Lecture 7 1.6

HWK06 due tonight in MyLab
HW04w, HW05w, HW06w due Gradescope

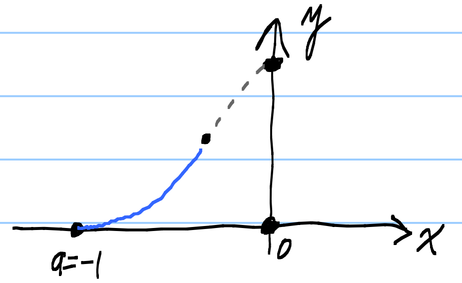
Change my zoom office hours to Wed. 7-8pm by email request

Dog-rabbit prob. $v_D = v_R$ $a=-1$

$$\frac{d^2y}{dx^2} = \underbrace{\frac{\sqrt{1+\left(\frac{dy}{dx}\right)^2}}{-x}}$$

Let $p = \frac{dy}{dx}$.

$\frac{dy}{dx}=0$ when $x=a=-1$



no y on this side

Rate out:

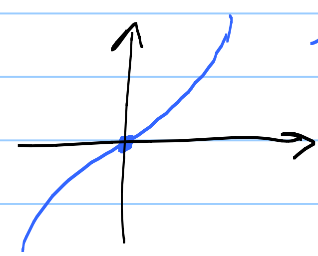
$$\left(\begin{matrix}\text{gals/min}\\ \text{out}\end{matrix}\right)\left(\frac{S(t)}{V(t)}\right)$$

V might

Get $\frac{dp}{dx} = \frac{\sqrt{1+p^2}}{-x}$

$$\int \frac{dp}{\sqrt{1+p^2}} = -\int \frac{dx}{x}$$

$$\sinh u = \frac{e^u - e^{-u}}{2}$$



$$\sinh^{-1} p = -\ln|x| + C$$

or $\ln\left(p+\sqrt{1+p^2}\right)$

Know $\frac{dy}{dx} = p = 0$ when $x=a=-1$

$$\underbrace{\sinh^{-1} 0}_{0} = -\underbrace{\ln|-1|}_{0} + C$$

See $C=0$

Now $p = \sinh\left(-\ln|x|\right)$

$$= \frac{1}{2}\left(e^{-\ln|x|} - e^{-(-\ln|x|)}\right)$$

$$= \frac{1}{2}\left(\frac{1}{|x|} - |x|\right)$$

Note $x<0$ on path

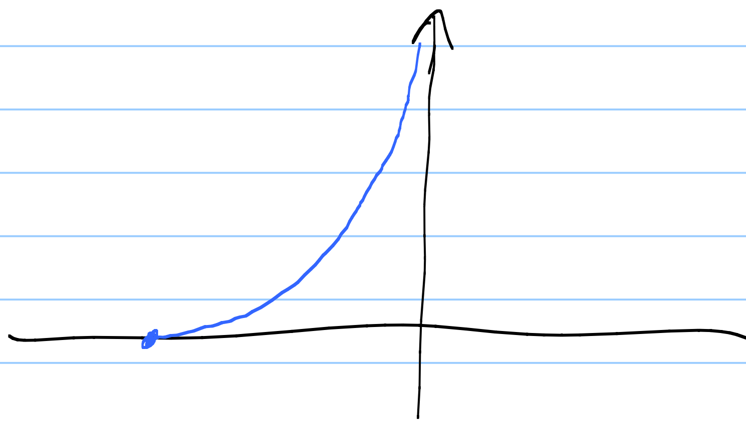
$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{-x} - (-x)\right) = \frac{1}{2}x - \frac{1}{2}\cdot\frac{1}{x}$$

Aha!

$$y = \int \frac{1}{2}x - \frac{1}{2}\frac{1}{x}\,dx = \frac{1}{4}x^2 - \frac{1}{2}\ln|x| + C$$

Know $y=0$ when $x=a=-1$; Get $C=-\frac{1}{4}$.

$$\boxed{y = \frac{1}{4}x^2 - \frac{1}{2}\ln(-x) - \frac{1}{4}}$$



1.6 A) $\frac{d^2y}{dx^2} = \underbrace{F\left(x, \frac{dy}{dx}\right)}_{\text{no } y \text{ on this side}}$ Let $p = \frac{dy}{dx}$

$\frac{dp}{dx} = F(x, p)$ Solve for $p = f(x)$.

Finally $\frac{dy}{dx} = p = f(x)$. Integrate

B) $\frac{d^2y}{dx^2} = \underbrace{F\left(y, \frac{dy}{dx}\right)}_{\text{no } x \text{ on this side}}$ Let $p = \frac{dy}{dx}$

Need one more step:

$$\frac{d^2y}{dx^2} = \frac{dp}{dx} = F(y, p)$$

hate this x!

Trick: Use chain rule: $\frac{dp}{dx} = \frac{dp}{dy} \cdot \underbrace{\frac{dy}{dx}}_{p}$

$$\boxed{\frac{dp}{dx} = p\frac{dp}{dy}}$$

New prob: $\underbrace{\underbrace{\frac{d^2y}{dx^2}}_{\frac{dp}{dx}}}_{p\frac{dp}{dy}} = F\left(y, \frac{dy}{dx}\right)$

$$\boxed{p\frac{dp}{dy} = F(y,p)}$$

New <u>first order ODE</u>: (Reduction of order).

Solve it. Get $p = g(y)$

$\frac{dy}{dx} = g(y)$ ← Separable!

Get y.

<u>Remark</u>: Do it for $F = ma$

Get Work-Energy equ!

Change of variables

EX;

$$\frac{dy}{dx} = (\underbrace{x+y+8}_{v})^2$$

$$v = x + y + 8$$

unknown fcn of x

Step 1: Solve for y:

$$y = v - x - 8$$

Step 2: Diff:

$$\boxed{\frac{dy}{dx} = \frac{dv}{dx} - 1} - 0$$

Step 3: Replace stuff in original ODE to get rid of y

$$\frac{dy}{dx} = (x+y+8)^2$$

$$\boxed{\frac{dv}{dx} - 1 = v^2}$$

separable!

$$\int \frac{dv}{1+v^2} = \int dx$$

$$\tan^{-1} v = x + C$$

Isolate v : $\underbrace{v}_{x+y+8} = \tan(x+c)$

Last step: Replace v by $x+y+8$. Solve for y.

$$y = -x - 8 + \tan(x+C)$$

Homogeneous first order ODEs:

$$\frac{dy}{dx} = F\left(\frac{y}{x}\right)$$

Method: Let $v = \frac{y}{x}$.

Step 1: Solve for y: $y = xv$

Step 2: Diff: $\frac{dy}{dx} = 1 \cdot v + x\frac{dv}{dx}$

$$\boxed{\frac{dy}{dx} = v + x\frac{dv}{dx}}$$

Step 3: Plug in: $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$

$$v + x\frac{dv}{dx} = F(v)$$

Separable!

$$\int \frac{dv}{F(v)-v} = \int \frac{dx}{x} = \ln|x| + C$$

Solve. Get $\underset{\frac{y}{x}}{\underset{\uparrow}{v}} = f(x)$

Finally

$$\text{So} \quad y = x f(x)$$

EX:

$$\frac{dy}{dx} = e^{y/x} + \frac{y}{x}$$

$$v + x\frac{dv}{dx} = e^{v} + v$$

$$x\frac{dv}{dx} = e^{v}$$

$$\int e^{-v}\,dv = \int \frac{dx}{x}$$

$$-e^{-v} = \ln|x| + C$$

$$e^{-v} = -\ln|x| - C$$

$$\text{So} \quad -\underset{\frac{y}{x}}{\underset{\uparrow}{v}} = \ln\left(-\ln|x| - C\right)$$

$$y = -x\ln\left(-\ln|x| - C\right)$$

EX: $\dfrac{x-y}{x+y} \cdot \dfrac{\frac{1}{x}}{\frac{1}{x}} = \dfrac{1-\left(\frac{y}{x}\right)}{1+\left(\frac{y}{x}\right)}$ homog!

EX: $\ln y - \ln x = \ln\frac{y}{x}$ homog!

EX: $\dfrac{x^6 + x^4 y^2 + x^3 y^3}{x^2 y^4 + y^6} \cdot \dfrac{\left(\frac{1}{x^6}\right)}{\left(\frac{1}{x^6}\right)}$

$= \dfrac{1 + \left(\frac{y}{x}\right)^2 + \left(\frac{y}{x}\right)^3}{\left(\frac{y}{x}\right)^4 + 1}$ homog.

Book: $(x^2 y^4 + y^6)\dfrac{dy}{dx} = (x^6 + x^2 y^4 + x^3 y^3)$

Bernoulli Equs:

Prob 19, 27

$$\frac{dy}{dx} + P(x)\, y = Q(x)\, y^n$$

1 here. Standard form important

Method: $\boxed{v = y^{1-n}}$ Plug into Standard form ODE

$$\frac{dv}{dx} + (1-n)P(x)\, v = (1-n)\, Q(x) \quad \text{Linear!}$$

Solve for v: Get $v = f(x)$

y^{1-n}

Last step: solve for y here