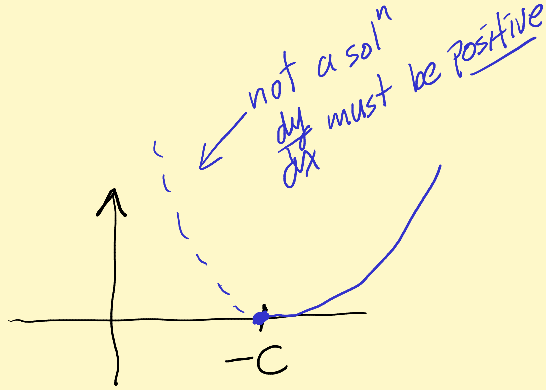
The classic example of an eqn $\frac{dy}{dx} = f(x,y)$ where f is continuous, but $\frac{\partial f}{\partial y}(x,y)$ is <u>not</u> where uniqueness fails.

$$\frac{dy}{dx} = 2\sqrt{|y|} \longleftarrow \begin{cases} f(x,y) \text{ continuous, but} \\ \text{when } y>0,\ \frac{\partial f}{\partial y} = \frac{1}{\sqrt{y}} \text{ is bad as } y \searrow 0. \end{cases}$$

$y>0$ case: $\displaystyle\int \frac{1}{2\sqrt{y}}\,dy = \int dx$

$$\sqrt{y} = x + C$$

$$y = (x+C)^2$$



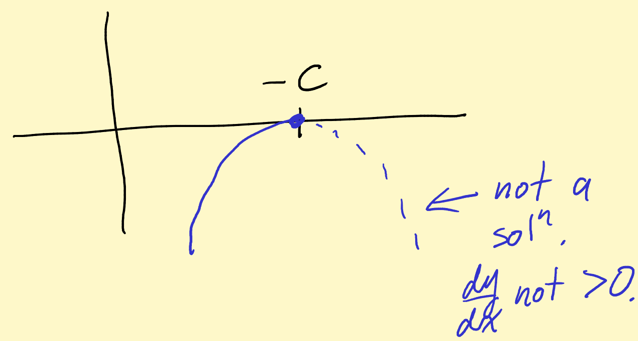
$y<0$ case: $\sqrt{|y|} = \sqrt{-y}$

$$\int \frac{1}{2\sqrt{-y}}\,dy = \int dx$$

$$-\sqrt{-y} = x + C$$

$$-y = (x+C)^2$$

$$y = -(x+C)^2$$



Missing sol^n from method: $y \equiv 0$

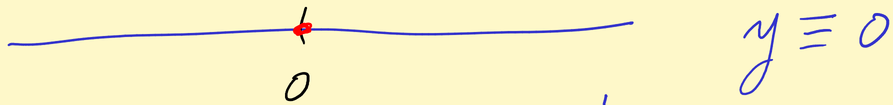
Observation: $y = \begin{cases} 0 & x \le -C \\ (x+C)^2 & x > -C \end{cases}$ is a sol^n

So is $y = \begin{cases} -(x+C)^2 & x \le -C \\ 0 & x > -C \end{cases}$

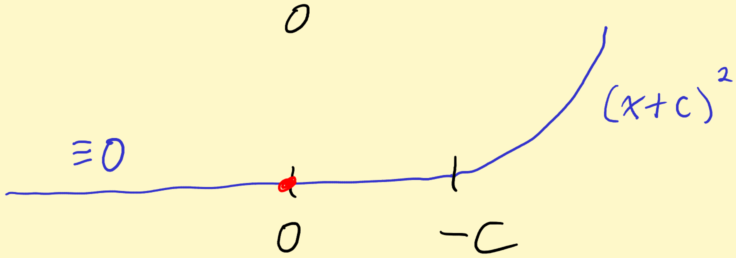
Solutions with $y(0)=0$ exist, but there are infinitely many of them!

Here are several:

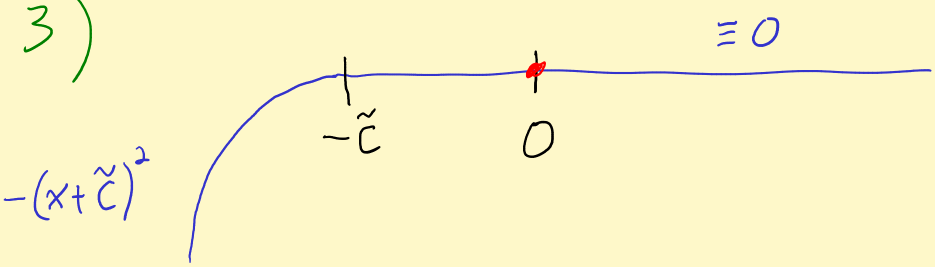
1)



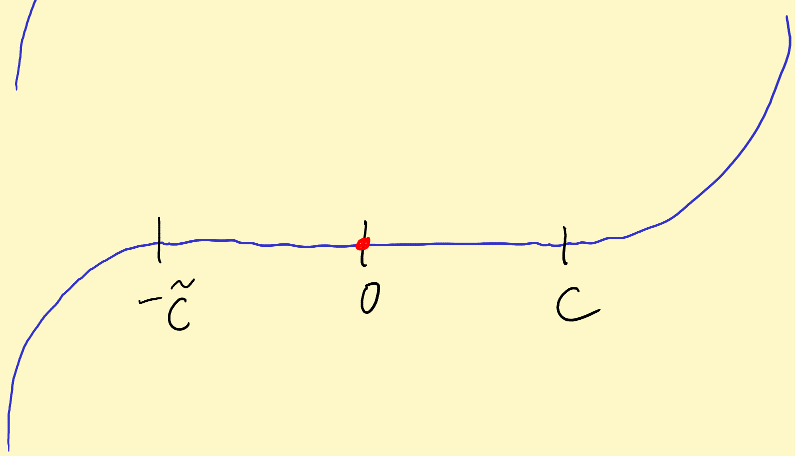
2)



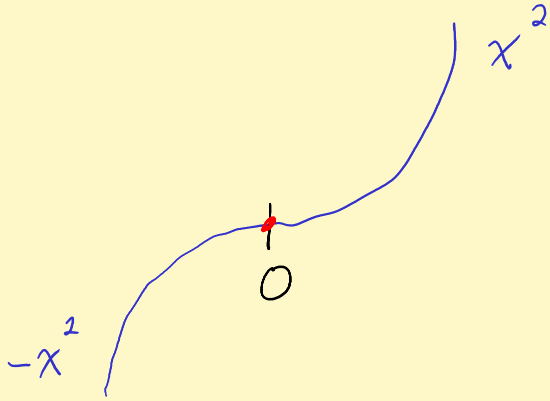
3)



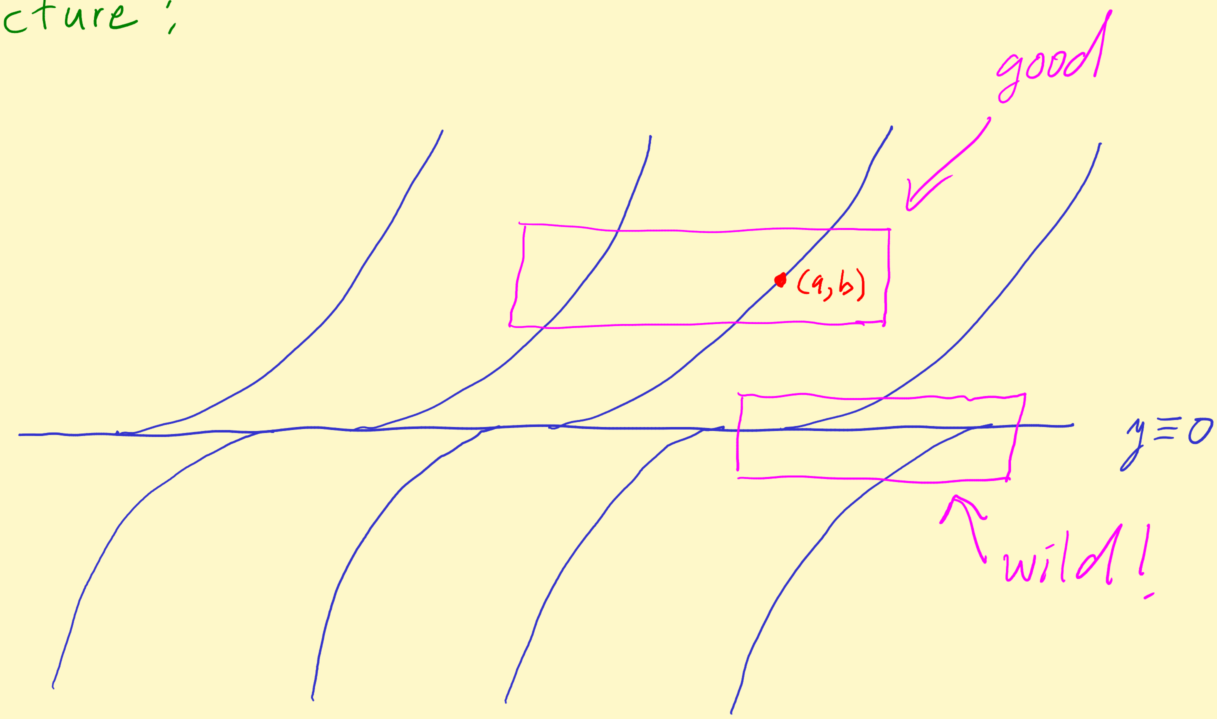
4)



5)



Big picture:



In the good box where $f(x,y) = 2\sqrt{y}$ and $\frac{\partial f}{\partial y} = \frac{1}{\sqrt{y}}$ $(y > 0)$, there is exactly one sol^n going through each pt (a,b) in the box.