

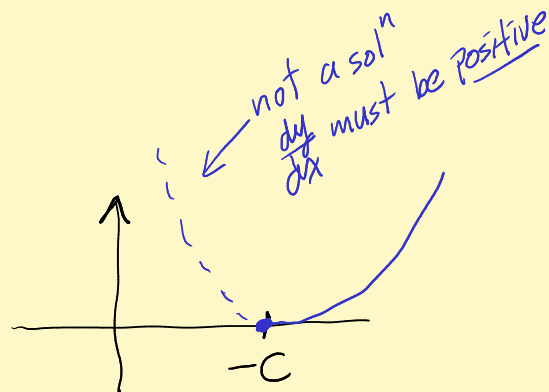
The classic example of an eqn $\frac{dy}{dx} = f(x, y)$ where f is continuous, but $\frac{\partial f}{\partial y}(x, y)$ is not where uniqueness fails.

$$\frac{dy}{dx} = 2\sqrt{|y|} \leftarrow \begin{cases} f(x, y) \text{ continuous, but} \\ \text{when } y > 0, \frac{\partial f}{\partial y} = \frac{1}{\sqrt{y}} \text{ is bad as } y \downarrow 0. \end{cases}$$

$$y > 0 \text{ case: } \int \frac{1}{2\sqrt{y}} dy = \int dx$$

$$\sqrt{y} = x + C$$

$$y = (x + C)^2$$



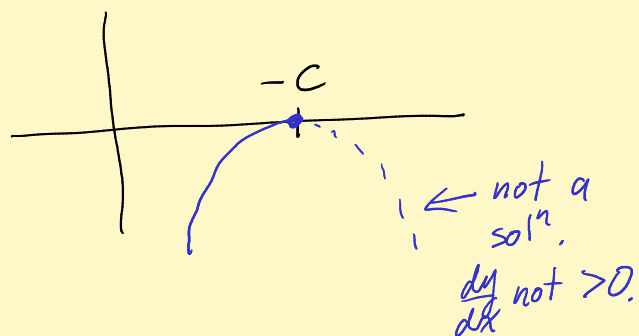
$$y < 0 \text{ case: } \sqrt{|y|} = \sqrt{-y}$$

$$\int \frac{1}{2\sqrt{-y}} dy = \int dx$$

$$-\sqrt{-y} = x + C$$

$$-y = (x + C)^2$$

$$y = -(x + C)^2$$



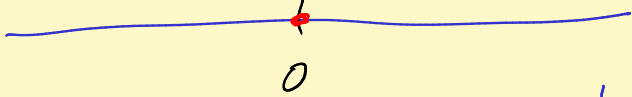
Missing solⁿ from method: $y \equiv 0$

Observation: $y = \begin{cases} 0 & x \leq -C \\ (x + C)^2 & x > -C \end{cases}$ is a solⁿ

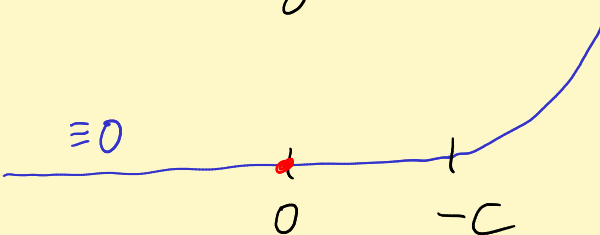
So is $y = \begin{cases} -(x + C)^2 & x \leq -C \\ 0 & x > -C \end{cases}$

Solutions with $y(0)=0$ exist, but there are infinitely many of them!

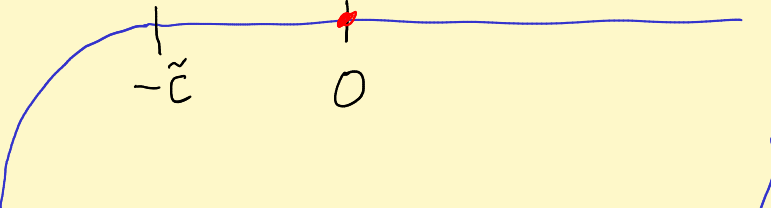
Here are several :

1)  $y \equiv 0$

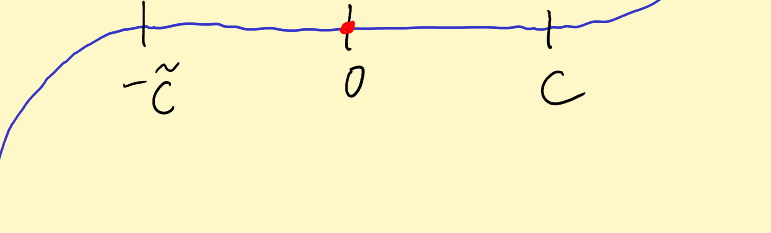
A horizontal blue line representing the function $y=0$. A red dot marks the point $(0,0)$ on the line.

2)  $(x+c)^2$

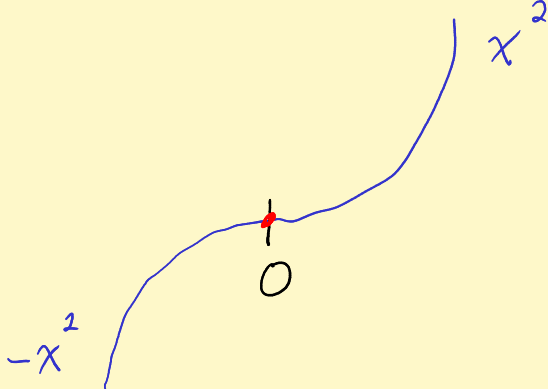
A horizontal blue line representing $y=0$ and a blue parabola $y=(x+c)^2$ opening upwards. A red dot marks the point $(0,0)$ on the line. The x-axis has tick marks at 0 and $-c$.

3)  $-(x+\tilde{c})^2$

A horizontal blue line representing $y=0$ and a blue parabola $y=-(x+\tilde{c})^2$ opening downwards. A red dot marks the point $(0,0)$ on the line. The x-axis has tick marks at $-\tilde{c}$ and 0 .

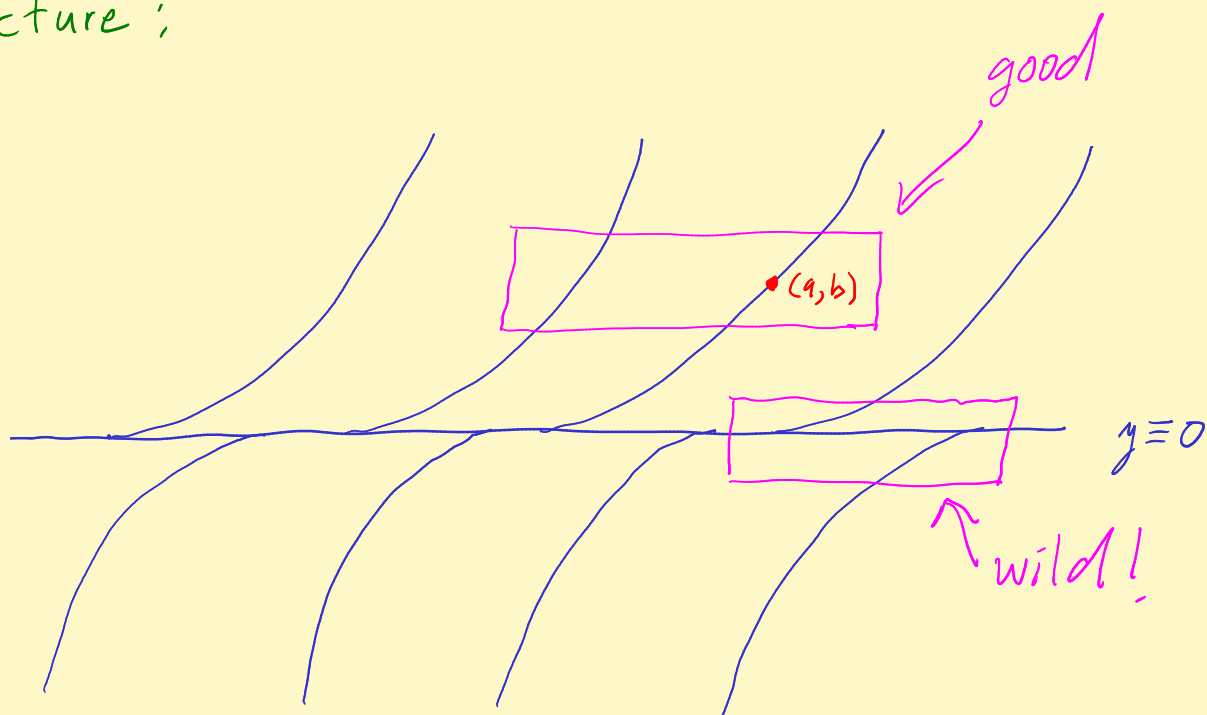
4) 

A horizontal blue line representing $y=0$ and a blue parabola opening upwards with its vertex at $x=-\tilde{c}$. A red dot marks the point $(0,0)$ on the line. The x-axis has tick marks at $-\tilde{c}$, 0 , and c .

5)  x^2 and $-x^2$

A blue parabola $y=x^2$ opening upwards and a blue parabola $y=-x^2$ opening downwards. A red dot marks the point $(0,0)$ where they intersect. The x-axis has a tick mark at 0 .

Big picture:



In the good box where $f(x,y) = 2\sqrt{y}$ and $\frac{\partial f}{\partial y} = \frac{1}{\sqrt{y}}$ ($y > 0$),
 there is exactly one solⁿ going through each pt (a,b) in
 the box.