Trig Facts: Derivatives of sin, cos, sec, tan and their Integrals.

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\int \frac{du}{\sqrt{1+u^2}} = \sinh^{-1} u + C$$

don't have to memorize.

$$\int \frac{2x-3}{x^2+2x+5}\,dx$$

$u = x+1 \qquad du = dx$

$x = u-1$

$\frac{1}{2}\cdot 2$

$$\int \frac{2x-3}{(\underbrace{x+1}_{u})^2+4}\,dx = \int \frac{2(u-1)-3}{u^2+2^2}\,du$$

dv

$$= \int \frac{2u}{u^2+2^2}\,du - 5\int \frac{1}{u^2+2^2}\,du$$

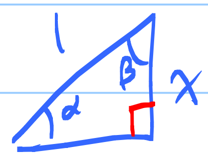
v

$$= \ln(u^2+4) - 5\,\frac{1}{2}\tan^{-1}\frac{u}{2} + C$$

$$= \ln\left((x+1)^2+4\right) - \frac{5}{2}\tan^{-1}\left(\frac{x+1}{2}\right) + C$$

Know $\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$ and integral

$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$ and integral



$$\underbrace{\alpha+\beta}_{\frac{\pi}{2}} = \sin^{-1}x + \cos^{-1}x$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$|e| < 3$ in estimates for error.

$e^c < e^{10} < 3^{10}$ in an error estimate

$$R_N(x) = \frac{f^{(N+1)}(c)}{(N+1)!}(x-a)^{N+1}$$

where c is between a and x.

$$\left| e^x - \sum_{n=0}^{N} \frac{x^n}{n!} \right| = \left| R_N(x) \right| = \frac{e^c}{(N+1)!}|x|^{N+1}$$

$$\left| e^{10} - \sum_{n=0}^{N} \frac{10^n}{n!} \right| \le \frac{e^{10}}{(N+1)!} 10^{N+1} \le \frac{3^{10}\, 10^{N+1}}{(N+1)!} < 10^{-6}$$

want

Limit comparison of $\sum_{1}^{\infty} \frac{\ln n}{n^{8/7}}$ and $\sum_{1}^{\infty} \frac{1}{n^p}$.

$$\lim_{n\to\infty} \frac{\left(\ln n / n^{8/7}\right)}{\left(1/n^p\right)} = \lim_{n\to\infty} \frac{\ln n}{n^{(8/7-p)}}$$

need >0

$\lim = 0$ if $\frac{8}{7} - p > 0$.

$$\frac{8}{7} - p > 0 \qquad p < \frac{8}{7}.$$

But we also need $\sum \frac{1}{n^p}$ to converge, i.e., $p > 1$.

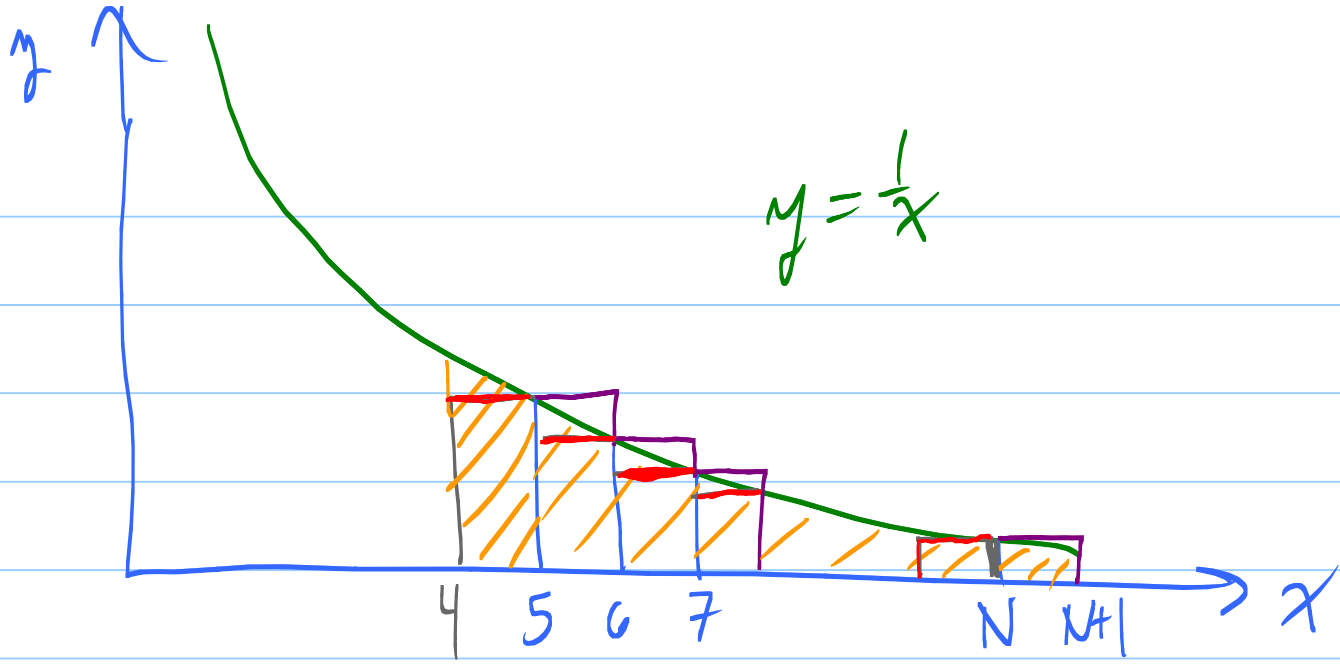
So to get convergence of $\sum \frac{\ln n}{n^{8/7}}$ via comparison with $\sum \frac{1}{n^p}$, need $1 < p < \frac{8}{7}$.

Exam 2 covers Chaps. 8, 9, 10 from syllabus.

Know limits on p. 556.

Geometric Series: $a + ar + ar^2 + ar^3 + \cdots = \frac{a}{1-r}$ if $|r| < 1$.

Don't memorize error estimates for Rec, Trap, Simpson's, Euler's Method formula.



Boom!

$$\sum_{n=5}^{N} \frac{1}{n} > \int_{5}^{N+1} \frac{1}{x}\,dx = \ln(N+1) - \ln 5$$

$$\sum_{n=5}^{N} \frac{1}{n} < \int_{4}^{N} \frac{1}{x}\,dx = \ln N - \ln 4$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \leftarrow \text{Conditionally convergent.}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\, n}{n^2+1}$$