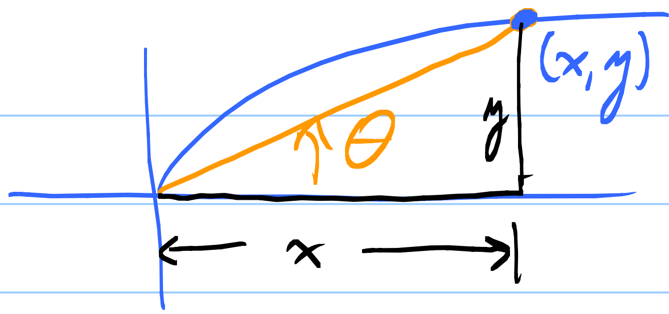
635: 32

$y=\sqrt{x}\,,\quad x=y^2$



$$\tan\theta = \frac{y}{x} = \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}} \qquad x = \frac{1}{\tan^2\theta} = \cot^2\theta$$

$=$ repeat, using $x=y^2$. $\quad y=$ eee

643: 37.

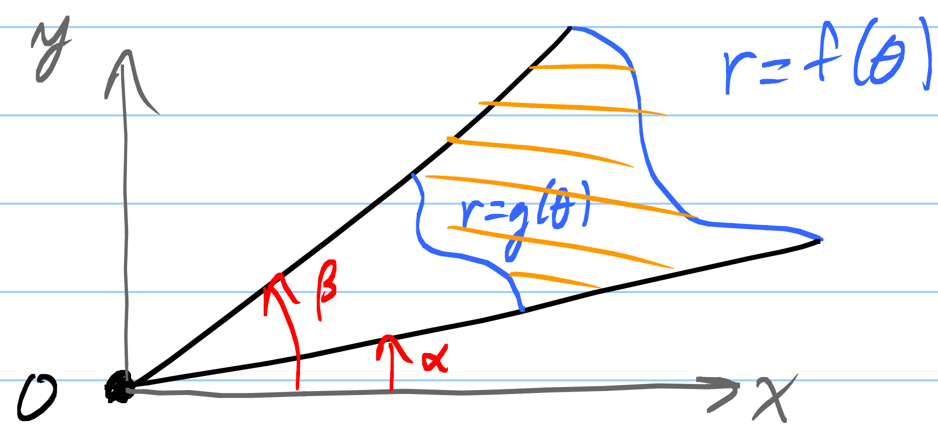
$$\bar{x} = \frac{\sum m_n \tilde{x}_n}{\sum m_n} = \frac{\rho\int f(t)\sqrt{f'^2+g'^2}\,dt}{\rho\int\sqrt{f'^2+g'^2}\,dt}$$

$(\tilde{x}_n, \tilde{y}_n)$

$m_n = \rho\Delta s$

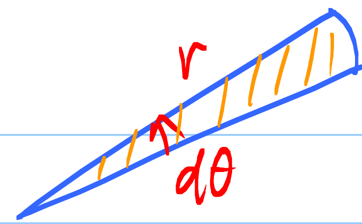
$\bar{y}$ = same thing with $g(t)$ here

Area between polar curves:



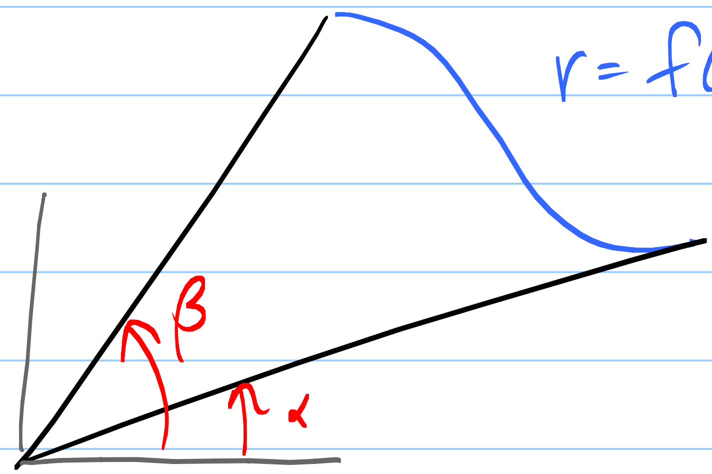
$$\text{Area} = \int_{\alpha}^{\beta} \frac{1}{2}\left[f(\theta)^2 - g(\theta)^2\right] d\theta$$

Think: $A = \int \frac{1}{2} r^2 \, d\theta$



$dA = \frac{1}{2} r^2 d\theta$

Prob: Find the length of a polar curve.

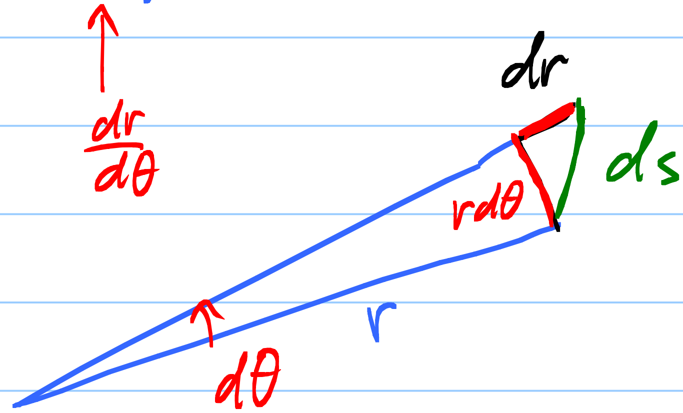


$$\begin{cases} x = r\cos\theta = f(\theta)\cos\theta \\ y = r\sin\theta = f(\theta)\sin\theta \end{cases}$$ ← parametric form of curve

11.1 $$L = \int_\alpha^\beta \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta$$

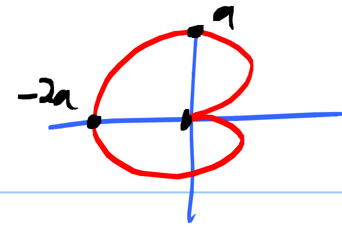
$$= \int_\alpha^\beta \sqrt{\underbrace{f(\theta)}_{r}{}^2 + \underbrace{f'(\theta)}_{\frac{dr}{d\theta}}{}^2}\, d\theta$$

How to remember:



$$ds = \sqrt{(r\,d\theta)^2 + (dr)^2}$$
$$= \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

Cardioid: $r = a(1-\cos\theta)$



$$L = \int_0^{2\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{a^2(1-\cos\theta)^2 + (a\sin\theta)^2}\, d\theta$$

$$= a\int_0^{2\pi} \sqrt{1 - 2\cos\theta + \underbrace{\cos^2\theta + \sin^2\theta}_{1}}\, d\theta$$

$$= \sqrt{2}\,a \int_0^{2\pi} \sqrt{1-\cos\theta}\, d\theta$$

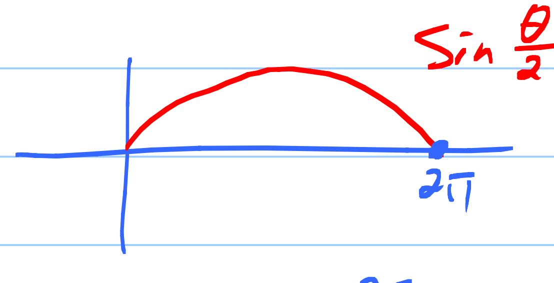
Trick: Use
$$\sin^2 \underset{\uparrow \atop \frac{\theta}{2}}{A} = \frac{1}{2}(1-\cos \underbrace{2A}_{\theta})$$

$$\boxed{1-\cos\theta = 2\sin^2\frac{\theta}{2}}$$

$$= \sqrt{2}\,a \int_0^{2\pi} \sqrt{2\sin^2\frac{\theta}{2}}\, d\theta$$

$$= 2a \int_0^{2\pi} \left|\sin\frac{\theta}{2}\right| d\theta$$

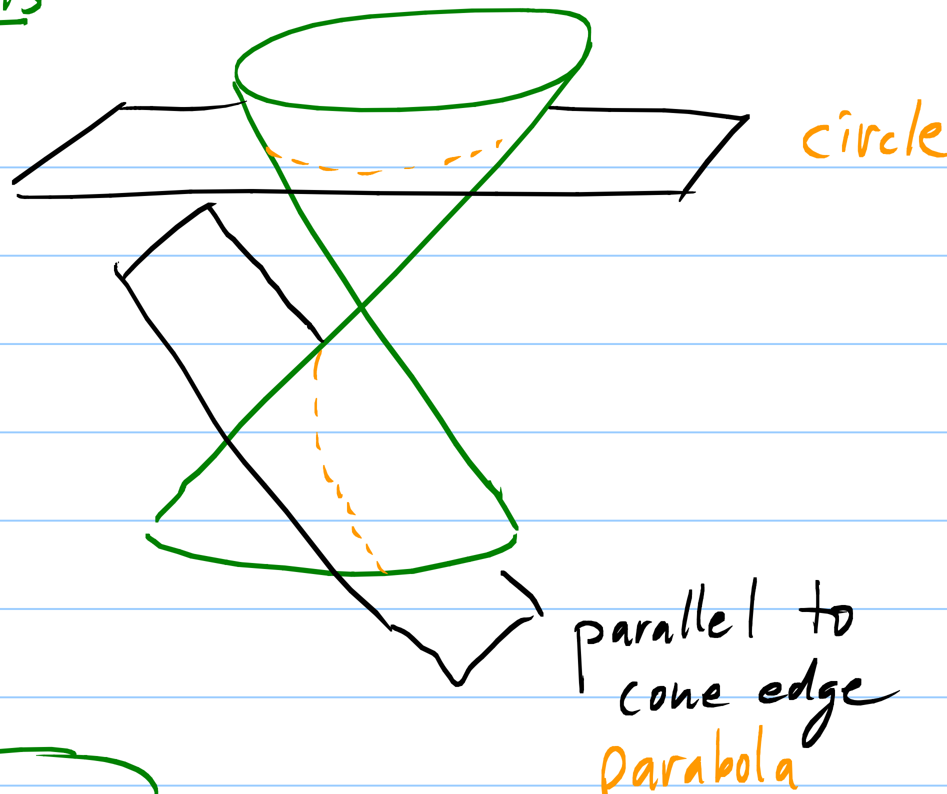
positive on $(0, 2\pi)$

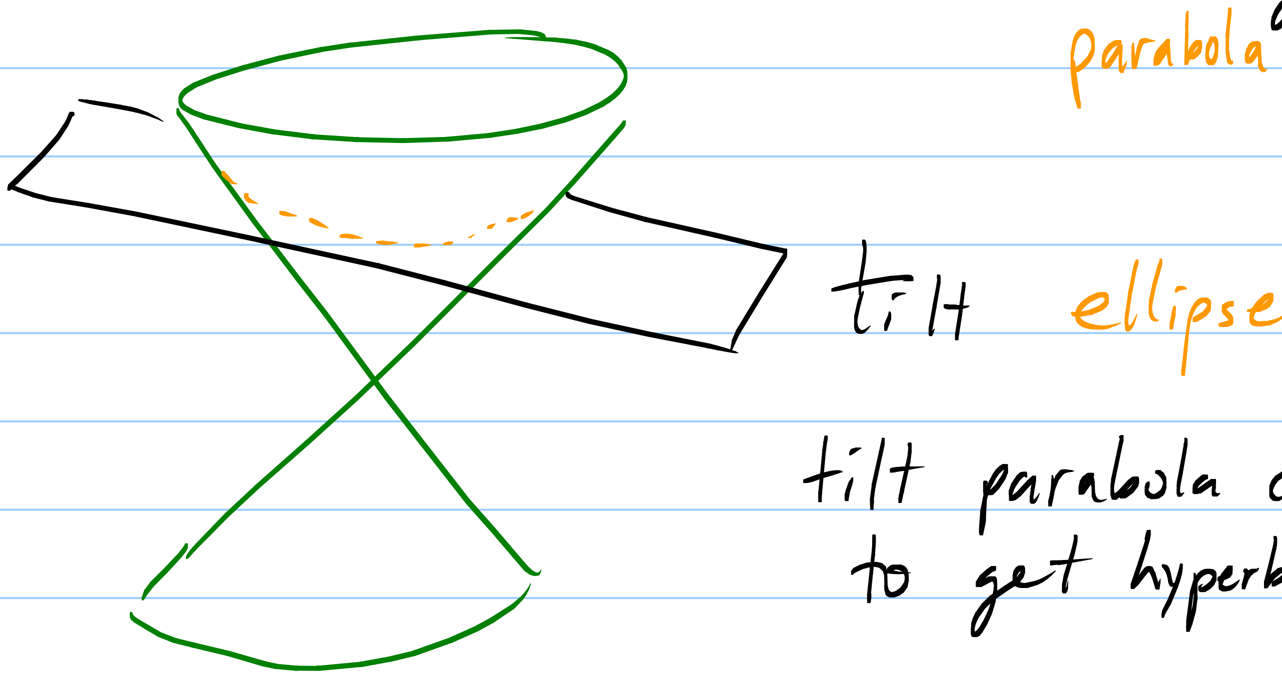


$$= 2a\int_0^{2\pi} \sin\frac{\theta}{2}\, d\theta = 2a\left[-2\cos\frac{\theta}{2}\right]_0^{2\pi}$$

$$= 2a\left[-2(-1) - (-2\cdot 1)\right] = \underline{\underline{8a}}$$

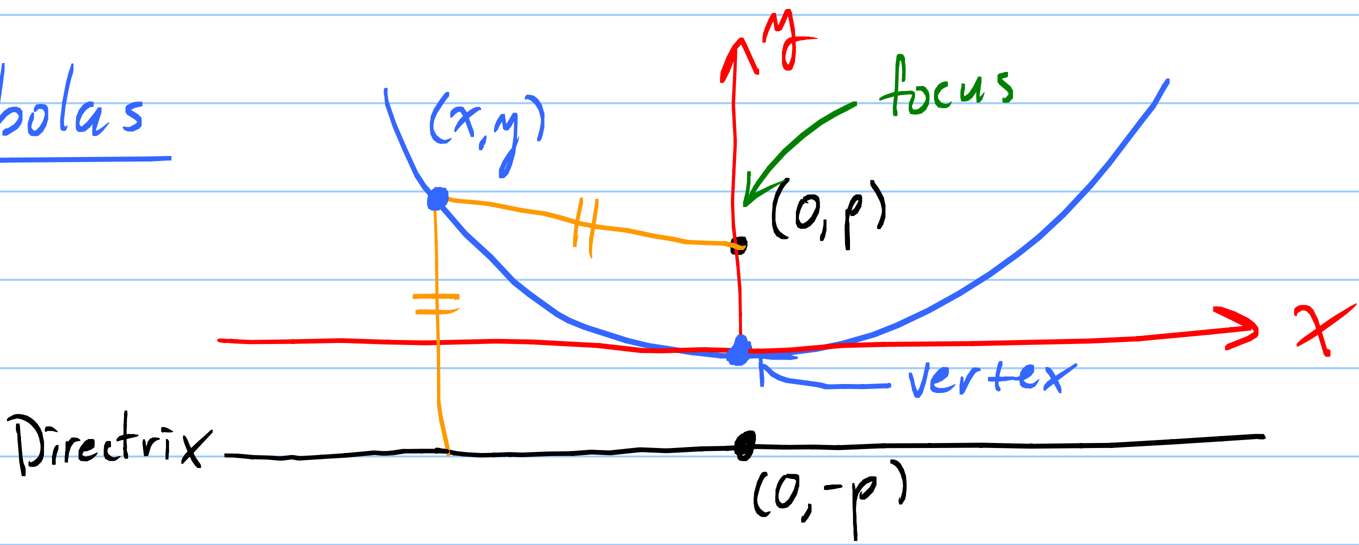
11.6 Conic Sections





tilt parabola case to get hyperbola

Parabolas



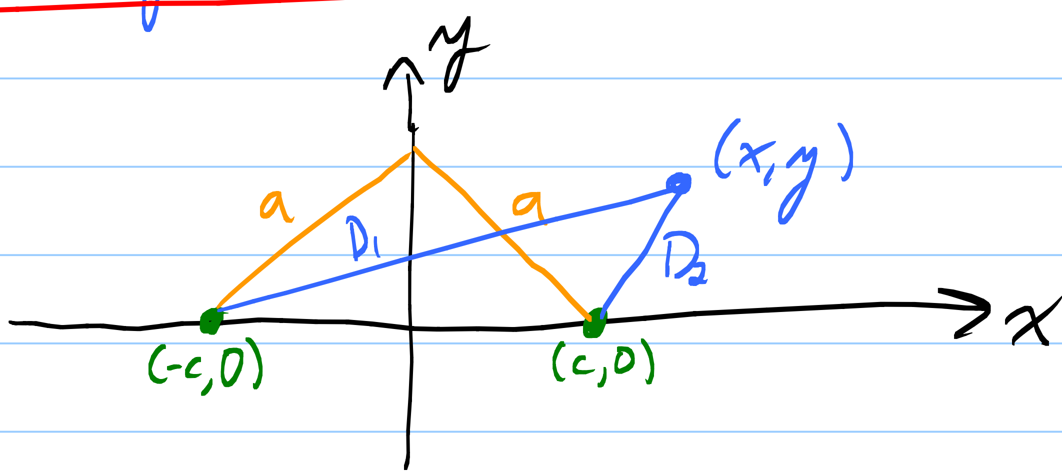
$$\underbrace{y+p}_{\text{point to line dist}} = \underbrace{\sqrt{(x-0)^2+(y-p)^2}}_{\text{point to focus}}$$

$$(y+p)^2 = x^2 + (y-p)^2$$

$$\cancel{y^2} + 2py + \cancel{p^2} = x^2 + \cancel{y^2} - 2py + \cancel{p^2}$$

$$\boxed{4py = x^2}$$

Ellipse:



$$D_1 + D_2 = 2a$$

$$\sqrt{(x-(-c))^2+y^2} + \sqrt{(x-c)^2+y^2} = 2a$$

$$\sqrt{(x+c)^2+y^2} = 2a - \sqrt{(x-c)^2+y^2}$$

$$(x+c)^2 + \cancel{y^2} = 4a^2 - 4a\sqrt{(x-c)^2+y^2} + (x-c)^2 + \cancel{y^2}$$

$$4a\sqrt{(x-c)^2+y^2} = 4a^2 + (x-c)^2 - (x+c)^2$$

$$= 4a^2 + \cancel{x^2} - 2xc + \cancel{c^2} \quad \cancel{-x^2} - 2cx - \cancel{c^2}$$

$$= 4a^2 - 4cx$$

$$a\sqrt{(x-c)^2+y^2} = a^2 - cx$$

$$a^2\left[(x-c)^2+y^2\right] = a^4 - 2a^2cx + c^2x^2$$

$$a^2x^2 - \cancel{2a^2cx} + a^2c^2 + a^2y^2 = a^4 - \cancel{2a^2cx} + c^2x^2$$

$$(a^2-c^2)x^2 + a^2y^2 = a^4 - a^2c^2 = a^2(a^2-c^2)$$

$$\frac{x^2}{a^2} + \frac{y^2}{\underbrace{a^2-c^2}_{b^2}} = 1$$