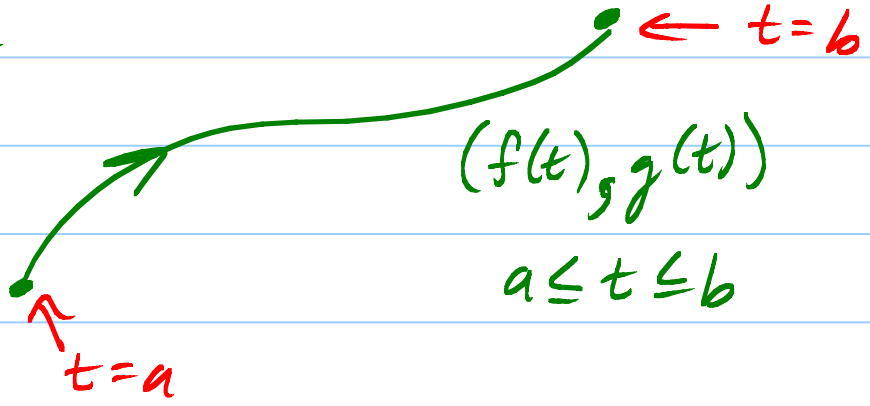
HWK 13 on Home Page. Due Wed. after break.

Parametric curves:

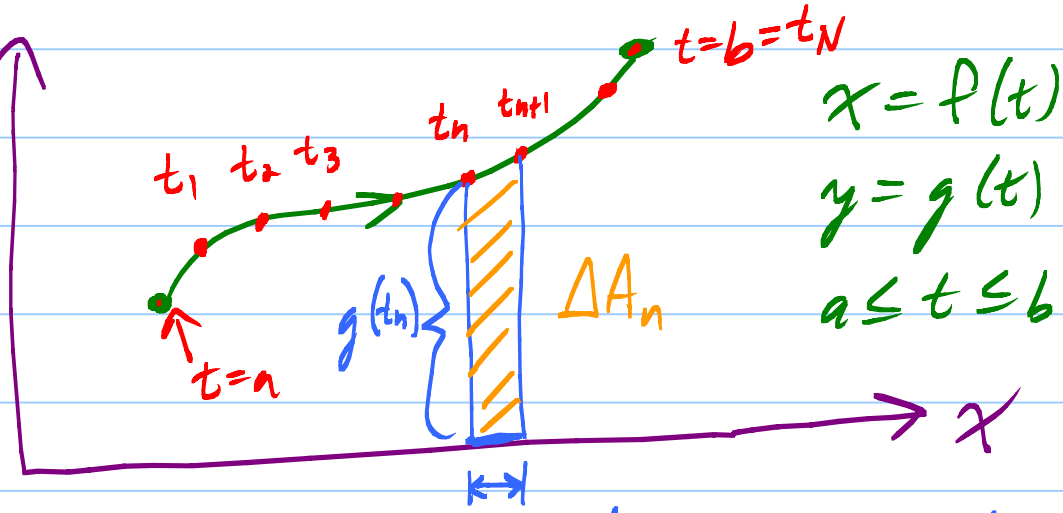
$x = f(t)$

$y = g(t)$



Think: $dx = \frac{dx}{dt}\,dt$ $\qquad dy = \frac{dy}{dt}\,dt$

$$ds = \sqrt{(dx)^2 + (dy)^2} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

Area: 

$$\Delta x \approx \frac{dx}{dt}\Delta t = f'(t_n)\Delta t$$

$$\Delta A_n \approx g(t_n)\left(f'(t_n)\,\Delta t\right)$$

$$A \approx \sum_{n=1}^{N} \Delta A_n \qquad \text{Take limit:}$$

$$A = \int_a^b g(t)\,f'(t)\,dt$$

How to remember formula:

$$A = \int y\,dx = \int y\left(\frac{dx}{dt}\,dt\right)$$

$$= \int_a^b g(t)\,f'(t)\,dt$$

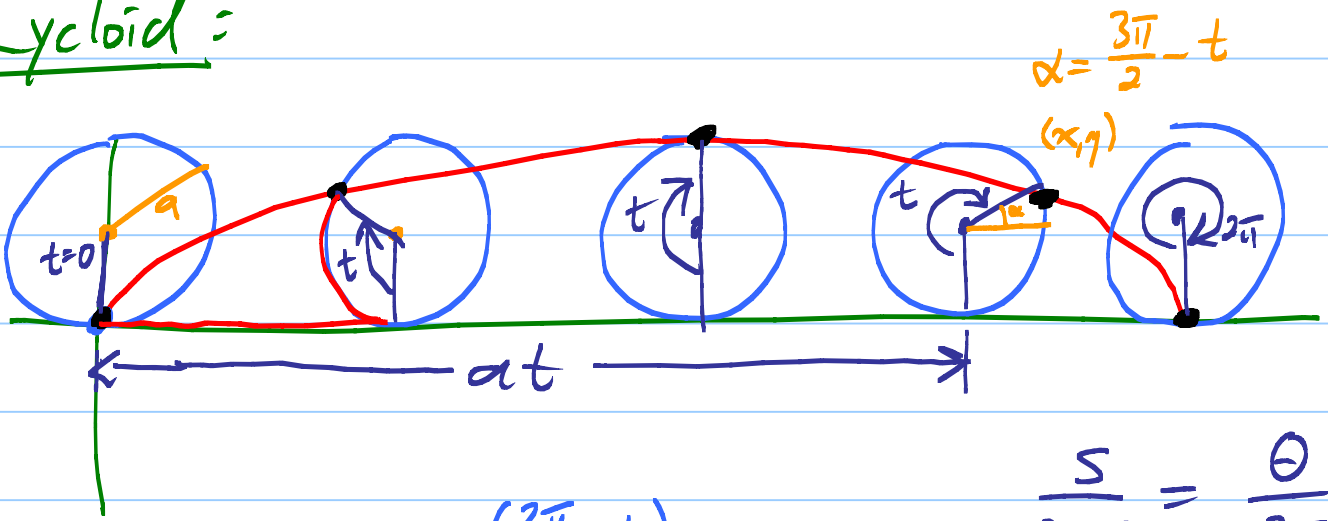
Center of mass of a uniform wire:

$$\bar{x} = \frac{\sum m_n x_n}{\sum m_n} = \frac{\rho \int_a^b f(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt}{\rho \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt}$$

($\frac{dx}{dt}$ is $f'(t)$, $\frac{dy}{dt}$ is $g'(t)$)

$\bar{y}$ = Same thing with g in place of f.

Cycloid:



$$\begin{cases} x = at + a\cos\left(\frac{3\pi}{2} - t\right) \\ y = a + a\sin\left(\frac{3\pi}{2} - t\right) \end{cases}$$

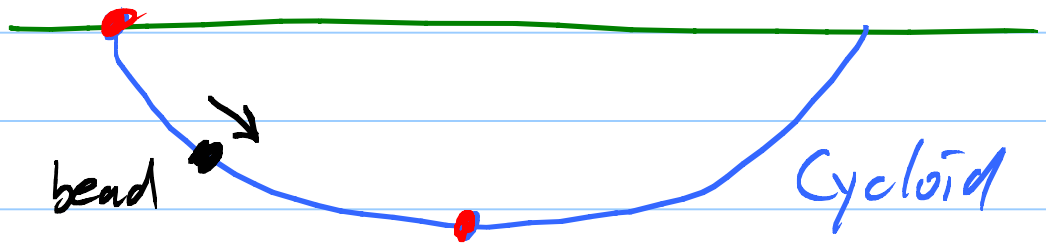
$$\frac{s}{2\pi r} = \frac{\theta}{2\pi}$$

$$s = \theta r$$

$$\begin{cases} x = at - a\sin t \\ y = a - a\cos t \end{cases}$$

Brachistochrone & Tautochrone!

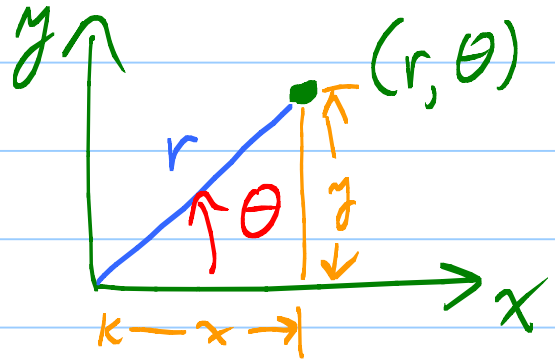
Flip it under x-axis:



Shortest time!

No matter where you start the bead, it takes same amount of time!

11.3 Polar coordinates:



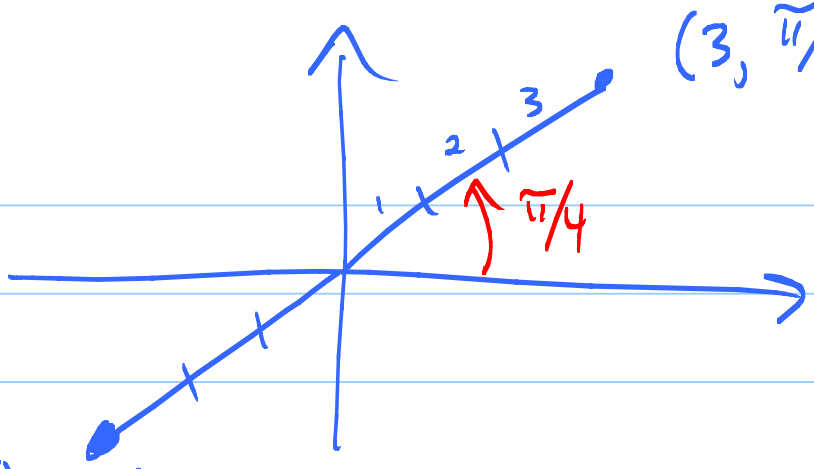
$x = r\cos\theta$

$y = r\sin\theta$

Other way: $r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}\frac{y}{x}$ ← pick the one you like best.

A pain: Can describe a point in multiple ways.



$(3, \pi/4) = (3, 2\pi + \frac{\pi}{4})$

$(-3, \frac{\pi}{4}) = (3, \pi + \frac{\pi}{4})$

Cardioid: $r = a(1 - \cos\theta)$