MATH 181 Exam I solutions.

1. $$\lim_{x\to 0} \frac{3x-\sin 3x}{x^3} \overset{\left(\frac{0}{0}\right)}{\underset{L'H}{=}} \lim_{x\to 0} \frac{3-3\cos 3x}{3x^2}$$

$$\overset{\left(\frac{0}{0}\right)}{\underset{L'H}{=}} \lim_{x\to 0} \frac{9\sin 3x}{6x} \overset{\left(\frac{0}{0}\right)}{\underset{L'H}{=}} \lim_{x\to 0} \frac{27\cos 3x}{6}$$

$$= \frac{27}{6} = \frac{9}{2}$$

2. $$\int x^2 (\underbrace{x+1}_{u})^{3/2}\,dx \qquad u = x+1 \leftarrow x = u-1$$

$$du = dx$$

$$\int (u-1)^2 u^{3/2}\,du = \int (u^2-2u+1)u^{3/2}\,du$$

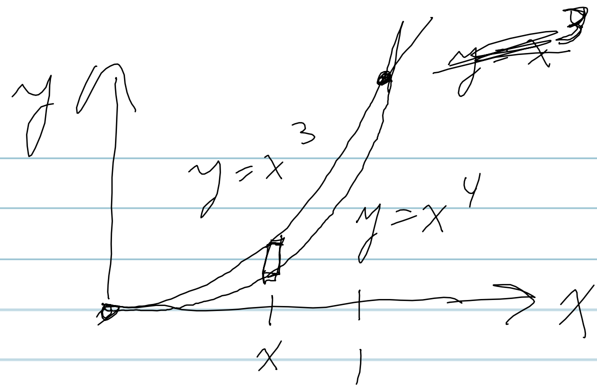
$$= \int u^{7/2} - 2u^{5/2} + u^{3/2}\,du \qquad \left[\int u^p\,du = \frac{1}{p+1}u^{p+1} + C\right]$$

$$= \frac{2}{9}u^{9/2} - 2\cdot\frac{2}{7}u^{7/2} + \frac{2}{5}u^{5/2} + C$$

$$= \frac{2}{9}(x+1)^{9/2} - \frac{4}{7}(x+1)^{7/2} + \frac{2}{5}(x+1)^{5/2} + C$$

3.

3.

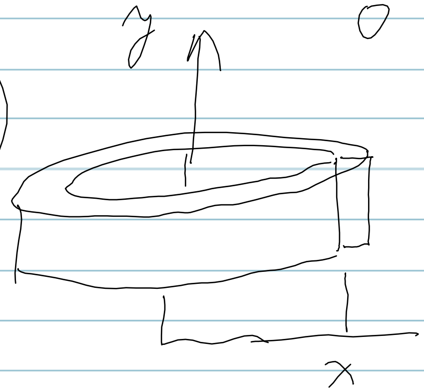


a)

$$\pi\left((x^3)^2 - (x^4)^2\right)\Delta x$$

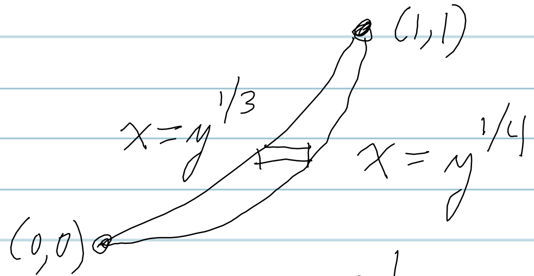
$$V = \int_0^1 \pi\left((x^3)^2 - (x^4)^2\right)\,dx$$

b)



$$(2\pi x)(x^3 - x^4)\Delta x$$

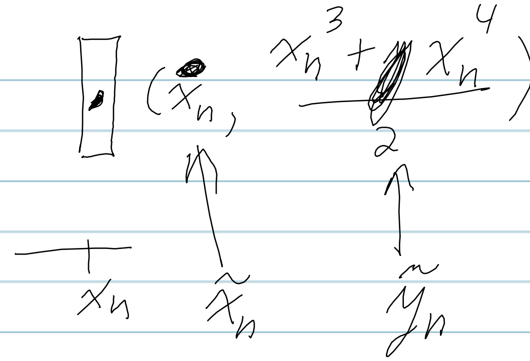
$$V = \int_0^1 (2\pi x)(x^3 - x^4)\,dx$$



$$\pi\left((y^{1/4})^2 - (y^{1/3})^2\right)\Delta y$$

$$V = \int_0^1 \pi\left((y^{1/4})^2 - (y^{1/3})^2\right)\,dy$$

c) $$\bar{x} = \frac{\sum \tilde{x}_n m_n}{\sum m_n}$$



$$\bar{y} = \frac{\sum \tilde{y}_n m_n}{\sum m_n}$$

$$\bar{x} = \frac{\int_0^1 x \left[(x^3 - x^4)\right] dx}{\int_0^1 (x^3 - x^4)\, dx}$$

$$\bar{y} = \frac{\int_0^1 \left(\frac{x^3 + x^4}{2}\right) \left[(x^3 - x^4)\right] dx}{\int_0^1 (x^3 - x^4)\, dx}$$

4. $$I = \int_3^7 e^{(\underbrace{2x+1}_{u})^2} dx = c \int_a^b e^{u^2} du$$

$u = 2x + 1$

$du = 2dx$

$\boxed{dx = \frac{1}{2} du}$

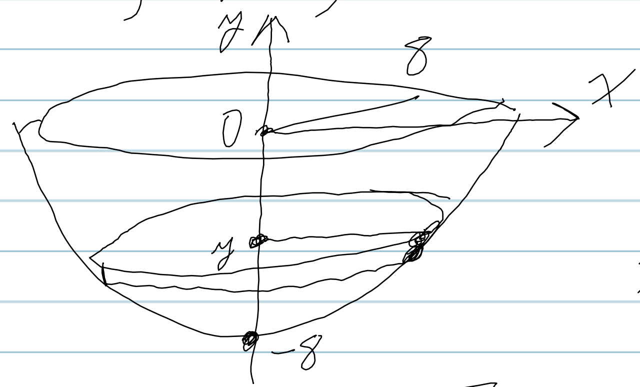
When $x = 3$; $u = 2 \cdot 3 + 1 = 7$

When $x = 7$; $u = 2 \cdot 7 + 1 = 15$

$$I = \int_{x=3}^{7} e^{(2x+1)^2} dx = \int_{u=7}^{15} e^{u^2} \left(\frac{1}{2} du\right)$$

$c = \frac{1}{2}$, $a = 7$, $b = 15$.

5.



$62.4\ lbs/ft^3$

$$x^2 + y^2 = 8^2 = 64$$

$$y = -\sqrt{64 - x^2}$$

$$x = \sqrt{64 - y^2}$$

$$\Delta W = (62.4)\Big(\underbrace{\pi\left(\sqrt{64-y^2}\right)^2 \Delta y}_{\text{Volume}}\Big)\underbrace{(0-y)}_{h}.$$

$$W = \int_{-8}^{0} (62.4)\,\pi\left(\sqrt{64-y^2}\right)^2 (0-y)\,dy$$