Lecture 11 2.3 Mechanics

Exam 1: Tues. Oct. 5 8:00-9:00 pm

HWK10 due tonight in MyLab

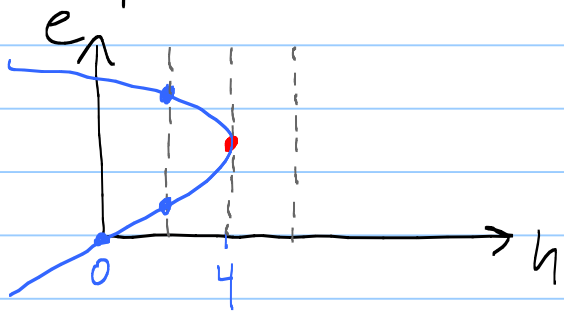
Bifurcation diagram: EX: $\frac{dy}{dt} = \underbrace{y(4-y)-h}_{\text{Equil. sol'ns } = 0}$

$e(4-e)-h=0$

Solve for e: Get

$$\boxed{e = 2 \pm \sqrt{4-h}}$$

Graph e vs h:

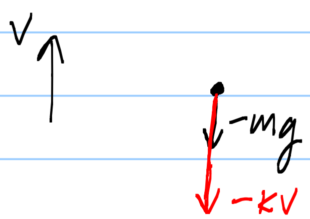


h=4 Bifurcation pt.

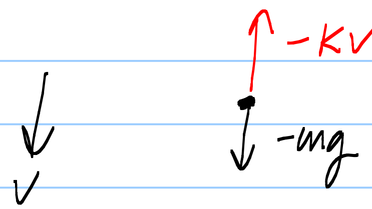
EX: Stokes' Law: Friction force $= -kv$

Laminar flow (slow).

Up:



Down:

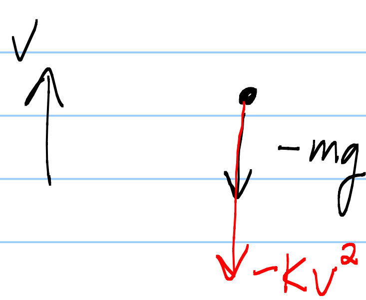


F = ma

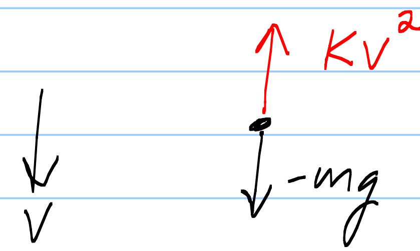
$$\boxed{-mg - kv = m\frac{dv}{dt}}$$

Higher velocity with turbulence: Friction $\propto v^2$

Up:



Down:



Two separate problems!

Up: $F = ma$

$$-mg - kv^2 = m\frac{dv}{dt}$$ Separable!

$$\int \frac{dv}{g+\frac{k}{m}v^2} = -\int dt = -t + C$$

Use $\int \frac{1}{1+u^2}\,du = \text{Tan}^{-1}u + C$

Get $(\text{Const.})\,\text{Tan}^{-1}\big((\text{Const.})\,v\big) = -t + C$

Solve for v: $\underbrace{v}_{dy/dt} = \sqrt{\frac{g}{k}}\,\text{Tan}\big(\underbrace{eee}_{At+B}\big)$

$$\int \text{Tan}\,u\,du = \text{Ln}|\sec u| + C$$

Get y using $\int \text{Tan}\,u\,du$ formula. See book p. 96.

Down: $F = ma$

$$-mg + kv^2 = m\frac{dv}{dt}$$ Separable

$$\int \frac{dv}{g-\frac{k}{m}v^2} = -\int dt = -t + C$$

Use $\int \frac{1}{1-u^2}\,du = \text{Tanh}^{-1}u + C$

Get $(\text{const.})\,\text{Tanh}^{-1}\big((\text{const.})\,v\big) = -t + C$

Solve for $v = \sqrt{\frac{g}{k}}\,\text{Tanh}\big(\underbrace{eeee}_{At+B}\big)$

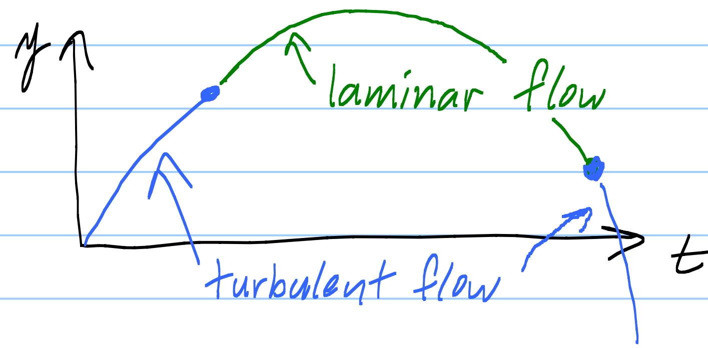
$\overbrace{\frac{dy}{dt}}^{v}$

Use $\int \tanh u \, du = \ln(\cosh u) + C$

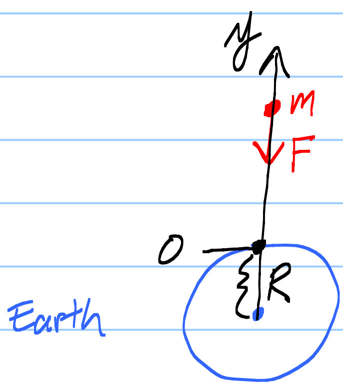
Get $y = (\text{const}) \ln\left(\cosh\left[(\text{const.})t + c\right]\right)$

[Do Up prob. Find peak. Get new initial conds there.
Now do down prob.

Closer to real life:



Shoot a rock into space:



$F = ma$ ← $a = \frac{d^2y}{dt^2}$. Let $v = \frac{dy}{dt}$ (Reduction of order)

$$-\frac{GMm}{(y+R)^2} = m\frac{dv}{dt}$$

Too many vars! y, v, t

Work-Kinetic energy trick: Use chain rule:

$$\frac{dv}{dt} = \frac{dv}{dy} \cdot \frac{dy}{dt} = v\frac{dv}{dy}$$

Aha! $$-\frac{GMm}{(y+R)^2} = mv\frac{dv}{dy}$$ ← Separable!

$$-\int \frac{GMm}{(y+R)^2}\,dy = \int mv\,dv = \frac{1}{2}mv^2 + C$$

Physics: $\underbrace{-\int_0^y \frac{GMm}{(y+R)^2}\,dy}_{\text{work}} = \underbrace{\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2}_{\Delta(K.E.)}$

$$\underbrace{-\left[\frac{-GMm}{(y+R)}\right]_0^y}_{\Delta(\text{Potential Energy})}$$

Math class: $-\int \frac{GMm}{(y+R)^2}\,dy = \frac{1}{2}mv^2 + C$

$$\frac{GMm}{y+R} = \frac{1}{2}mv^2 + C$$

Shoot rock with velocity v_0 when $y=0$ at surface.

Get $\frac{GMm}{0+R} = \frac{1}{2}mv_0^2 + C$

$$\boxed{C = \frac{GMm}{R} - \frac{1}{2}mv_0^2}$$

$$\frac{GMm}{(y+R)} = \frac{1}{2}mv^2 + \left(\frac{GMm}{R} - \frac{1}{2}mv_0^2\right)$$

Δ(pot eng) $=$ Δ(K.E.)

Solve for v:

$$\boxed{v^2 = \frac{2GM}{(y+R)} - \frac{2GM}{R} + v_0^2}$$

$v = \frac{dy}{dt}$ Oy!

$$\underbrace{\int \frac{dy}{\sqrt{\frac{2GM}{(y+R)} - \frac{2GM}{R} + v_0^2}}}_{\text{Big hairy fcn of } y!} = \int dt = t + C$$

Can only solve for y as fcn of t numerically

Question: When does rock turn around and head down?

$\frac{dy}{dt} = v = 0$ at top.

Hmmm.

$$v^2 = \frac{2GM}{y+R} - \frac{2GM}{R} + v_0^2$$

$$0 = \frac{2GM}{y_{top}+R} - \frac{2GM}{R} + v_0^2$$

$$\boxed{\frac{2GM}{y_{top}+R} = \frac{2GM}{R} - v_0^2}$$

Hmmm. $v_0^2 < \frac{2GM}{R}$, get y_{top}.

$$v_0^2 \geq \frac{2GM}{R}.$$ $\left(+ \text{ on left}\right) = \left(\begin{matrix}\leq 0 \text{ on} \\ \text{right}\end{matrix}\right)$

no sol^n!

Famous velocity: Escape velocity:

$$v_e = \sqrt{\frac{2GM}{R}}$$

Cool thing: When $v_0 = \sqrt{\frac{2GM}{R}}$, can calculate "hairy" integral <u>and</u> solve for y.