8.1 Integration by Parts:

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\underbrace{\int \frac{d}{dx}(uv)\,dx}_{uv} = \int u \underbrace{\frac{dv}{dx}\,dx}_{dv} + \int v \underbrace{\frac{du}{dx}\,dx}_{du}$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

EX: $\int \underbrace{x}_{u}\ \underbrace{\sin x\,dx}_{dv}$

$u = x$ $\quad$ $dv = \sin x\,dx$

$du = dx$ $\quad$ $v = \int \sin x\,dx$

$= -\cos x$

pick nicest cleanest antiderivative

$$= uv - \int v\,du$$

$$= (x)(-\cos x) - \int (-\cos x)\,(dx)$$

$$= -x\cos x + \int \cos x\,dx$$

$$= -x\cos x + \sin x + C$$

EX: $\int \underbrace{x}_{u}\ \underbrace{e^{2x}\,dx}_{dv}$

$u = x$ $\quad$ $dv = e^{2x}\,dx$

$du = dx$ $\quad$ $v = \frac{1}{2}e^{2x}$

$$= uv - \int v\,du = (x)\left(\tfrac{1}{2}e^{2x}\right) - \int \tfrac{1}{2}e^{2x}\,dx$$

$$= \tfrac{1}{2}xe^{2x} - \tfrac{1}{4}e^{2x} + C$$

EX: $\int x^2 e^{2x}\,dx$

$u = x^2 \qquad dv = e^{2x}\,dx$

$du = 2x\,dx \qquad v = \tfrac{1}{2}e^{2x}$

$$= x^2\left(\tfrac{1}{2}e^{2x}\right) - \int \underbrace{\left(\tfrac{1}{2}e^{2x}\right)}_{v}\underbrace{(2x\,dx)}_{du}$$

Do int. by parts again!

$$= \tfrac{1}{2}x^2e^{2x} - \left[\tfrac{1}{2}xe^{2x} - \tfrac{1}{4}e^{2x}\right] + C$$

Tabular Integration: $\int \underset{f(x)}{x^3}\,\underset{g(x)}{e^{2x}}\,dx$

	f(x) and its derivatives		g(x) and its integrals	
u →	x^3	$+$	e^{2x}	
	$3x^2$	$-$	$\tfrac{1}{2}e^{2x}$	← v
	$6x$	$+$	$\tfrac{1}{4}e^{2x}$	
	6	$-$	$\tfrac{1}{8}e^{2x}$	
	0	$+$	$\tfrac{1}{16}e^{2x}$	
	$\vdots$	done	eeee	

$$\int x^3 e^{2x}\,dx = x^3\left(\tfrac{1}{2}e^{2x}\right) - 3x^2\left(\tfrac{1}{4}e^{2x}\right) + (6x)\left(\tfrac{1}{8}e^{2x}\right) - 6\left(\tfrac{1}{16}e^{2x}\right) + C$$

Choosing u and v:

Good u: $\frac{du}{dx}$ is easier and better than u.

$u = x^n \qquad \frac{du}{dx} = nx^{n-1}$ ← one degree less!

$u = \ln x \qquad \frac{du}{dx} = \frac{1}{x}$

$u = \tan^{-1} x \qquad \frac{du}{dx} = \frac{1}{1+x^2}$

Good $dv = g(x)dx$ $\int g(x)dx$ is no worse than $g(x)$

$dv = \sin 3x\,dx \qquad v = \frac{1}{3}\cos 3x$

$dv = e^{2x}\,dx \qquad v = \frac{1}{2}e^{2x}$

EX: $\int \underbrace{\ln x}_{u}\,\underbrace{dx}_{dv}$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}dx \qquad v = \int dx = x$

$$= uv - \int v\,du = (\ln x)\cdot(x) - \int x\left(\frac{1}{x}dx\right)$$

$$= x\ln x - \int 1\,dx = x\ln x - x + C$$

EX: $\int \underbrace{\operatorname{Tan}^{-1} x}_{u} \underbrace{dx}_{dv}$ $\qquad u = \operatorname{Tan}^{-1} x \qquad dv = dx$

$du = \frac{1}{1+x^2}\,dx \qquad v = x$

$$= x\operatorname{Tan}^{-1} x - \frac{1}{2}\int 2x \cdot \frac{1}{\underbrace{1+x^2}_{u}}\,dx$$

$$= x\operatorname{Tan}^{-1} x - \frac{1}{2}\ln\left(1+x^2\right) + C$$

EX: $I = \int \underbrace{e^x}_{u} \underbrace{\cos x\,dx}_{dv}$ $\qquad u = e^x \qquad dv = \cos x\,dx$

$du = e^x dx \qquad v = \sin x$

$$= uv - \int v\,du = e^x \sin x - \int \sin x\, e^x\,dx$$

Darn!

Do Int. by Parts again!

$$I = e^x \sin x - \int \underbrace{e^x}_{u} \underbrace{\sin x\,dx}_{dv}$$

$du = e^x dx \qquad v = -\cos x$

$$I = e^x \sin x - \left[e^x(-\cos x) - \int (-\cos x)e^x\,dx\right]$$

$$I = e^x \sin x + e^x \cos x - \underbrace{\int e^x \cos x\,dx}_{I}$$

Solve for I!

$$I = \frac{1}{2}e^{x}\sin x + \frac{1}{2}e^{x}\cos x + C$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}\left(a\sin bx - b\cos bx\right) + C$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}\left(a\cos bx + b\sin bx\right) + C$$

Definite Integrals:

$$\int_a^b u\,dv = (uv)\Big|_a^b - \int_a^b v\,du$$