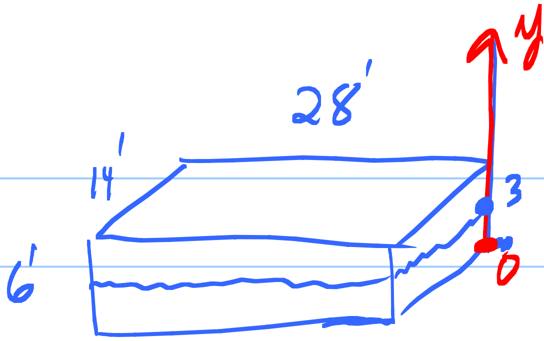
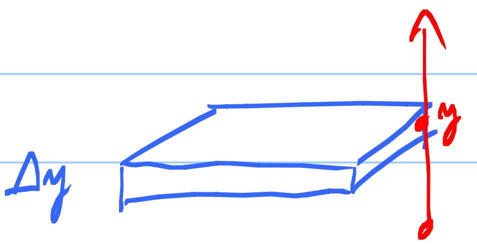


$62.4 \ \text{lbs}/ft^3$

$$\Delta W = mgh$$



$$\Delta W = (14 \cdot 28)\,\Delta y \cdot (62.4)\,[6-y]$$

$$W = \sum \Delta w = \int_0^3 (14 \cdot 28 \cdot 62.4)\,(6-y)\,dy$$

PP. 11. $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

$$\left| f(x) - \underbrace{P_N(x)}_{\sum_{n=0}^{N} \frac{f^{(n)}(0)}{n!} x^n} \right| = \left| \underbrace{R_N(x)}_{\frac{f^{(n+1)}(c)}{(n+1)!} x^{n+1}} \right|$$

Tricky point: $1 - \frac{x^2}{2!} = P_2(x)$ <u>and</u> $P_3(x)$

use this one!

$$\left| \cos x - \left(1 - \frac{x^2}{2!}\right) \right| = \left| R_3(x) \right| = \left| \frac{\frac{d^4}{dx^4} \cos x \Big|_{x=c}}{4!} x^4 \right|$$

$$= \left|\frac{\sin c}{4!}\right| x^4 \le \frac{x^4}{4!} \le \frac{(1/2)^4}{4!} \text{ if } |x| \le \tfrac{1}{2}.$$

$$\frac{1}{\underbrace{2\cdot 3\cdot 4}\cdot 16} < \frac{1}{200} \checkmark$$

$2\cdot3\cdot4\cdot16 = 384$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$\left| \sin x - \underbrace{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!}\right)}_{\text{alt. series}} \right| < \frac{x^7}{7!}$$

alt. series $\sum_{n=1}^{N} (-1)^{n+1} a_n$

a_{N+1} alt. series error estimate

or use Taylor's Thm with remainder.

13. $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$. Try limit comparison with a p-series $\sum \frac{1}{n^p}$.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\left(\frac{\ln n}{n^2}\right)}{\left(\frac{1}{n^p}\right)} = \lim_{n\to\infty} \frac{\ln n}{n^{2-p}} = 0 \text{ if } \begin{array}{l} 2-p>0 \\ p<2 \end{array}$$

and $\sum \frac{1}{n^p}$ converges if $p > 1$.

Pick p between 1 and 2, say $p = \frac{3}{2}$.

Conclude that $\sum_{1}^{\infty} \frac{\ln n}{n^2}$ converges.

More standard problem: $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3+1}}$

Compare to $\frac{1}{\sqrt{n}}$. $\sum \frac{1}{\sqrt{n}}$ diverges $(p=\frac{1}{2})$.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\left(\frac{n}{\sqrt{n^3+1}}\right)}{\left(\frac{1}{\sqrt{n}}\right)} = \lim_{n\to\infty} \frac{1}{\left(1+\frac{1}{n^3}\right)^{1/2}} = 1$$

diverges too

Think: $\sqrt{12n^4+7n^3+n^2+1} \sim \sqrt{12n^4}$ for big n.

$$\frac{n}{\sqrt{n^3+1}} = \frac{n}{n^{3/2}\left(1+\frac{1}{n^3}\right)^{1/2}} = \frac{1}{\sqrt{n}\left(1+\frac{1}{n^3}\right)^{1/2}}$$

13. $\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{n}$

Try alt. series test $\sum (-1)^{n+1} a_n$

1) $a_n > 0$ for $n > N$?

2) $a_n \to 0$ as $n \to \infty$?

3) $a_N > a_{N+1} > a_{N+2} > \cdots$?

Oops! $\frac{2^n}{n} \to \infty$ as $n \to \infty$.

N-th term test: If the terms do not tend to zero, then the series diverges.

$$\lim_{x\to\infty} \frac{2^x}{x} = \lim_{x\to\infty} \frac{e^{x\ln 2}}{x} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}} \lim_{x\to\infty} \frac{\ln 2\, e^{x\ln 2}}{1} = \infty$$

13. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+1}}$

$\frac{1}{\sqrt{n^2+1}} > 0$ ✓

$\lim_{n\to\infty} \frac{1}{\sqrt{n^2+1}} = 0$ ✓

$\frac{1}{\sqrt{1^2+1}} > \frac{1}{\sqrt{2^2+1}} > \frac{1}{\sqrt{3^2+1}} > \cdots$ ✓

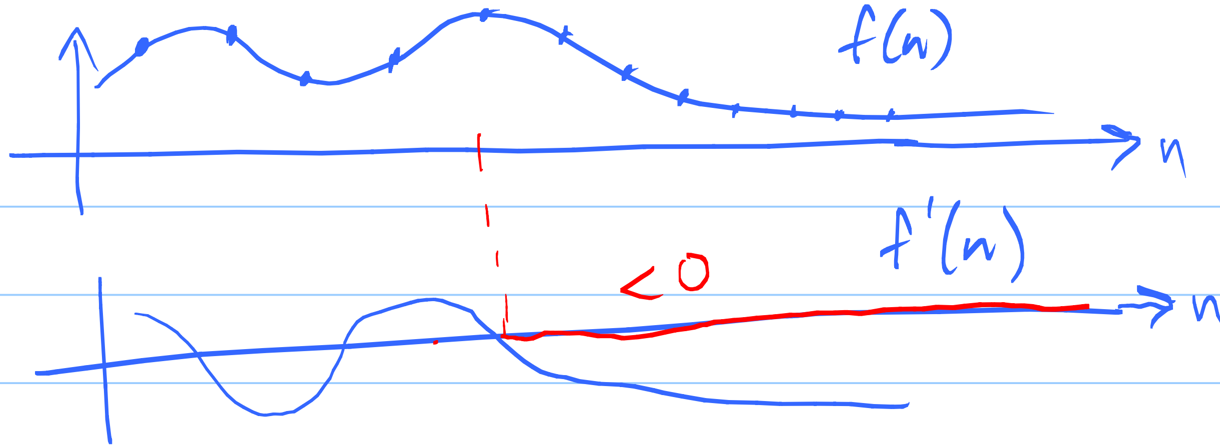
Alt. Series Test: It converges.

Absolutely? $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$. Limit comparison with $\sum \frac{1}{n}$, which diverges. See $\sum \frac{1}{\sqrt{n^2+1}}$ diverges. Given series above is <u>conditionally convergent</u>.

Warning: Alt. Series is a <u>one-way</u> test. If conditions hold, get convergence. <u>But</u> if conditions fail, can't conclude divergence.

<u>Important tool:</u> $a_n = f(n)$

Too see a_n's are decreasing for $n > N$, check $f'(n)$ and see that it is < 0 for $n > N$.



Ex: $\sum (-1)^n \frac{n^2}{e^n}$ $\qquad f(n) = \frac{n^2}{e^n}$

$$f'(n) = \frac{2ne^n - n^2e^n}{e^{2n}} = \frac{n(2-n)}{e^n} < 0 \text{ if } n > 2$$

1. $\frac{n^2}{e^n} \to 0$ as $n \to \infty$ ✓
2. decreasing after $n = 2$ ✓
3. $\frac{n^2}{e^n} > 0$ ✓

Absolute conv.? Ratio test $\left|\frac{u_{n+1}}{u_n}\right| = \left|\frac{(-1)^{n+1} \frac{(n+1)^2}{e^{n+1}}}{(-1)^n \frac{n^2}{e^n}}\right|$

$$= \frac{(n+1)^2}{n^2} \frac{1}{e} = \left[1 + \frac{1}{n}\right]^2 \frac{1}{e} \to \frac{1}{e} \text{ as } n \to \infty$$

$e \approx 2.7\ldots$ so $\frac{1}{e} < 1$. Absolute Convergence.