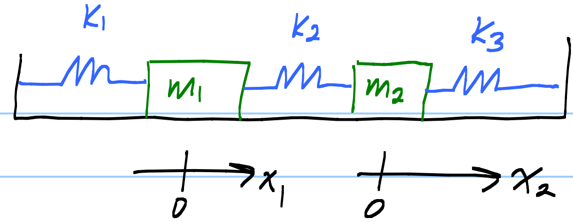
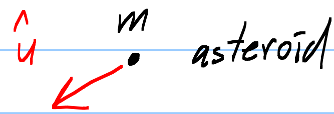
Lecture 22 4.1 Systems

HW 21 due in MyLab



EX: $x_1 = \left(\sin \frac{\sqrt{3}-1}{2} t\right)\left(\sin \frac{\sqrt{3}+1}{2} t\right)$

$$x_2 = \left(\cos \frac{\sqrt{3}-1}{2} t\right)\left(\cos \frac{\sqrt{3}+1}{2} t\right)$$



$$m\vec{a} = \vec{F}$$

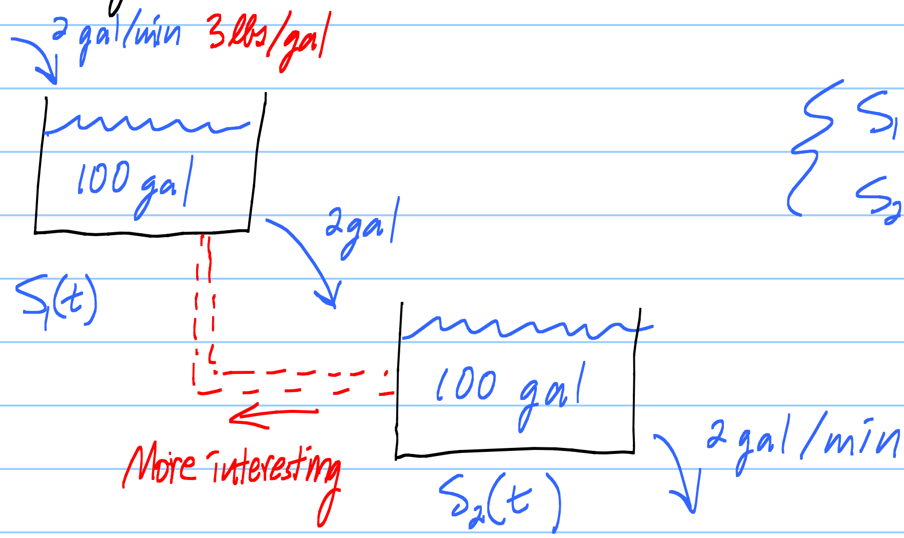
$$\vec{r} = x_1\hat{i} + x_2\hat{j} + x_3\hat{k}$$

$$\hat{u} = \frac{-\vec{r}}{|\vec{r}|}$$

$$\begin{cases} m\ddot{x}_1 = \dfrac{GMm}{|\vec{r}|^2} \cdot \left(\dfrac{-x_1}{|\vec{r}|}\right) \\ m\ddot{x}_2 = \dfrac{GMm}{|\vec{r}|^2} \cdot \left(-\dfrac{x_2}{|\vec{r}|}\right) \\ m\ddot{x}_3 = \dfrac{GMm}{|\vec{r}|^2} \cdot \left(-\dfrac{x_3}{|\vec{r}|}\right) \end{cases}$$

Thing in denom: $|\vec{r}|^3 = (x_1^2 + x_2^2 + x_3^2)^{3/2}$

EX:



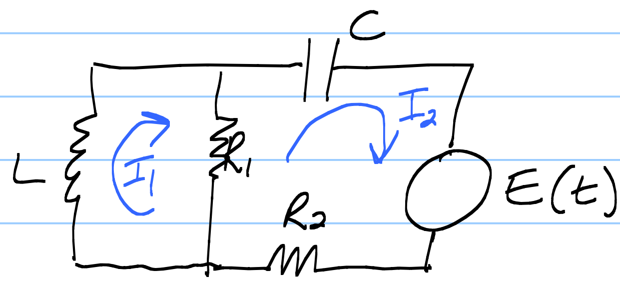
$$\begin{cases} S_1(0) = 0 \\ S_2(0) = 0 \end{cases}$$

$$\frac{dS_1}{dt} = (\text{Rate in}) - (\text{Rate out})$$
$$= 2\cdot 3 - 2\left(\frac{S_1(t)}{100}\right)$$

$$\begin{cases} S_1' = 6 - \frac{1}{50} S_1 \\ S_2' = \frac{1}{50} S_1 - \frac{1}{50} S_2 \end{cases}$$

$$\frac{dS_2}{dt} = 2\left(\frac{S_1(t)}{100}\right) - 2\left(\frac{S_2(t)}{100}\right)$$

EX:



Voltage drops:

$$RI, \quad L\frac{dI}{dt}, \quad \frac{1}{C}Q$$

Aha! $I_2 = \frac{dQ}{dt}$

(A) $\quad L\frac{dI_1}{dt} + R_1(I_1 - I_2) = 0$

(2) $\quad \frac{1}{C}Q - E(t) + R_2 I_2 + R_1(I_2 - I_1) = 0$

(B): $\quad \frac{d}{dt}$ (2) $\qquad$ (A) + (B) first order system.

First order systems say it all!

EX: $\quad y'' + P(x)y' + Q(x)y = F(x) \quad (*)$

Standard method: Let $\begin{cases} u_1 = y \\ u_2 = y' \end{cases}$ ← stop

$[u_3 = y'' \text{ if } 3^{rd} \text{ order} \ldots]$

$$\begin{cases} u_1' = y' = u_2 \\ u_2' = y'' = -P(x)\underbrace{y'}_{u_2} - Q(x)\underbrace{y}_{u_1} + F(x) \end{cases}$$ from ODE $(*)$

$$\begin{cases} u_1' = u_2 \\ u_2' = -Pu_2 - Qu_1 + F \end{cases}$$

Ex: $\begin{cases} \ddot{x}_1 = 2x_1 - x_2 \\ \ddot{x}_2 = x_1 - 2x_2 \end{cases}$

Let $\begin{cases} u_1 = x_1 \\ u_2 = \dot{x}_1 \end{cases}$

$\begin{cases} u_3 = x_2 \\ u_4 = \dot{x}_2 \end{cases}$

$$\begin{cases} u_1' = \dot{x}_1 = u_2 \\ u_2' = \ddot{x}_1 = 2x_1 - x_2 = 2u_1 - u_3 \\ u_3' = \dot{x}_2 = u_4 \\ u_4' = \ddot{x}_2 = x_1 - 2x_2 = u_1 - 2u_3 \end{cases}$$

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix}' = \begin{pmatrix} u_2 \\ 2u_1 - u_3 \\ u_4 \\ u_1 - 2u_3 \end{pmatrix}$$

$$\vec{u}' = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & -2 & 0 \end{bmatrix} \vec{u}$$

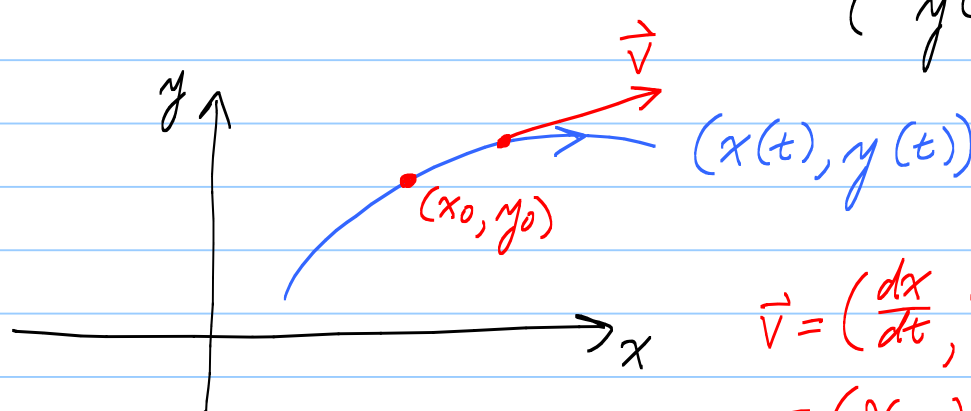
First order 4x4 linear system with const coeff!

Meaning of sol^{n}s:

$$\begin{cases} \dfrac{dx}{dt} = f(x,y) \\ \dfrac{dy}{dt} = g(x,y) \end{cases}$$

"Autonomous"
No t on RHS

$\begin{cases} x(0) = x_0 \\ y(0) = y_0 \end{cases}$



$$\vec{v} = \left(\frac{dx}{dt}, \frac{dy}{dt}\right) = \left(f(x,y), g(x,y)\right)$$

"Wind" vector field.

Sol^{n}s represent path of a dandelion fuzz in wind.

EX:
$$\begin{cases} x_1' = 4x_1 + 2x_2 & (A) \\ x_2' = 3x_1 - x_2 & (B) \end{cases} \qquad \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\vec{x}\,' = \mathbb{A}\,\vec{x}$$

Hmmm. Solve (A) for x_2:

$$\boxed{x_2 = \tfrac{1}{2}x_1' - 2x_1} \quad \text{box \#1}$$

Plug box 1 into (B):

$$\underbrace{\left(\tfrac{1}{2}x_1' - 2x_1\right)'}_{x_2'} = 3x_1 - \underbrace{\left(\tfrac{1}{2}x_1' - 2x_1\right)}_{x_2}$$

$$\tfrac{1}{2}x_1'' - 2x_1' + \tfrac{1}{2}x_1' - 5x_1 = 0$$

$$\boxed{x_1'' - 3x_1' - 10x_1 = 0}$$

$$r^2 - 3r - 10 = 0$$

$$(r+2)(r-5) = 0$$

$$r = -2, 5$$

$$x_1 = c_1e^{-2t} + c_2e^{5t}$$

Box #1 to get x_2: $\quad x_2 = \tfrac{1}{2}x_1' - 2x_1$

$$x_2 = \frac{1}{2}\left(-2c_1e^{-2t} + 5c_2e^{5t}\right) - 2c_1e^{-2t} - 2c_2e^{5t}$$

$$x_2 = -3c_1e^{-2t} + \frac{1}{2}c_2e^{5t}$$

$$\boxed{\vec{x} = c_1\begin{pmatrix}1\\-3\end{pmatrix}e^{-2t} + c_2\begin{pmatrix}1\\\frac{1}{2}\end{pmatrix}e^{5t}}$$