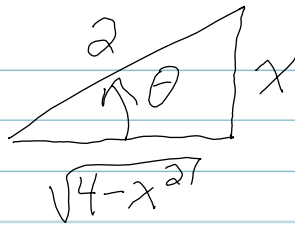


$$c) I = \int \frac{x^2}{\sqrt{4-x^2}} dx$$



$$x = 2 \sin \theta$$

$$dx = 2 \cos \theta d\theta$$

$$\sqrt{4-x^2} = 2 \cos \theta$$

$$I = \int \frac{(2 \sin \theta)^2}{(2 \cos \theta)} (2 \cos \theta d\theta)$$

$$= 4 \int \sin^2 \theta d\theta = 4 \int \frac{1}{2} (1 - \cos 2\theta) d\theta$$

$$= 2\theta - \sin 2\theta + C$$

$$= 2 \sin^{-1} \frac{x}{2} - \sin \left(2 \sin^{-1} \frac{x}{2} \right) + C$$

Better way: $\sin 2\theta = 2 \sin \theta \cos \theta$
 $= 2 \frac{x}{2} \frac{\sqrt{4-x^2}}{2}$

$$I = 2 \sin^{-1} \frac{x}{2} - \frac{1}{2} x \sqrt{4-x^2} + C$$

2. a) $\frac{\text{degree 6 } \checkmark}{x^2(x-1)^3(x^2+2x+2)} = \frac{A}{x^2} + \frac{B}{x} +$

$$\frac{C}{(x-1)^3} + \frac{D}{(x-2)^2} + \frac{E}{(x-1)} + \frac{Fx+G}{x^2+2x+2}$$

$$b) \frac{4x^2 + 2x + 1}{x(x^2 + 1)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 1}$$

$$4x^2 + 2x + 1 = A(x^2 + 1) + (Bx + C)x$$

$$4x^2 + 2x + 1 = \underbrace{(A+B)}_4 x^2 + \underbrace{C}_2 x + \underbrace{A}_1$$

$$A = 1$$

$$C = 2$$

$$A + B = 4 \leftarrow B = 4 - A = 4 - 1 = 3$$

$$\frac{1}{x} + \frac{3x + 2}{x^2 + 1}$$

$$3. a) \sum_{n=1}^{\infty} \frac{n}{n^2 + n + 1}$$

$$\underbrace{\frac{n}{n^2 + n + 1}}_{a_n} \sim \underbrace{\frac{n}{n^2}}_{b_n} = \frac{1}{n} \quad \text{for big } n.$$

$$\text{Limit comparison: } \lim_{n \rightarrow \infty} \frac{a_n}{b_n} =$$

$$\lim_{n \rightarrow \infty} \frac{\left(\frac{n}{n^2 + n + 1} \right)}{\left(\frac{1}{n} \right)} = \lim_{n \rightarrow \infty} \frac{n^2}{n^2 + n + 1} \cdot \frac{\left(\frac{1}{n^2} \right)}{\left(\frac{1}{n^2} \right)} =$$

$$\lim_{n \rightarrow \infty} \frac{1}{1 + \frac{1}{n} + \frac{1}{n^2}} = \frac{1}{1+0+0} = 1$$

$\sum_{n=1}^{\infty} \frac{1}{n}$ Diverges. Hence, so does

the given series by Limit Comparison.

b) $\sum_{n=1}^{\infty} \frac{n^3}{3^n}$ Ratio Test: $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}$

$$= \lim_{n \rightarrow \infty} \frac{\left[\frac{(n+1)^3}{3^{n+1}} \right]}{\left[\frac{n^3}{3^n} \right]} = \lim_{n \rightarrow \infty} \frac{1}{3} \left[\frac{n+1}{n} \right]^3$$

$$= \lim_{n \rightarrow \infty} \frac{1}{3} \left(\frac{1 + \frac{1}{n}}{1} \right)^3 = \frac{1}{3} \cdot 1 = \frac{1}{3} < 1$$

Ratio Test implies series converges.

Or Root Test $\sqrt[n]{a_n} = \left(\frac{n^3}{3^n} \right)^{1/n} = \frac{(n^{1/n})^3}{3}$

$$\rightarrow \frac{1^3}{3} = \frac{1}{3} < 1. \quad \checkmark$$

$$c) \sum_{n=1}^{\infty} (-1)^n \frac{n^2}{n(n+10)}$$

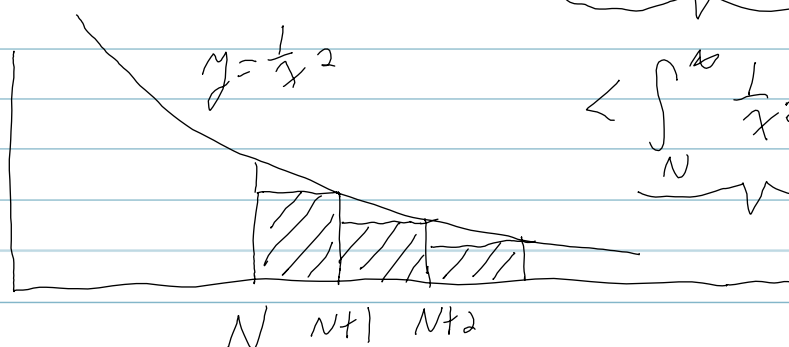
$$a_n = \frac{n^2}{n^2 + 10n} \cdot \frac{\left(\frac{1}{n^2}\right)}{\left(\frac{1}{n^2}\right)} = \frac{1}{1 + \frac{10}{n}} \rightarrow 1$$

$(-1)^n a_n$ DO NOT tend to zero!

n -th term test yields that series diverges.

$$4. a) \sum_{n=1}^{\infty} \frac{1}{n^2} \quad L = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\left| L - \sum_{n=1}^N \frac{1}{n^2} \right| = \underbrace{\sum_{n=N+1}^{\infty} \frac{1}{n^2}}_{\text{want}} < 10^{-2}$$



$$< \underbrace{\int_N^{\infty} \frac{1}{x^2} dx}_{\text{want}} < 10^{-2}$$

$$\lim_{B \rightarrow \infty} \left[\frac{-1}{x} \right]_N^B = \lim_{B \rightarrow \infty} \left(\frac{-1}{B} - \left(\frac{-1}{N} \right) \right) =$$

$$\frac{1}{N} \overset{\text{want}}{<} 10^{-2}$$

Need

$$\frac{1}{N} < \frac{1}{100}, \quad N > 100.$$

$$N = 101.$$

$$b) \sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{1}{n} = L$$

$$\left| \left(1 - \frac{1}{2} + \frac{1}{3} - \dots + (-1)^{N+1} \cdot \frac{1}{N} \right) - L \right| < \underbrace{a_{N+1}}_{\frac{1}{N+1}}$$

$\uparrow$
 $(-1)^{N+1} a_N$

want $< 10^{-2}$, $\frac{1}{N+1} < \frac{1}{100}$, $N+1 > 100$
 $N = 100$, $N > 99$

$$c) \sum_{n=1}^{\infty} \frac{1}{n!} = \text{Taylor Series } (e^x - 1) \text{ at } x=1.$$

$$e^x - 1 = \sum_{n=1}^{\infty} \frac{x^n}{n!}$$

$$|f(x) - P_N(x)| = |R_N(x)|$$

$$\left| (e-1) - \sum_{n=1}^N \frac{1}{n!} \right| = \left| \frac{f^{(N+1)}(c)}{(N+1)!} \underbrace{\left(\underset{x=1}{1} - \underset{c=0}{0} \right)^{N+1}} \right|$$

But $f(x) = e^x - 1$

$$f^{(N+1)}(x) = e^x$$

So $f^{(N+1)}(c) = e^c$

c between 0 and 1.

So $e^c < e^1 < 3$.

ERROR $< \frac{3}{(N+1)!} < \overset{\text{want}}{\frac{1}{100}}$

$$\frac{3}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} = \frac{1}{240} < \frac{1}{100}$$

$\uparrow$
 $N+1$

$N = 5$

5. $\sum_{n=1}^{\infty} \frac{x^n}{n 3^n}$

Ratio Test $\lim_{n \rightarrow \infty} \left| \frac{u_{n+1}}{u_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{\left(\frac{x^{n+1}}{(n+1) 3^{n+1}} \right)}{\left(\frac{x^n}{n 3^n} \right)} \right|$

$$= \lim_{n \rightarrow \infty} \frac{|x|}{3} \cdot \frac{n}{n+1} = \frac{|x|}{3} < 1 \text{ conv.}$$

$$\frac{|x|}{3} > 1 \text{ div.}$$

$|x| < 3 \leftarrow$ absolute convergence.

$|x| > 3 \leftarrow$ divergence.

$x = \pm 3$ delicate.

$x = 3 : \sum_{n=1}^{\infty} \frac{3^n}{n \cdot 3^n} = \sum_{n=1}^{\infty} \frac{1}{n} \quad \underline{\text{diverges.}}$

$x = -3 : \sum_{n=1}^{\infty} \frac{(-3)^n}{n \cdot 3^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \quad \text{Conditionally Convergent.}$