$$\int_0^{\pi/4} \sin^7 x\, dx = \int \underbrace{\sin^6 x}_{\substack{(\sin^2 x)^3 \\ (1-\cos^2 x)^3}} \underbrace{(\sin x\, dx)}_{-d\cos x}$$

$$= -\int_{u=1}^{\sqrt{2}/2} (1-u^2)^3\, du$$

256 : 28. $r = \sqrt{1+\sin 2\theta}$ $\quad 0 \le \theta \le \pi\sqrt{2}$

$$L = \int_0^{\pi\sqrt{2}} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$= \int_0^{\pi/\sqrt{2}} \sqrt{1+\sin 2\theta + \left[\tfrac{1}{2}(1+\sin 2\theta)^{-1/2} 2\cos 2\theta\right]^2}\, d\theta$$

$1-\sin^2 2\theta = (1-\sin 2\theta)(1+\sin 2\theta)$

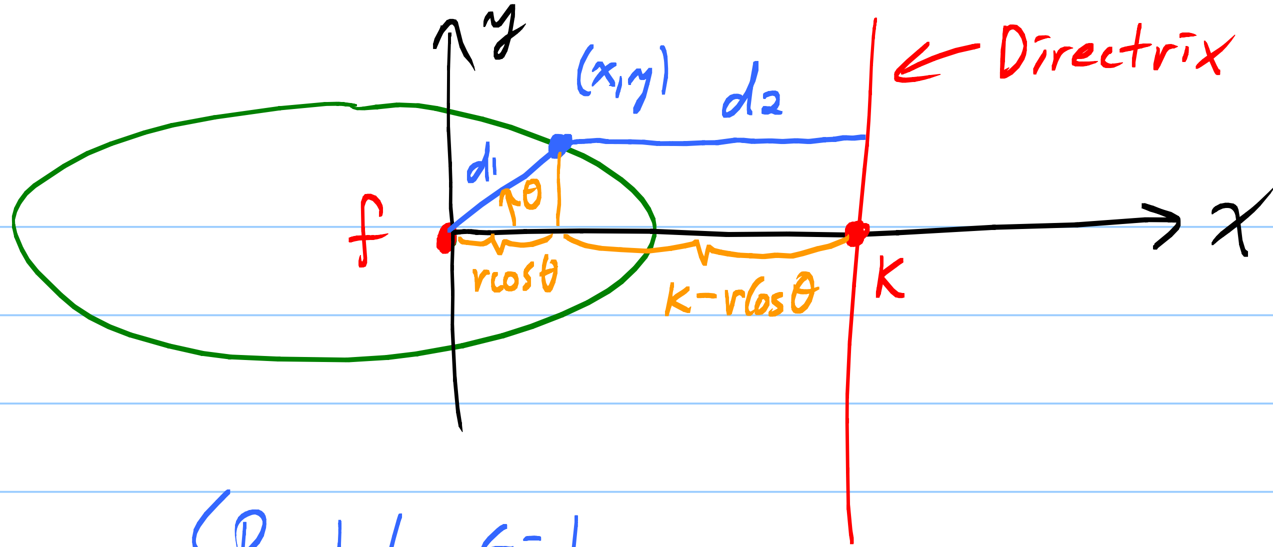
$$= \int_0^{\pi\sqrt{2}} \sqrt{1+\sin 2\theta + \frac{\cos^2 2\theta}{1+\sin 2\theta}}\, d\theta$$

Conic Sections in polar coordinates:

$$r = \frac{K\epsilon}{1+\epsilon\cos\theta}$$ one size fits all!

$0 < \epsilon < 1$ Ellipse. $\quad \epsilon = 1$ Parabola

[$\epsilon = 0$ circle ($r = $ const)] $\quad \epsilon > 1$ hyperbola



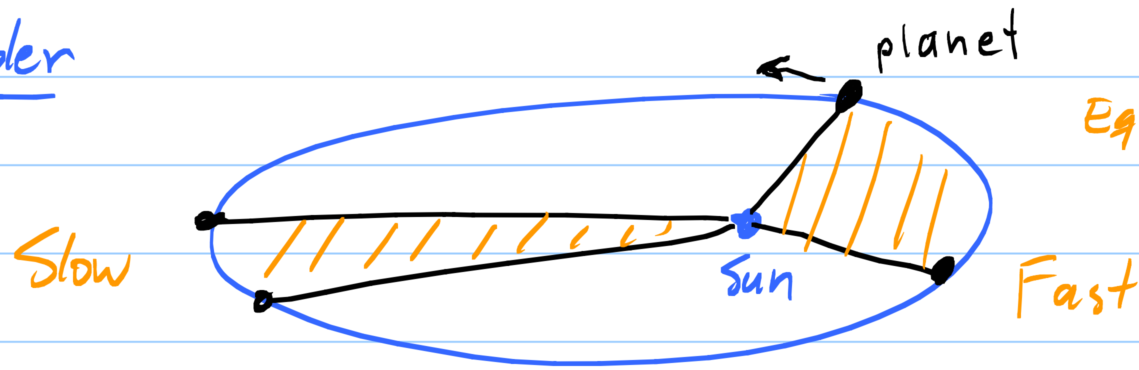
$$d_1 = \epsilon d_2 \begin{cases} \text{Parabola} & \epsilon = 1 \\ (0 < \epsilon < 1) & \text{ellipse} \\ 1 < \epsilon & \text{hyperbola} \end{cases}$$

$$d_1 = r = \epsilon d_2 = \epsilon (K - r\cos\theta)$$

$$r(1 + \epsilon\cos\theta) = \epsilon K$$

$$r = \frac{\epsilon K}{1 + \epsilon\cos\theta}$$

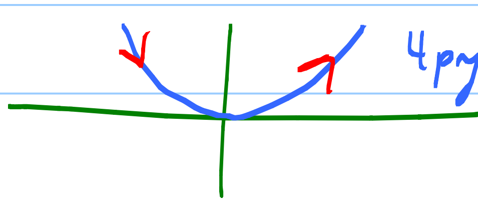
Kepler



Equal areas in equal time.

Conic Sections in parametric form:

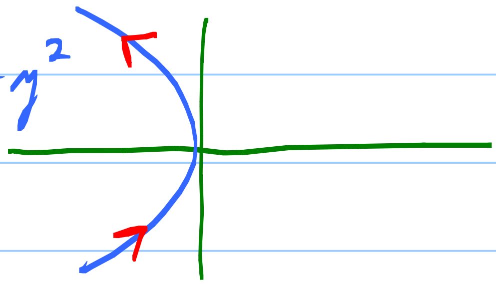
Parabolas:



$4py = x^2$ $\qquad$ $y = \frac{1}{4p} x^2$

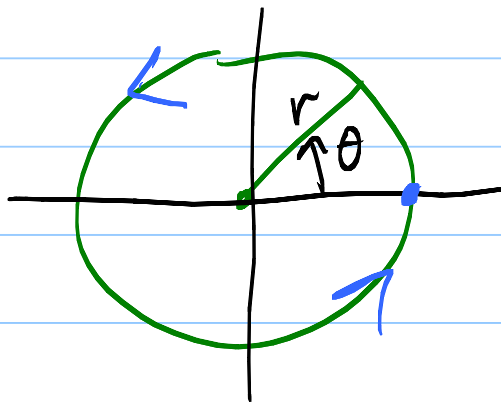
Kind of dumb: $\begin{cases} x = t \\ y = \frac{1}{4p} t^2 \end{cases}$ $\quad -\infty < t < \infty$

$-4px = y^2$



Not as dumb: $\begin{cases} x = -\frac{1}{4p} t^2 \\ y = t \end{cases}$ $\quad -\infty < t < \infty$

Circles:

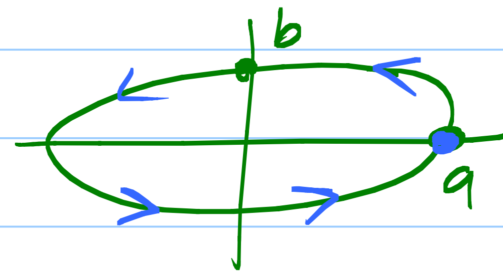


$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$

$0 \le \theta \le 2\pi$

Ellipses:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



$$\Big(\underset{\uparrow \atop \cos\theta}{\frac{x}{a}}\Big)^2 + \Big(\underset{\uparrow \atop \sin\theta}{\frac{y}{b}}\Big)^2 = 1$$

$$\cos^2\theta + \sin^2\theta = 1$$

$\begin{cases} x = a\cos\theta \\ y = b\sin\theta \end{cases}$

$0 \le \theta \le 2\pi$

Hyperbolas:

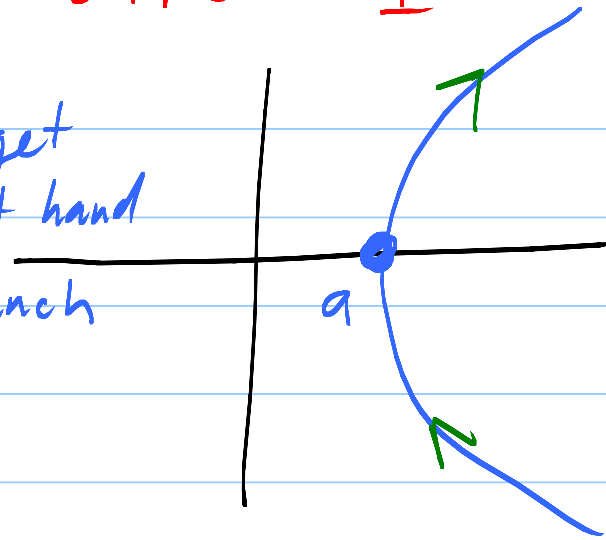
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Big(\underset{\uparrow \atop \cosh t}{\frac{x}{a}}\Big)^2 - \Big(\underset{\uparrow \atop \sinh t}{\frac{y}{b}}\Big)^2 = 1$$

$$\cosh^2 t - \sinh^2 t = 1$$

$$\begin{cases} x = a\cosh t \\ y = b\sinh t \end{cases}$$
$-\infty < t < \infty$

(only get right hand branch)



Other half: $x = -a\cosh t$
$y = b\sinh t$
$-\infty < t < \infty$

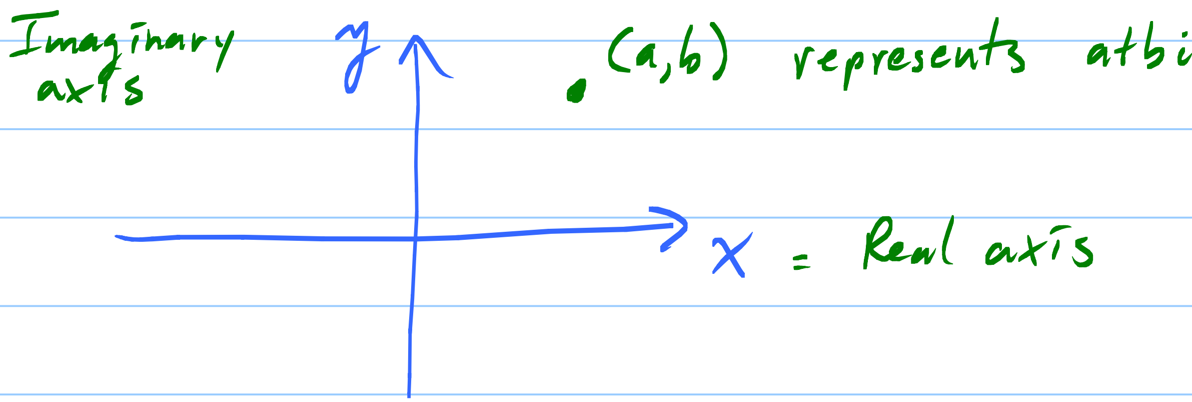
Complex numbers: $x^2 + 1 = 0$ $\quad x^2 = -1$.

"Imaginary number" i such that $i^2 = -1$.

$x^2 + 1 = 0$ has roots $x = \pm i$.

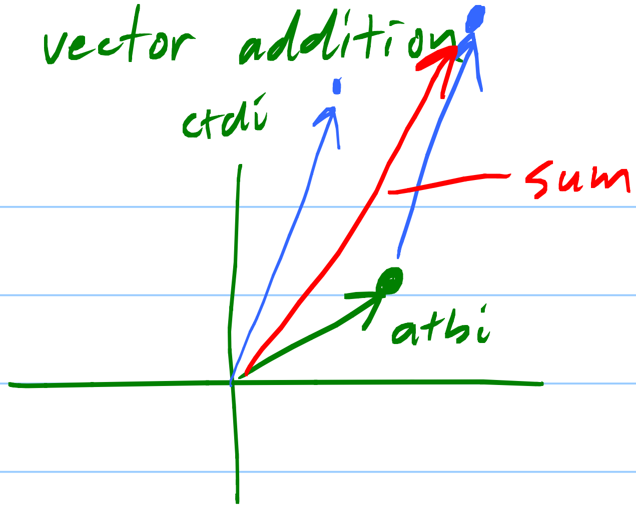
Add i to $\mathbb{R}$. Get complex numbers:

$\mathbb{C} = \{a + bi : a, b \in \mathbb{R}\}$.



Add them: $(a+bi) + (c+di) = (a+c) + (b+d)i$

Same as vector addition



<u>Multiply them</u>: $(a+bi)(c+di) = ac + adi + bci + bdi^2$

$i^2 = -1$

$= (ac-bd) + (ad+bc)i$

Geometric meaning? Later.

<u>Division?</u> $(a+bi)\underbrace{(x+iy)}_{\text{multiplicative inverse of } a+bi} = 1$

Look for x, y so that

$$\underbrace{(ax-by)}_{\text{need}=1} + \underbrace{(ay+bx)}_{\text{need}=0}i = 1 + 0i$$

want

Need $\begin{cases} ax - by = 1 \\ bx + ay = 0 \end{cases}$

Cramer's Rule:

$$x = \frac{\det\begin{bmatrix} 1 & -b \\ 0 & a \end{bmatrix}}{\det\begin{bmatrix} a & -b \\ b & a \end{bmatrix}} = \frac{a}{a^2+b^2}$$

not zero if $a+bi \neq 0$!

$$y = \frac{\det\begin{bmatrix} a & 1 \\ b & 0 \end{bmatrix}}{\det\begin{bmatrix} a & -b \\ b & a \end{bmatrix}} = \frac{-b}{a^2+b^2}$$

Notation: $(a+bi)^{-1} = \dfrac{a}{a^2+b^2} - \dfrac{b}{a^2+b^2}\, i$

$\dfrac{1}{(a+bi)}$

High school: $\dfrac{1}{a+bi} \cdot \dfrac{a-bi}{a-bi} = \dfrac{a-bi}{a^2+b^2}$