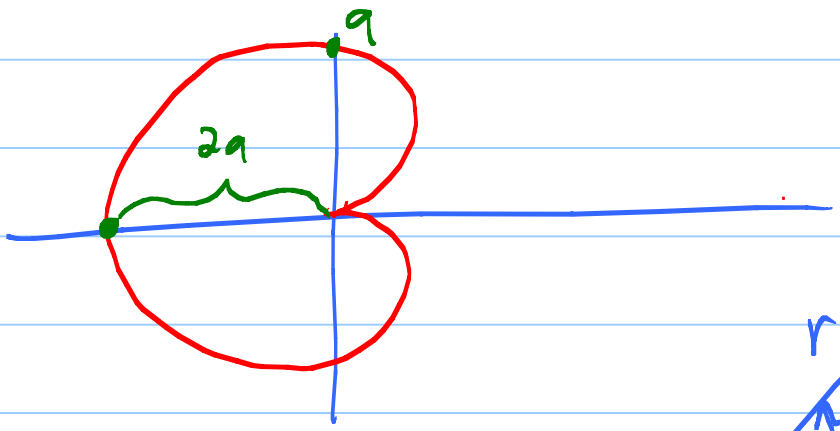
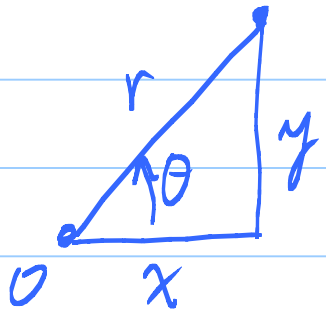
Polar Coords 11.3, 11.4

Cardioid: $r = a(1 - \cos\theta)$





$$\begin{cases} r = \sqrt{x^2+y^2} \\ \cos\theta = \dfrac{x}{r} = \dfrac{x}{\sqrt{x^2+y^2}} \end{cases}$$

$$r = a(1 - \cos\theta)$$

$$\sqrt{x^2+y^2} = a\left(1 - \frac{x}{\sqrt{x^2+y^2}}\right) \leftarrow \text{mult. by } \sqrt{x^2+y^2}$$

$$x^2+y^2 = a\left(\sqrt{x^2+y^2} - x\right)$$

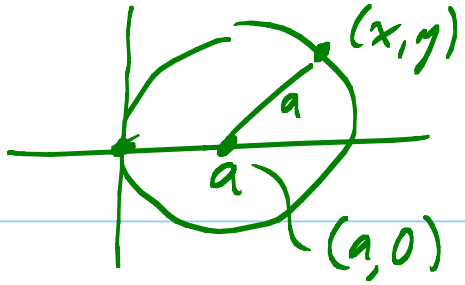
$$x^2+y^2+ax = a\sqrt{x^2+y^2} \leftarrow \text{square}$$

$$(x^2+y^2+ax)^2 = a^2(x^2+y^2) \leftarrow \text{quadratic in } y^2$$

$y^2 =$ Quad Formula

$y = \pm\sqrt{(\,) \pm \sqrt{}}$ ugh!

EX: Find polar eqn of



$$\sqrt{(x-a)^2+(y-0)^2} = a$$

$$(x-a)^2 + y^2 = a^2$$

$$(r\cos\theta - a)^2 + (r\sin\theta)^2 = a^2$$

$$r^2\cos^2\theta - 2ra\cos\theta + a^2 + r^2\sin^2\theta = a^2$$

$r^2\cos^2\theta + r^2\sin^2\theta = r^2$

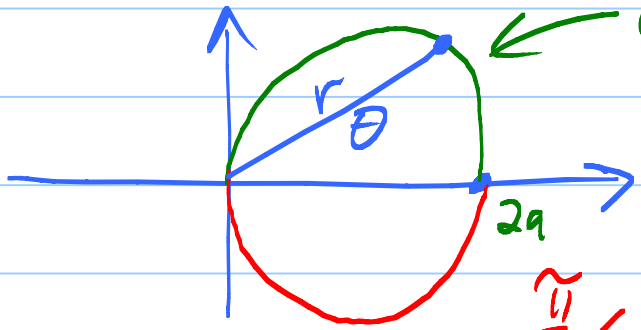
$$r(r - 2a\cos\theta) = 0$$

$r - 2a\cos\theta = 0$

$r = 0$ dumb soln

$$r^2 = 2ra\cos\theta$$

$$\boxed{r = 2a\cos\theta}$$



$0 < \theta < \frac{\pi}{2}$

$\pi < \theta < \frac{3\pi}{2}$ ← $r < 0$

$\frac{\pi}{2} < \theta < \pi$ ← $r < 0$

$\frac{3\pi}{2} < \theta < 2\pi$

Prob: Find the slope of a tangent line to a polar curve $r = f(\theta)$.

Idea: Describe curve by parametric equations using θ as parameter:

$$\begin{cases} x = r\cos\theta = f(\theta)\cos\theta \\ y = r\sin\theta = f(\theta)\sin\theta \end{cases}$$

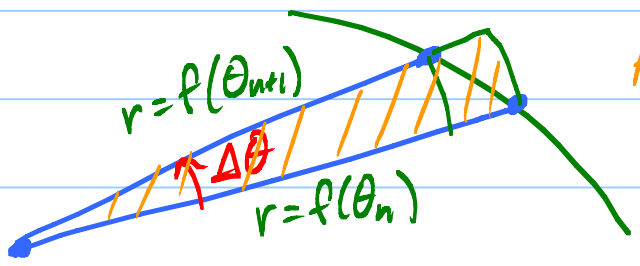
Recall: $$\frac{dy}{dx} = \frac{(dy/d\theta)}{\left(\frac{dx}{d\theta}\right)} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$$

Note: For the Cardioid, where $r = f(0) = 0$ when $\theta = 0$, get $$\left.\frac{dy}{dx}\right|_{\theta=0} = \frac{f'(0)\sin 0}{f'(0)\cos 0} = 0.$$

If $r = f(\theta_0) = 0$, do same thing. Get

$$\left.\frac{dy}{dx}\right|_{\theta=\theta_0} = \frac{\sin\theta_0}{\cos\theta_0} = \tan\theta_0.$$

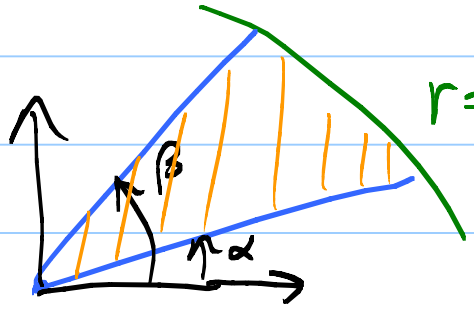
Problem: Find Area of the Cardioid.



$$\text{Area} = \frac{1}{2} f(\theta_n)^2 \Delta\theta$$

$$\frac{\Delta\theta}{2\pi} = \frac{A}{\pi r^2} \qquad A = \frac{1}{2} r^2 \Delta\theta$$

$$A = \lim_{\Delta\theta \to 0} \sum_{n=1}^{N} \frac{1}{2} f(\theta_n)^2 \Delta\theta = \int_{\alpha}^{\beta} \frac{1}{2} f(\theta)^2 d\theta$$



Cardioid: $A = \int_0^{2\pi} \frac{1}{2} (1 - \cos\theta)^2 \, d\theta$

$$= \frac{1}{2} \int_0^{2\pi} (1 - 2\cos\theta + \cos^2\theta) \, d\theta$$

$$= \frac{1}{2} \left[2\pi - 0 + \int_0^{2\pi} \frac{1}{2} (1 + \cos 2\theta) \, d\theta \right.$$

$$= \frac{1}{2} \left[2\pi - 0 + \frac{1}{2} \cdot 2\pi + 0 \right]$$

$$= \frac{3\pi}{2} \qquad [a = 1], \cdot a^2 \text{ otherwise.}$$