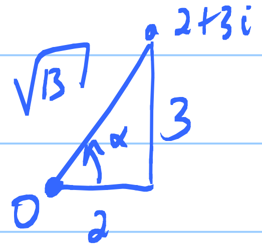
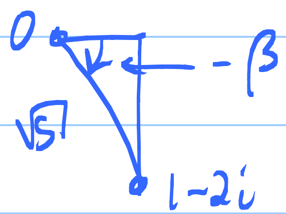
Please do evaluation for C. He (online). Me too.

Ap-34: 14. $(2+3i)(1-2i) =$ easy



$\alpha = \text{ArcTan}\,\frac{3}{2}$

$2+3i = \sqrt{13}\, e^{i\alpha}$ where $\alpha = \text{ArcTan}\,\frac{3}{2}$.



$\beta = \text{ArcTan}\,\frac{2}{1}$

$1-2i = \sqrt{5}\, e^{-i\beta}$

$(2+3i)(1-2i) = \sqrt{13}\cdot\sqrt{5}\; e^{i(\alpha-\beta)}$

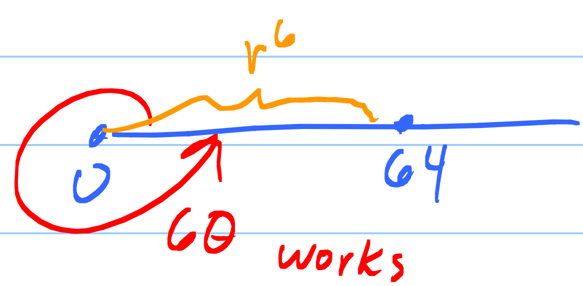
20. Find all z such that $z^6 = 64$.

<u>One way</u>: $z^6 - 64 = (z^3-8)(z^3+8)$

$= \left[(z-2)(z^2+2z+4)\right]\left[(z+2)(z^2-2z+4)\right]$

<u>Better way</u>: $z = re^{i\theta}$. Want $(re^{i\theta})^6 \overset{\text{want}}{=} 64$

$r^6 e^{i6\theta} = 64$

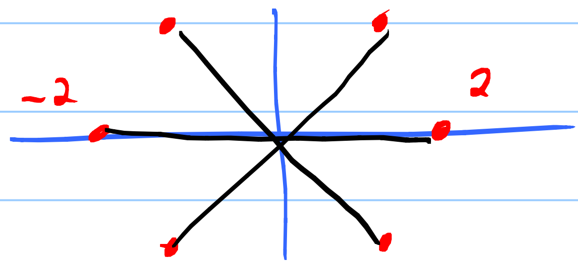


$r^6 = 64$ $\boxed{r=2}$

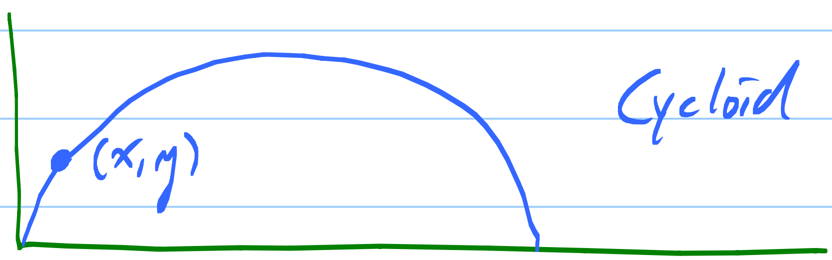
$6\theta = 0 + 2\pi n \quad n = 0, \pm 1, \pm 2, \ldots$

$\theta = \frac{\pi n}{3}, \quad n = 0, \pm 1, \pm 2, \ldots$

Six roots: $2e^{i\pi n/3} \quad n = 0, 1, 2, 3, 4, 5$



Practice Problems: 1. $\begin{cases} x(t) = t - \sin t \\ y(t) = 1 - \cos t \end{cases} \quad 0 \le t \le 2\pi$



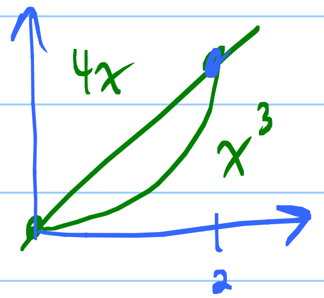
$$\frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} = \frac{\sin t}{1 - \cos t} = \frac{0}{0} \text{ at } t = 0.$$

Use L'H. See $\frac{dy}{dx} \to +\infty$ as $t \to 0$.

Or: $\frac{dx}{dy} = \frac{(dx/dt)}{(dy/dt)} \xrightarrow[\text{L'H}]{\frac{0}{0}} 0$. Vertical tangent.

Best: DQ for $\frac{dx}{dy} = \lim_{t\to 0} \frac{x(t)-0}{y(t)-0} = \lim_{t\to 0}\frac{x(t)}{y(t)} \underset{L'H}{\overset{0/0}{=}}$

$= \lim_{t\to 0} \frac{x'(t)}{y'(t)} \underset{L'H}{\overset{0/0}{=}} \frac{0}{1} = 0$

3. 

$S_2' = \int_0^2 2\pi x\sqrt{1+(3x^2)^2}\,dx$

$S_1' =$ cone part.

$S' = S_1 + S_2$.

5. $\frac{d}{dx}\left[x^{(x^2)}\right] = ?$ $\qquad y = x^{(x^2)}$

$\ln y = \ln x^{(x^2)} = x^2 \ln x$

Diff'tiate both sides: Logarithmic Diff'tion.

$\frac{1}{y}\frac{dy}{dx} = 2x\ln x + x^2\cdot\frac{1}{x}$

$\frac{dy}{dx} = \left(2x\ln x + x\right)\underbrace{x^{(x^2)}}_{y}$

OR $x^{(x^2)} = e^{\ln x^{(x^2)}} = e^{x^2\ln x}$ use chain rule.

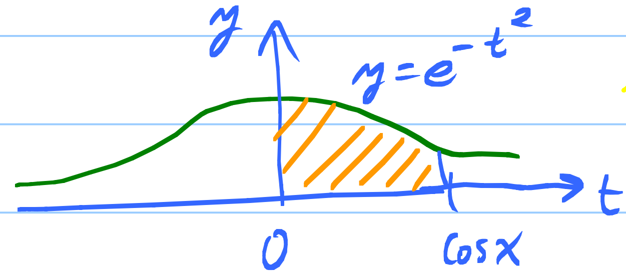
$$\frac{d}{dx}\left[2^{\sin^{-1}x}\right] = ?$$

$$y = 2^{\sin^{-1}x} = e^{(\sin^{-1}x)\ln 2}$$

$$\frac{dy}{dx} = \Big[\underbrace{e^{(\sin^{-1}x)\ln 2}}_{2^{\sin^{-1}x}}\Big] \cdot \frac{1}{\sqrt{1-x^2}} \cdot \ln 2$$

Fund. Thm. Calc. $\quad \frac{d}{dx}\int_a^x f(t)\,dt = f(x).$

$$\frac{d}{dx}\int_0^{\cos x} e^{-t^2}\,dt$$



$y = \int_0^u e^{-t^2}\,dt$ where $u = \cos x$.

Chain Rule! $\quad \frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$

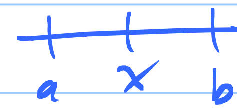
$$= e^{-u^2}(-\sin x)$$

$$= e^{-\cos^2 x}(-\sin x)$$

Old Final. 1. $\quad \int_{-x}^{x} \frac{(t+1)^2}{1+t^5}\,dt \qquad 0 \le x < 1$

$$\int_{-x}^{x} = \int_{-x}^{0} + \int_{0}^{x}$$

How to diff'tiate $\int_x^b f(t)\,dt$



$$\int_x^b = \int_a^b - \int_a^x$$

const.

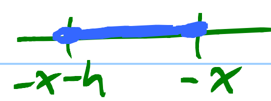
$$\frac{d}{dx}\int_x^b f(t)\,dt = -\frac{d}{dx}\int_a^x f(t)\,dt = -f(x)$$

Finally, $y=\int_{-x}^{0} f(t)\,dt = \int_u^0 f(t)\,dt$ where $u=-x$.

$$\frac{dy}{dx} = -f(u)\frac{dy}{dx} = -f(-x)(-1) = f(-x).$$

Ans: 1. $f(x)+f(-x) = \dfrac{(x+1)^2}{1+x^5} + \dfrac{(-x+1)^2}{1+(-x)^5}$

Nice thing to do: $F(x)=\int_{-x}^{0} f(t)\,dt$



$$\frac{F(x+h)-F(x)}{h} = \frac{\int_{-(x+h)}^{0} f(t)\,dt - \int_{-x}^{0} f(t)\,dt}{h}$$

$$= \frac{1}{h}\int_{-x-h}^{-x} f(t)\,dt = f(c)$$

MVT

for $\int$'s

where c is between $-x-h$ and $-x$.

Let $h \to 0$. Then c slides into $-x$.

Get $\lim_{h\to 0} DQ = f(-x)$.

PP. 8. $\frac{dP}{dt} = kP$ $\qquad P(t) = P_0 e^{kt}$

$t=0$ at 1900. $\qquad P_0 = 75$ million

So $P = 75e^{kt}$.

Given $P(50) = 150$ million $= 75e^{k\cdot 50}$

1950

$$2 = e^{50k}$$

$$\ln 2 = \ln e^{50k} = 50k$$

$$\boxed{k = \frac{1}{50}\ln 2}$$

$$P(90) = 75e^{(\frac{1}{50}\ln 2)90}$$

1990