MA 266-111 Steve Bell MATH 750
494-1497

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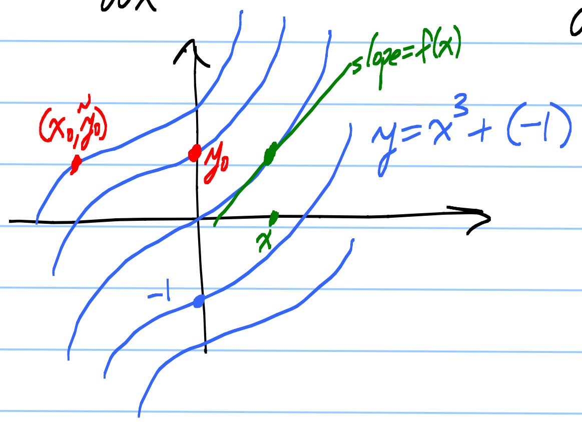
Easiest ODE: $\frac{dy}{dx} = f(x)$

$$y = \underbrace{\int f(x)\,dx}_{F(x) = \int_a^x f(t)\,dt} + C$$

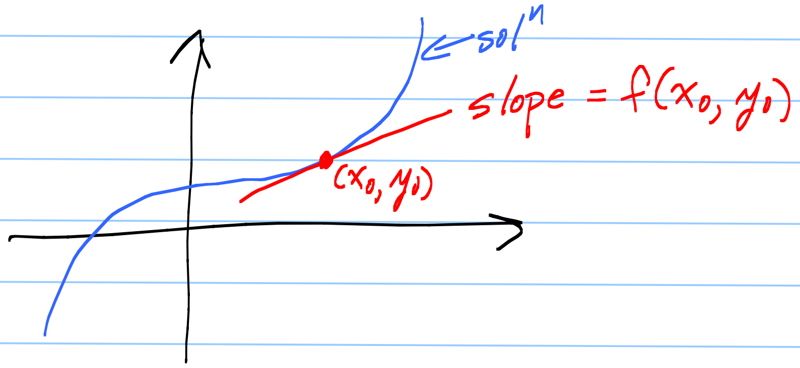
∞ many sol^{n}s!

EX: $\frac{dy}{dx} = 3x^2$ $\quad y = x^3 + C$



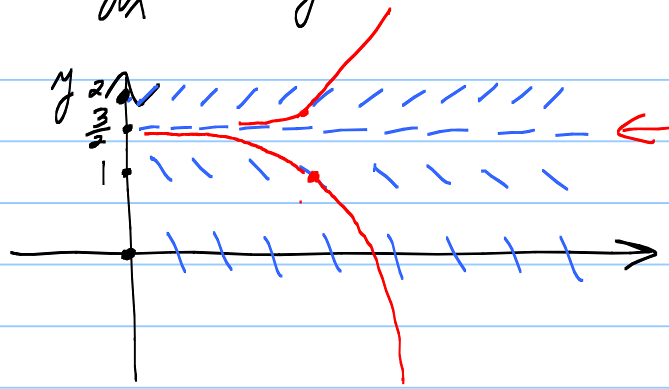
Pin down a sol^n by
$y(0) = y_0$

General first order ODE: $\frac{dy}{dx} = f(x, y)$



Direction field: Grid of pts (x_n, y_n)
Dashes of slope $f(x_n, y_n)$ at (x_n, y_n)

Ex: $\dfrac{dy}{dx} = 2y - 3$



$\leftarrow y = \frac{3}{2}$ is a sol^n

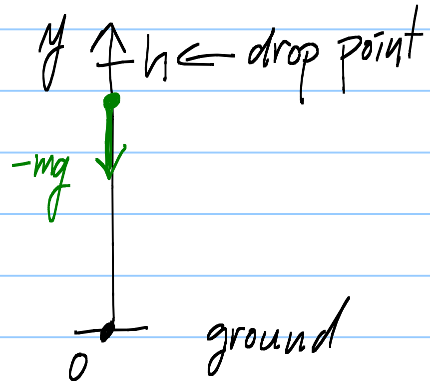
$\leftarrow$ isocline, slope $= -3$

horizontal lines

Sol^{n}s: Think: drop a cork in the flow

Newton's law: $F = ma = m\dfrac{d^2y}{dt^2} \longleftarrow$ second order ODE!

Ex: Drop a ball



$F = ma$

$-mg = m\dfrac{d^2y}{dt^2} = m\dfrac{dv}{dt}$

where $v = \dfrac{dy}{dt}$

$$\boxed{\frac{dv}{dt} = -g}$$

$v = -gt + C$

Aha! Velocity $= 0$ at moment of drop

$v(0) = -g\cdot 0 + C \overset{\text{want}}{=} 0$

$\boxed{C = 0}$

$$\boxed{v = -gt}$$

But $v = \boxed{\dfrac{dy}{dt} = -gt}$

$$y = \int -gt\,dt + K$$

$$= -\frac{1}{2}gt^2 + K$$

Get K from $y(0) = h$

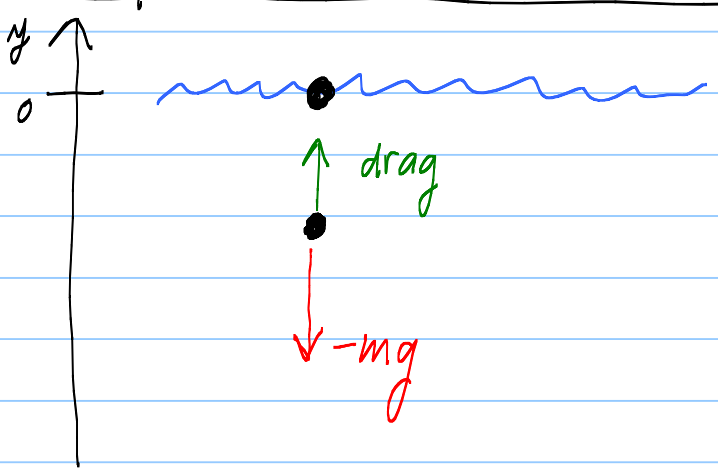
$$y(0) = -\frac{1}{2}g\cdot 0^2 + K \overset{\text{want}}{=} h$$

$$\boxed{K = h}$$

Sol^n: $\boxed{y = -\frac{1}{2}gt^2 + h}$

$$y = -\frac{1}{2}gt^2 + \underset{\substack{\uparrow\\ \text{initial}\\ \text{velocity}}}{v_0}\, t + \underset{\substack{\uparrow\\ \text{initial}\\ \text{height}}}{y_0}$$

Drop a cannonball in ocean:



Stokes' law:

drag $= -cv$

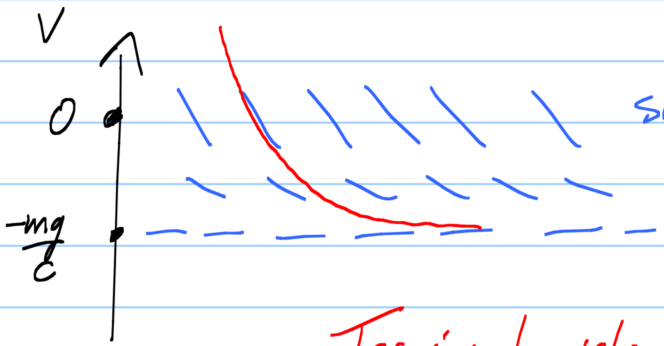
$$\underbrace{F}_{\text{net force}} = ma$$

$$-mg - cv = m\frac{d^2y}{dt^2}$$ Second order ODE

$v = \frac{dy}{dt}$

$$-mg - cv = m\frac{dv}{dt}$$ ← first order

$$\boxed{\frac{dv}{dt} = -g - \frac{c}{m}v}$$



slope $-g$

slope $= 0$

Terminal velocity $-\frac{mg}{c}$