Lecture 10   2.2 Autonomous first order ODE

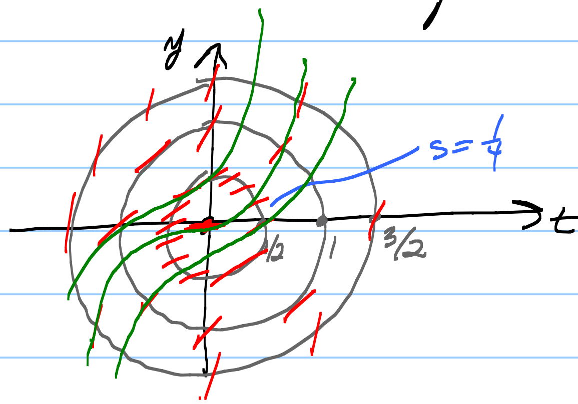
HW09 due tonight in MyLab

HW08w due tonight, Gradescope

EX: $\frac{dy}{dt} = t^2 + y^2$ ← t on RHS, not autonomous

Isoclines: Lines of constant slope of tangent lines of sol^{n}s

$t^2 + y^2 = s$ ← circles $t^2 + y^2 = r^2$ $\boxed{s = r^2}$

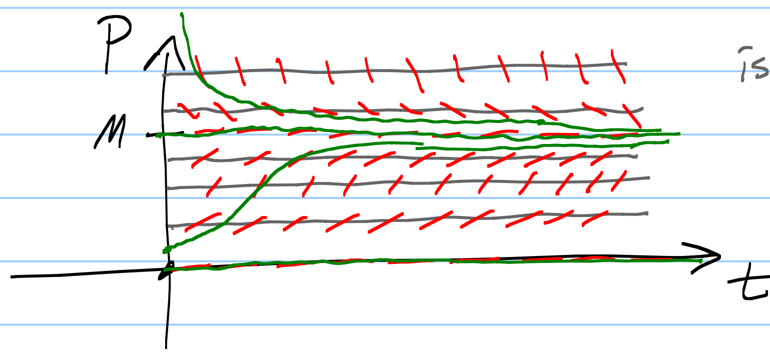


dfield (java version)

$\frac{dy}{dt} = f(y)$ ← no t on RHS: Autonomous

Isoclines: $f(y) = s$ $y = f^{-1}(s)$ ← horiz lines!

Population prob: $\frac{dp}{dt} = [k(M-P)]P = s$



isoclines $s = \text{const}$

Name for $P \equiv M$, $P \equiv 0$ sol^{n}s: Equillibrium sol^{n}s

Def^n: $\frac{dy}{dt} = f(y)$. Find zeroes of $f(y)$.

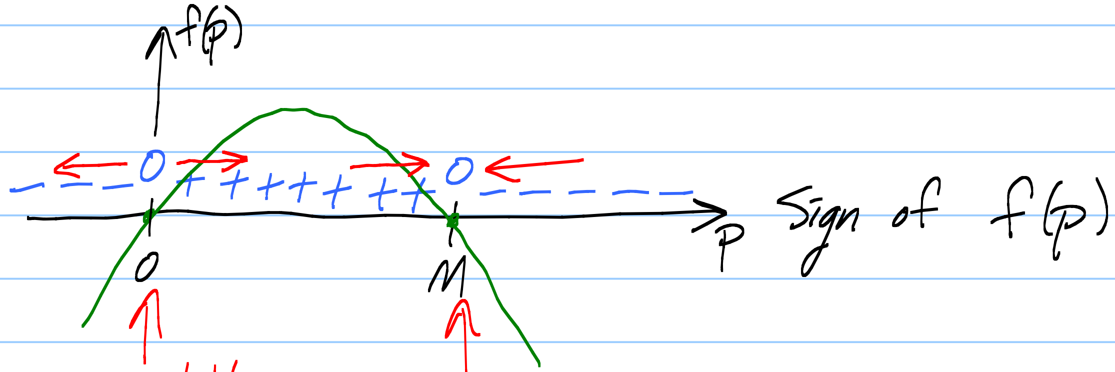
Say z is a zero.

Then $y \equiv z$ is a sol^n: $\underbrace{\frac{d}{dt}[z]}_{0} = \underbrace{f(z)}_{0}$ ✓

$y \equiv z$ for each zero are equillib. sol^{n}s.

Important diagram:

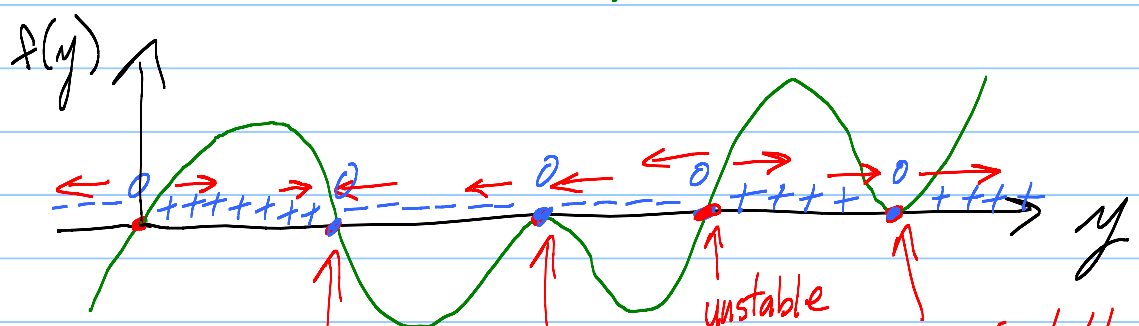
$$\frac{dp}{dt} = k(M-p)p \leftarrow f(p)$$



0 unstable
(top of a hill)

M is stable equillib.
(bottom of a hill)

General case: $\frac{dy}{dt} = f(y)$



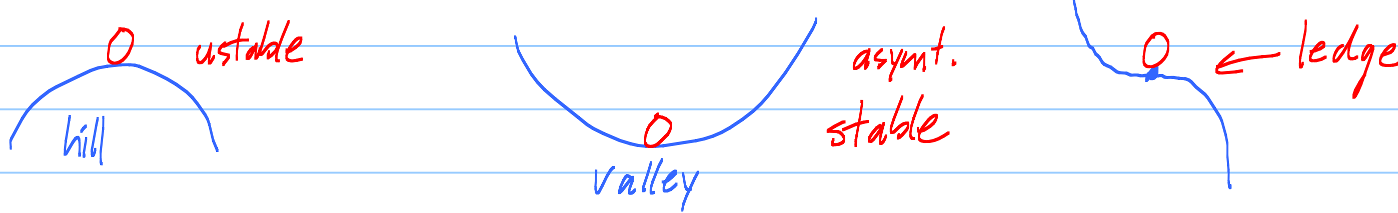
unstable

semi stable
(unstable)

"Asymptotically" stable
(gets pushed back)

semi stable
(unstable!)

Something stable but not asymptotically stable: The solar system:
One extra rock on moon doesn't rattle apart solar system.

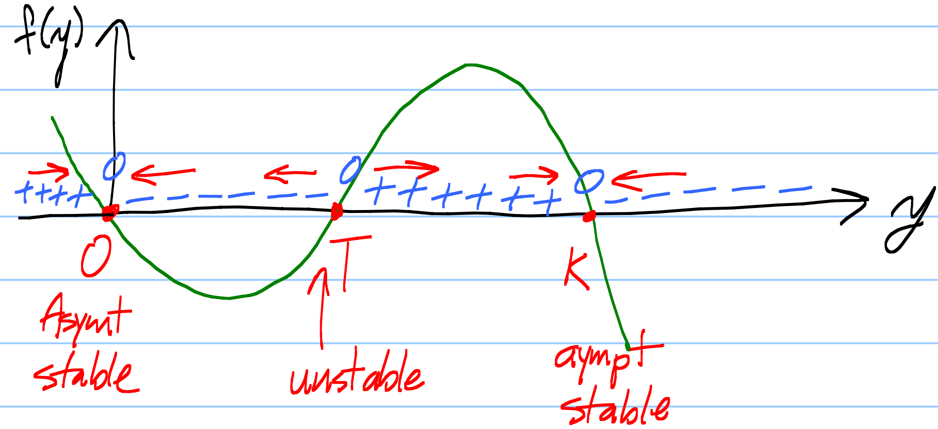


semi stable

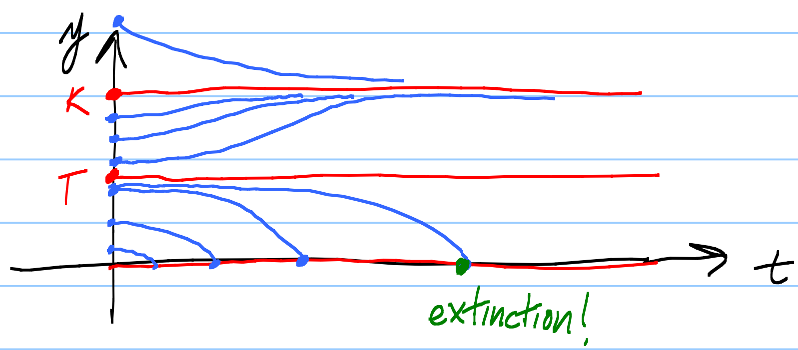
EX: Carrier Pigeons

$$\frac{dy}{dt} = -ry\left(1-\frac{y}{T}\right)\left(1-\frac{y}{K}\right)$$

$f(y)$



cubic



Where are the inflection points? Where $\frac{d^2y}{dt^2}$ changes sign.

$+$ concave up

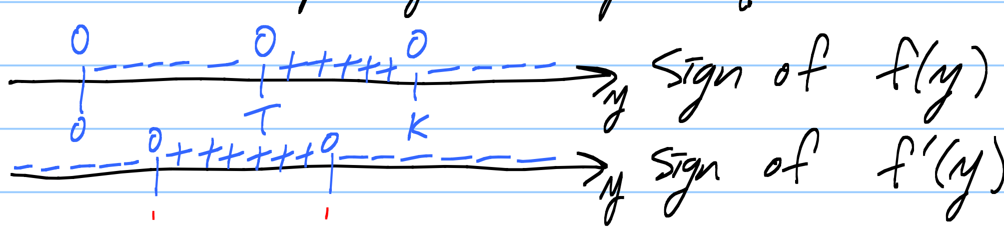
$-$ concave down

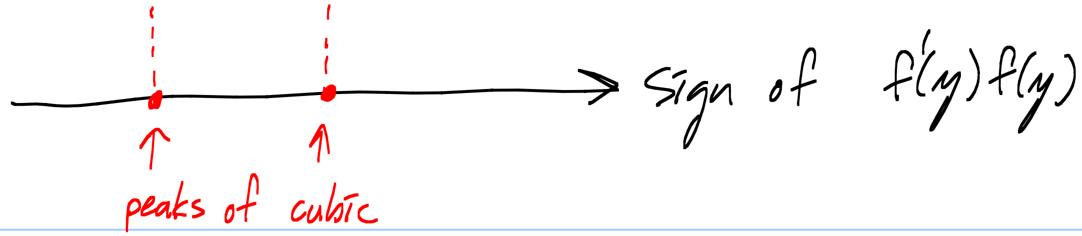
$$\frac{dy}{dt} = f(y)$$

$$\frac{d^2y}{dt^2} = \frac{d}{dt} f(y) = f'(y) \underbrace{\frac{dy}{dt}}_{= f(y)} = \underline{\underline{f'(y)\, f(y)}}$$

chain rule

Aha! Inflection pts fall on horizonal lines $y = ?$ where $f'(y) f(y)$ changes sign.





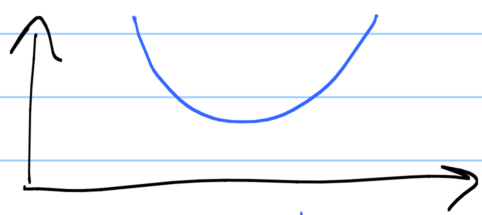
"Bifurcation points":

<u>Ex</u>: $\frac{dy}{dt} = y(4-y) - h$

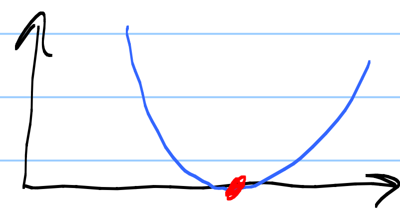
h ← parameter (constant)

Equilib pts: $y(4-y) - h = 0$ solve for y

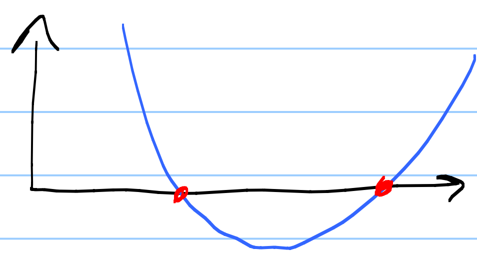
Roots: quadratic eqn: two roots



no roots



one root



two roots

Behavior depends delicately on h.
Bifurcation pts where h makes <u>changes</u>.