MATH 181

Practice Problems

1. Determine the equation of the tangent line to the curve given parametrically by $x(t) = t - \sin t$ and $y(t) = 1 - \cos t$, $0 \le t \le 2\pi$, at $t = 0$. Find the length of this curve from $t = 0$ to $t = 2\pi$. Hint: $2 - 2\cos t = 4\sin^2(t/2)$
2. Sove the differential equation $\sqrt{y+1}\,\frac{dy}{dx} = \frac{1}{x^2}$ with initial condition $y(-3) = 3$.
3. The region bounded by $y = 4x$ and $y = x^3$ in the first quadrant is rotated about the y-axis. Use the *cylindrical shell slicing* method to compute the volume of the solid. Also, set up an integral which gives the total surface area of the solid. (Do not evaluate this integral.)
4. If a curve is defined parametrically by $x(t) = -\cos^3 t$ and $y(t) = \sin^3 t$, $0 \le t \le 2\pi$, then find all points on the curve where the slope of the tangent line equals 1.
5. Find the derivatives of $x^{(x^2)}$, and $2^{\sin^{-1} x}$. Compute $\dfrac{d}{dx}\displaystyle\int_0^{\cos x} e^{-x^2}\,dx$.
6. Calculate $\int \frac{2\,dx}{(x-1)(x^2+1)}$ and $\int \frac{dx}{\sqrt{2-4x+4x^2}}$.
7. Evaluate $\int_0^\infty xe^{-x}\,dx$.
8. Assuming that the population of the U.S. was 75 million in 1900 and 150 million in 1950 and that the population grows exponentially, compute the size of the population in 1990.
9. What is the area of the region lying in the first quadrant and bounded by the polar curve $r = \cos\theta + \sin\theta$?
10. Find the foci of the ellipse $x^2 - 2x + 4y^2 = 3$.
11. Show that $|\cos x - (1 - x^2/2)| < \frac{1}{200}$ if $|x| \le \frac{1}{2}$.
12. Compute the Taylor series of $f(x) = \frac{1}{3-x}$ at $a = 1$.
13. Determine whether these series converge absolutely, conditionally, or diverge. Explain. $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$ $\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{n}$ $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+1}}$
14. Find the interval of convergence of the series

$$1 + \frac{x+2}{3\cdot 1} + \frac{(x+2)^2}{3^2\cdot 2} + \cdots + \frac{(x+2)^n}{3^n\cdot n} + \cdots$$

15. Evaluate $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$. Hint: Telescoping series, partial fractions.
16. Assuming $|x| > 1$, show that $\frac{1}{1-x} = -\frac{1}{x} - \frac{1}{x^2} - \frac{1}{x^3} - \cdots$