MATH 181, Exam I

(20) **1.** Compute a) $\displaystyle\int_0^2 \frac{x^2}{\sqrt{x^3+1}}\,dx$ and b) $\displaystyle\int x^5\sqrt{x^3+1}\,dx$.

(20) **2.** Suppose

$$F(x) = \int_0^{\sqrt{x}} e^{t^2}\,dt \qquad \text{for } x > 0.$$

Compute $F'(x)$. What is $\displaystyle\lim_{x\to\infty} \frac{x}{F(x)}$?

(20) **3.** Fill in the blank of the punch line of these two famous theorems.

a) Suppose f is continuous on $[a,b]$ and differentiable on (a,b). There exists a point c with $a < c < b$ such that $f(b) - f(a) =$ ____________________.

b) Suppose f is continuous on $[a,b]$. There exists a point c with $a < c < b$ such that $\int_a^b f(x)\,dx =$ ____________________.

(20) **4.** Explain why the method of cylindrical shells for computing the volume of a solid of revolution implies the Theorem of Pappus.

(20) **5.** The ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

can be described as the graph of $y = \pm\frac{b}{a}\sqrt{a^2 - x^2}$ or parametrically via

$$x = a\cos t \qquad \text{and} \qquad y = b\sin t, \qquad 0 \le t \le 2\pi.$$

Write down the following integrals, but DO NOT TRY TO COMPUTE THEM.

a) Write down a definite integral with respect to x that gives the volume of the solid obtained by revolving the top half of the ellipse around the x-axis.

b) Write down a definite integral with respect to x that gives the volume of the solid obtained by revolving the part of the ellipse in the first quadrant about the y-axis.

c) Write down a quotient of definite integrals with respect to x that gives the y-coordinate of the center of mass of a uniform metal plate in the shape of the top half of the ellipse.

d) Write down a definite integral with respect to t that gives the perimeter of the ellipse.

e) Write down a definite integral of any kind that gives the surface area of the ellipsoid obtained by revolving the ellipse about the x-axis.

Remember, DO NOT compute the integrals.