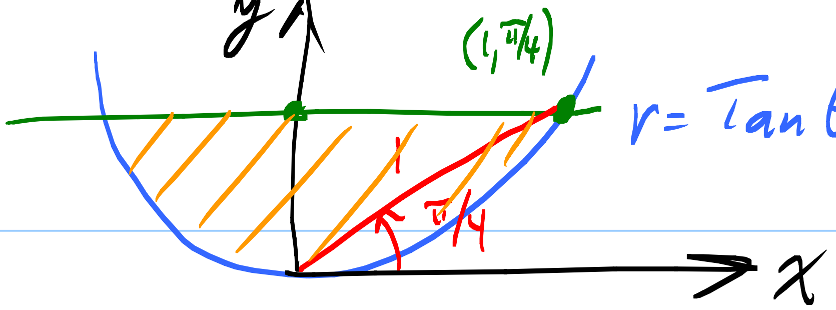
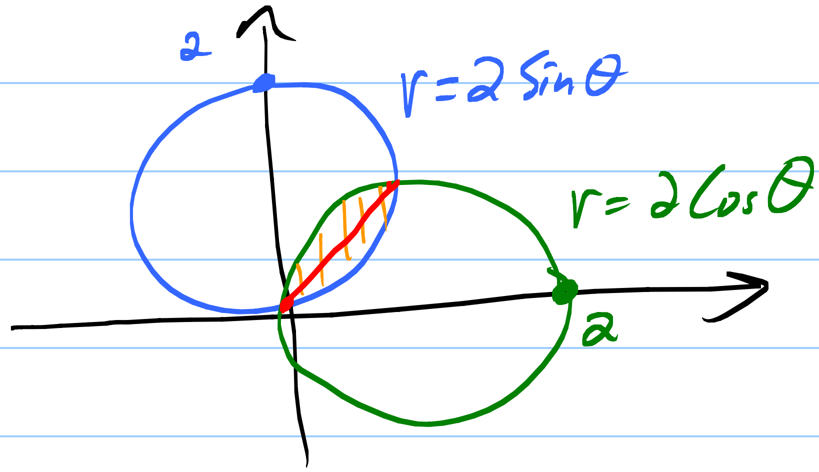
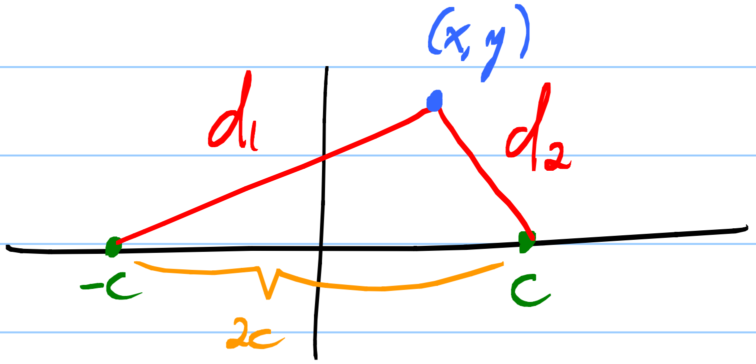
656 : 19.



9.



Hyperbolas :



$$d_1 - d_2 = \text{const.} = 2a$$

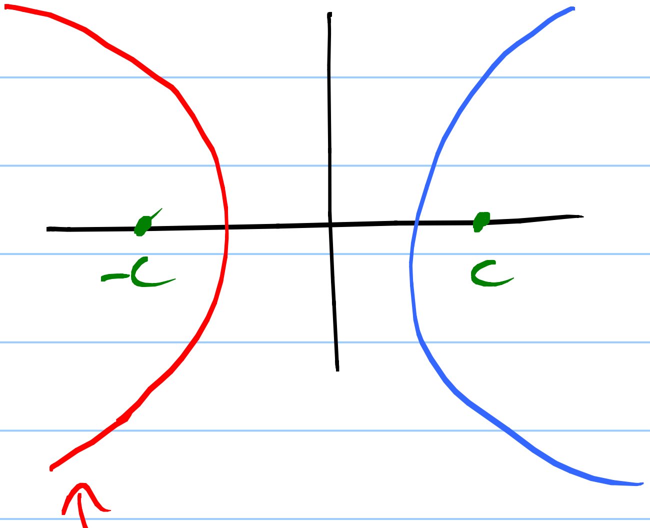
$$\sqrt{(x+c)^2 + y^2} - \sqrt{(x-c)^2 + y^2} = 2a$$

Algebra : $$\frac{x^2}{a^2} - \frac{y^2}{\underbrace{c^2 - a^2}_{\substack{>0 \\ \text{call it } b^2.}}} = 1$$

b>0 : why : $2c + d_2 > d_1$ (Δ fact)

$$2c > d_1 - d_2 = 2a$$

So $c > a$ and $\underbrace{c^2 - a^2}_{b^2} > 0$.



get this half by setting $d_2 - d_1 = 2a$

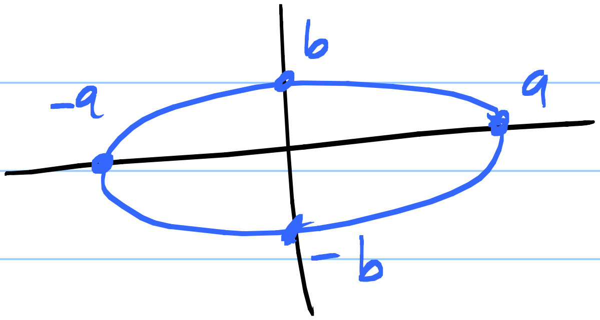
So $d_1 - d_2 = \pm 2a$ gives both sides.

Hyperbola eqn: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Ellipse eqn: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

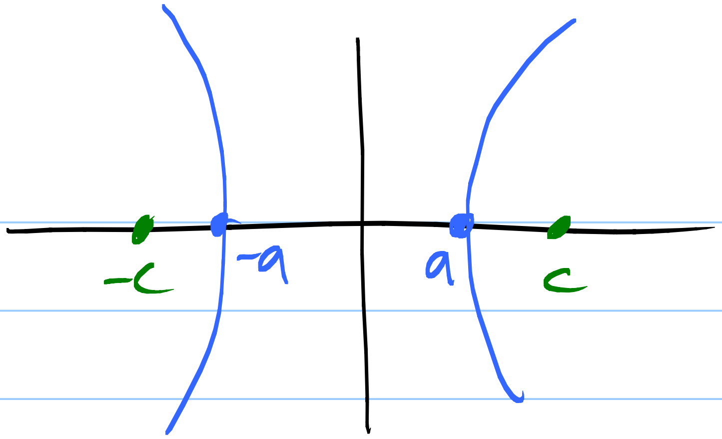
$y = 0$: $x = \pm a$

$x = 0$: $y = \pm b$



What is b all about for hyperbolas?

$y = 0$: $\frac{x^2}{a^2} - \frac{0^2}{b^2} = 1$ $x^2 = a^2$ $x = \pm a$.



b shows up when think about $\lim_{x \to \infty} y(x)$.

Fact: $\lim_{B \to \infty} \underbrace{B - \sqrt{B^2 - 1}}_{>0 \text{ if } B>1.} = 0$

Feel: $\sqrt{B^2-1} \approx \sqrt{B^2} = B$

Think: $\left(B - \sqrt{B^2-1}\right) \cdot \left(\frac{B+\sqrt{B^2-1}}{B+\sqrt{B^2-1}}\right)$

$$\frac{B^2 - (B^2-1)}{B+\sqrt{B^2-1}} = \frac{1}{B+\sqrt{B^2-1}} \to 0$$

as $B \to \infty$.

OR $B - \sqrt{B^2-1} = B\left(1 - \sqrt{1 - \frac{1}{B^2}}\right)$

$$= \frac{1-\sqrt{1-\frac{1}{B^2}}}{\left(\frac{1}{B}\right)} = \frac{1-\sqrt{1-x}}{x}$$

as $x = \frac{1}{B} \to 0$. Use L'H.

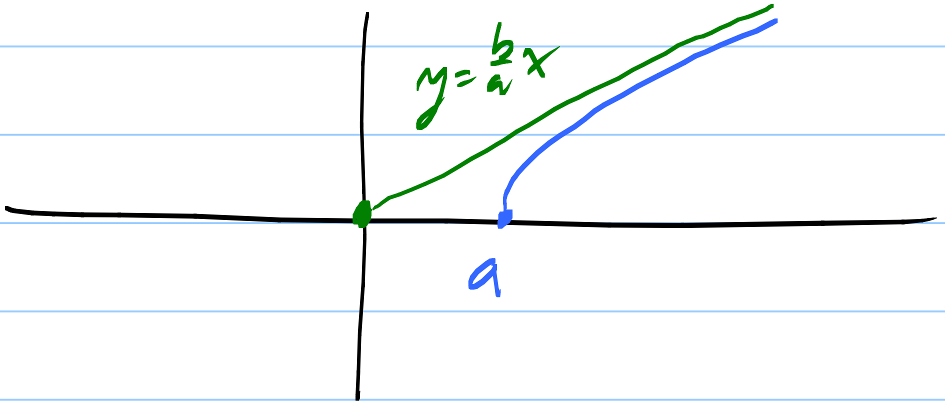
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

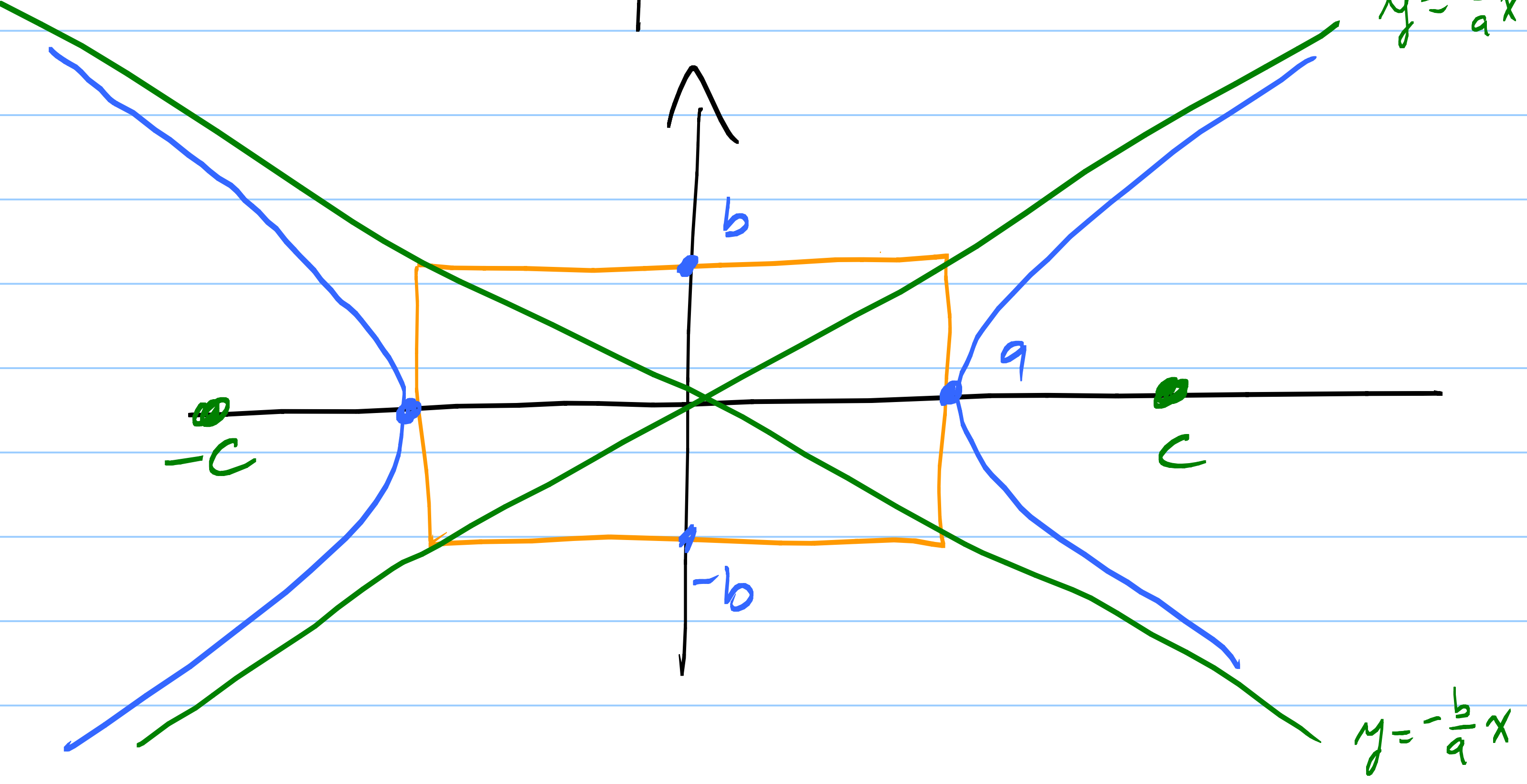
$$y = \pm b\sqrt{\frac{x^2}{a^2} - 1}$$

Plus case:

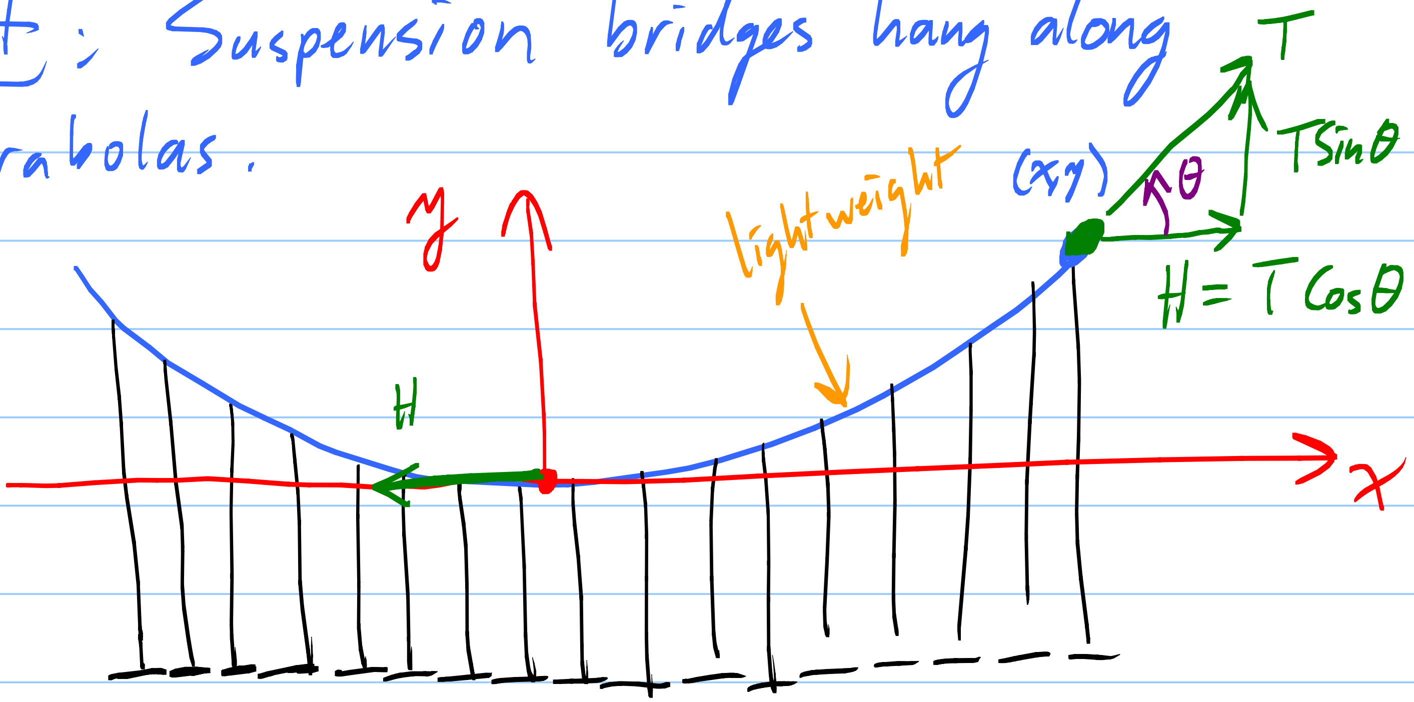
$$y = b\sqrt{\frac{x^2}{a^2} - 1} < b\sqrt{\frac{x^2}{a^2}} < b\frac{x}{a}$$

Take $B = \frac{x}{a}$. These get closer and closer as $x \to \infty$.





Fact: Suspension bridges hang along parabolas.



$H = T\,\mathrm{Cos}$ ← net horizontal forces = 0

$T\,\mathrm{Sin}\,\theta$ = weight held up by cable

$= (\rho x) g$

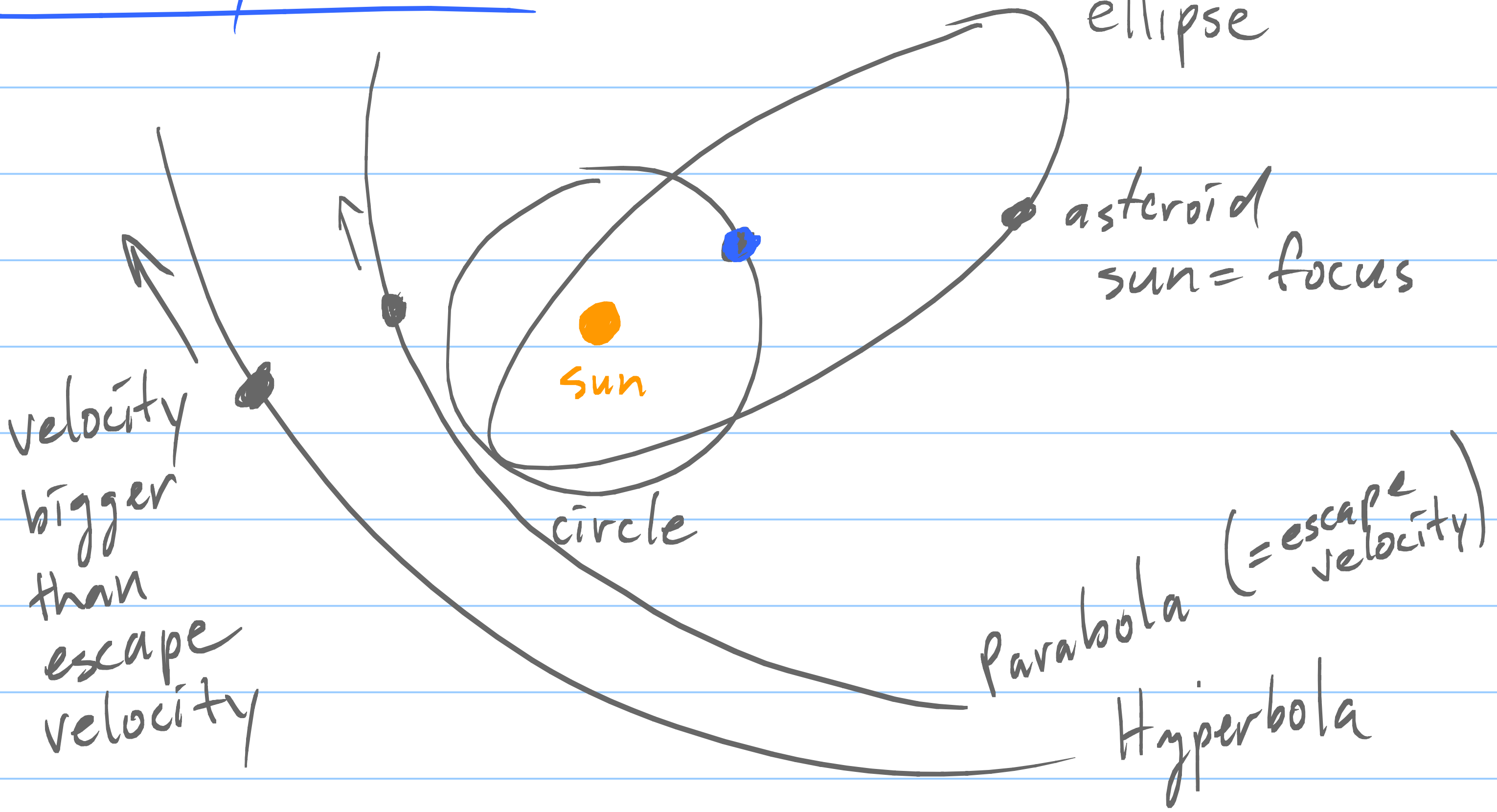
$$\frac{dy}{dx} = \mathrm{Tan}\,\theta = \frac{T\,\mathrm{Sin}\,\theta}{T\,\mathrm{Cos}\,\theta} = \frac{(\rho x) g}{H} = Kx$$

$$y = \int Kx\,dx = \tfrac{1}{2}Kx^2 + C$$

$y(0) = 0$ gives $c = 0$.

Cable is in shape of parabola $y = \frac{1}{2}Kx^2$.

Planetary motion:



Polar coordinates:

$$r = \frac{K\epsilon}{1 - \epsilon \cos\theta}$$

One size fits all!

$$\epsilon = \frac{c}{a}$$

$$\underset{\substack{\uparrow\\ \text{circle}}}{0} < \underset{\substack{\uparrow\\ \text{ellipse}}}{\epsilon} < \underset{\substack{\uparrow\\ \text{parabola}}}{1} < \underset{\substack{\uparrow\\ \text{hyperbola}}}{\epsilon}$$

Complex numbers. A.7 next week.