{x^2}\,dx$$

Primitive Method: Bisection Method:



Read 4.7 (Newton's Method), 8.6 (Numerical Integration)

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$x_\infty = x_\infty - \frac{f(x_\infty)}{f'(x_\infty)} \Leftarrow \text{See } f(x_\infty) = 0$$

[need $f'(x_\infty) \neq 0$].

$$x_\infty = \lim_{n \to \infty} x_n$$

<u>Rectangle Rule</u>: $|RS - I| \leq \frac{M(b-a)^2}{N}$, $M = \max_{[a,b]} |f'|$

<u>Trapezoid Rule</u>: $|TS - I| \leq \frac{m(b-a)^3}{12N^2}$, $m = \max_{[a,b]} |f''|$

$$I = \int_0^{1/\sqrt{2}} \underbrace{\sqrt{1-x^2}}_{f(x)}\, dx \qquad f(x) = (1-x^2)^{1/2}$$

$$f'(x) = \tfrac{1}{2}(1-x^2)^{-1/2}(-2x)$$

$$= -x(1-x^2)^{-1/2}$$

$$f''(x) = -(1-x^2)^{-1/2} - x\left(-\tfrac{1}{2}\right)(1-x^2)^{-3/2}(-2x)$$

$$= -(1-x^2)^{-1/2}\left[1 + \frac{1}{1-x^2}\right]$$

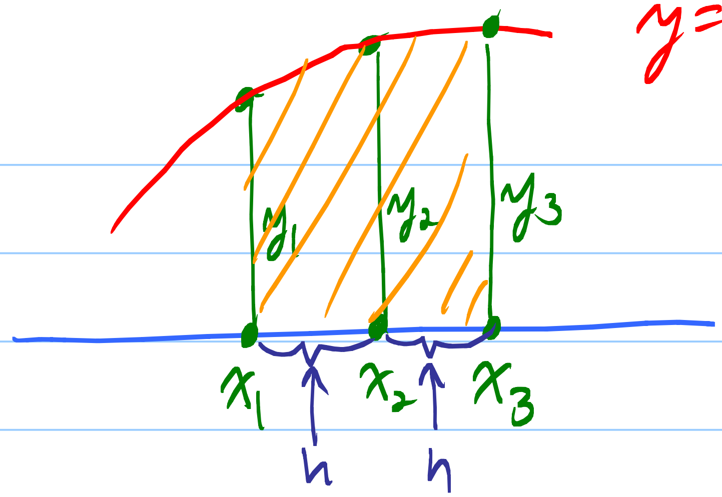
If $0 \leq x \leq \frac{1}{\sqrt{2}}$, then $|f''(x)| \leq \frac{1}{\left(1-\left(\frac{1}{\sqrt{2}}\right)^2\right)^{1/2}} \cdot 3 \leq 3\sqrt{2}$

$$|TS - I| \leq \frac{3\sqrt{2}\,(1/\sqrt{2})^3}{12N^2} \overset{\text{want}}{<} 10^{-6}$$

$\Leftarrow$ easy to see what N needs to be.

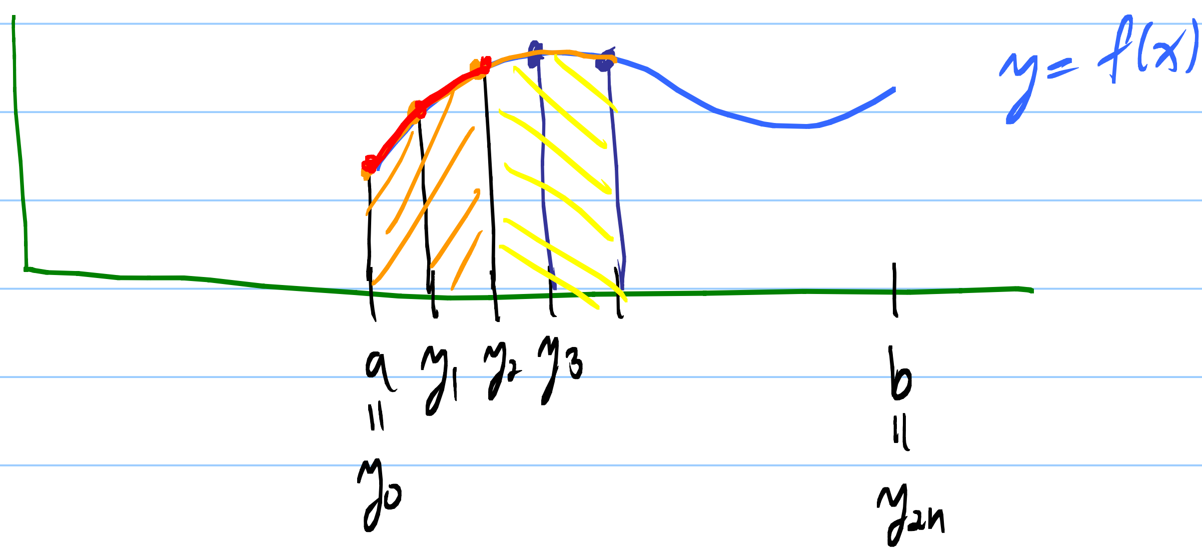
Simpson's Rule:

$y = Ax^2 + Bx + C$



Plug (x_1, y_1), (x_2, y_2), (x_3, y_3) into $y = Ax^2 + Bx + C$. Get 3 eqns in 3 unknowns A, B, C. Solve for A, B, C.

$$\text{Area} = \int_{x_1}^{x_3} Ax^2 + Bx + C \, dx = \frac{h}{3}\left(y_1 + 4y_2 + y_3\right)$$



$$I \approx \frac{h}{3}\left(y_0 + 4y_1 + y_2\right) + \frac{h}{3}\left(y_2 + 4y_3 + y_4\right) + \cdots + \frac{h}{3}\left(y_{2n-2} + 4y_{2n-1} + y_{2n}\right)$$

$$SR = \frac{h}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 4y_{2n-1} + y_{2n}\right]$$

Fact: $$\left| SR - I \right| \le \frac{M\,(b-a)^5}{180\,N^4},$$

$N = 2n$

$M = \max_{[a,b]} |f^{(4)}|$

ODD Numbers: $2k-1$: $k = 1, 2, \dots, n$

$1, 3, 5, \dots, 2n-1$

EVEN Numbers: $2k$: $k = 1, 2, \dots, n-1$

$2, 4, 6, \dots, 2n-2$

MAPLE:

SR = h/3*[f(a) + f(b) +

4*sum(f(a+(2k-1)h), k=1..n) +

2*sum(f(a+(2k)h), k=1..(n-1))]