470: 17.

$$\int \frac{x^3}{\sqrt{x^2+4}}\,dx = \frac{1}{2}\int \frac{x^2 \cdot 2x\,dx}{\sqrt{\underbrace{x^2+4}_{u}}} \qquad du = 2x\,dx$$

$$x^2 = u-4$$

Partial Fractions:

$$I = \int \frac{1}{x^2-5x+6}\,dx$$

$$\frac{1}{x^2-5x+6} = \frac{1}{\underbrace{(x-2)(x-3)}_{\text{denom}}} = \frac{A}{x-2} + \frac{B}{x-3}$$

$$1 = A(x-3) + B(x-2)$$

$$0 \cdot x + 1 = \underbrace{(A+B)}_{0}x + \underbrace{(-3A-2B)}_{1}$$

$$\begin{cases} A+B=0 \\ -3A-2B=1 \end{cases} \qquad \begin{matrix} A=-1 \\ B=1 \end{matrix}$$

$$I = \int \frac{-1}{x-2} + \frac{1}{x-3}\,dx = -\ln|x-2| + \ln|x-3| + C$$

"Cover up method"

$$\frac{1}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$$

Multiply by $(x-3)$:

$$\frac{1}{x-2} = A\,\frac{x-3}{x-2} + B$$

Let $x=3$:

$$\frac{1}{3-2} = 0 + B \qquad \boxed{B=1}$$

Cover up $x-2$. Let $x=2$. Get A.

8.5 Tables of Integrals and CAS.

(see MAPLE demo)

Compute a) $\displaystyle\int \frac{dx}{x^2+2x+1} = \int \frac{1}{(\underbrace{x+1}_{u})^2}\,dx = \frac{-1}{x+1} + C$

b) $\displaystyle\int \frac{dx}{x^2+2x+6}$

c) $\displaystyle\int \frac{dx}{x^2+2x-3}$

b) : $b^2-4ac = 2^2 - 4\cdot 6 = -20 < 0$ complex roots.

Partial Fractions dead-ends:

$$\frac{1}{x^2+2x+6} = \frac{Ax+B}{x^2+2x+6} \qquad A=0,\ B=1.$$

Key: Complete the square.

$$x^2+2x+6 = (x+1)^2 + (5)$$

$\frac{1}{2}2$

$$= (x+1)^2 + (\sqrt{5})^2$$

$$I = \int \frac{1}{(\underbrace{x+1}_{u})^2+(\sqrt{5})^2}\,dx \qquad du = dx$$

$$= \int \frac{1}{u^2+(\sqrt{5})^2}\,du = \frac{1}{\sqrt{5}}\tan^{-1}\frac{u}{\sqrt{5}} + C$$

$$= \frac{1}{\sqrt{5}} \operatorname{Tan}^{-1} \frac{(x+1)}{\sqrt{5}} + C$$

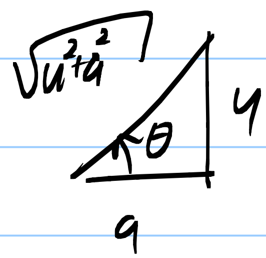
Know $\displaystyle\int \frac{dx}{x^2+1} = \operatorname{Tan}^{-1} x + C$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$$

$$\int \frac{1}{u^2+a^2}\, du$$

$u = a \operatorname{Tan}\theta$

$du = a \sec^2\theta \, d\theta$



$\sqrt{u^2+a^2} = a\sec\theta$

$$= \int \frac{a\sec^2\theta\, d\theta}{(a\sec\theta)^2} = \frac{1}{a}\int d\theta$$

$$= \frac{1}{a}\theta + C$$

$\frac{u}{a} = \operatorname{Tan}\theta$

$\theta = \operatorname{Tan}^{-1}\frac{u}{a}$

$$= \frac{1}{a}\operatorname{Tan}^{-1}\frac{u}{a} + C$$

---

c) $\displaystyle\int \frac{dx}{x^2+2x-3}$

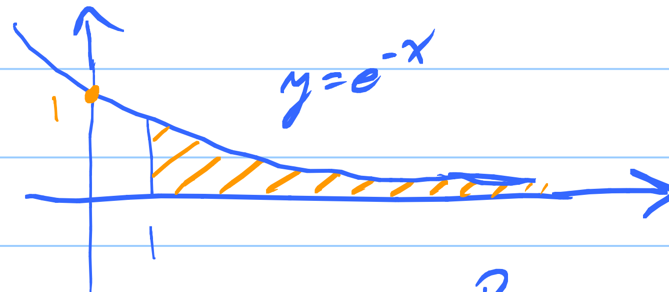
$$\frac{1}{x^2+2x-3} = \frac{1}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1}$$

etc.

Remark: Or complete the square. Look up integral in a table. Get $\tanh^{-1}\ldots$. That's correct too.

8.7 Improper Integrals.

EX: $\displaystyle\int_1^\infty e^{-x}\,dx =$



$$= \lim_{B\to\infty}\int_1^B e^{-x}\,dx = \lim_{B\to\infty}\left[-e^{-x}\right]_1^B$$

$$= \lim_{B\to\infty}\left(\frac{1}{e} - e^{-B}\right) = \frac{1}{e}$$

EX: $\displaystyle\int_1^\infty \frac{1}{x^p}\,dx$ For what p is this finite?

Case $p=1$: $\displaystyle\int_1^\infty \frac{1}{x}\,dx = \lim_{B\to\infty}\int_1^B \frac{1}{x}\,dx$

$$= \lim_{B\to\infty}\left[\ln x\right]_1^B = \lim_{B\to\infty}\left[\ln B - \underbrace{\ln 1}_{0}\right] = \infty$$

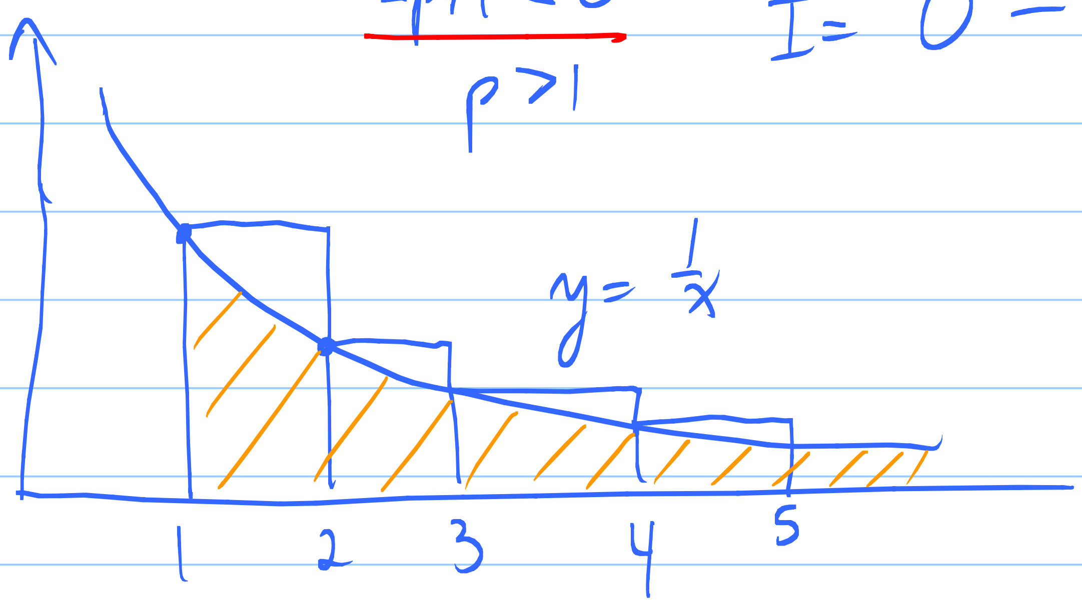
Case $p \neq 1$: Get $\displaystyle\int_1^\infty x^{-p}\,dx = \lim_{B\to\infty}\left[\frac{1}{-p+1}x^{-p+1}\right]_1^B$

Two cases: $\underline{-p+1 > 0}$. Ouch! Blows up.

$p<1$. $I = \infty$.

$\underline{-p+1 < 0}$ $\quad I = 0 - \frac{1}{-p+1} = \frac{1}{p-1}$

$p > 1$



$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots > \int_1^\infty \frac{1}{x}\,dx = \infty$$

Famous series diverges even though terms go to zero.