Review for Exam 2

Exam 2 Tues Nov 9, 8:00-9:00 pm WALC 1055

13. A particular solution, y_p, of $\underbrace{y'' - 4y' + 3y}_{L[y]} = 2t + e^t$ is

A. $\frac{2}{3}t + \frac{8}{9} - \frac{1}{2}te^t$ **B.** $\frac{2}{3}t + \frac{1}{2} - \frac{1}{2}te^t$ **C.** $\frac{1}{3}t + \frac{1}{2} - \frac{1}{2}te^t$ **D.** $\frac{1}{3}t + \frac{1}{2} - \frac{1}{2}e^t$ **E.** $t^2 + e^t$

$r^2 - 4r + 3 = 0$

$(r-3)(r-1) = 0$

$r = 1, 3$

$y_c = c_1 e^t + c_2 e^{3t}$

$y_p = At + B + t[Ce^t]$

$y_p' = A + Ce^t + Cte^t$

$y_p'' = 2Ce^t + Cte^t$

$$\underbrace{(2Ce^t + Cte^t)}_{y_p''} - 4\underbrace{(A + Ce^t + Cte^t)}_{y_p'} + 3\underbrace{(At + B + Cte^t)}_{y_p} = 2t + e^t$$

want

$$\underbrace{-2C}_{=1}e^t + \underbrace{(3A)}_{=2}t + \underbrace{(-4A + 3B)}_{=0} = 2t + e^t$$

14. If $y'' + 5y' + 6y = 24e^t,\ y(0) = 0,\ y'(0) = 0,$ then $y(1) =$

A. $-e^{-2} + 6e^{-3} + e$ **B.** $-8e^{-2} + 6e^{-3} + e$ **C.** $8e^{-2} + e^{-3} + e$ **D.** $-8e^{-2} + 6e^{-3} + 2e$
E. 0

$r^2 + 5r + 6 = 0$

$(r+2)(r+3) = 0$

$r = -2, -3$

$y_c = c_1 e^{-2t} + c_2 e^{-3t}$

$y_p = Ae^t$ Get $A = 2$.

Gen^l Sol^n: $\begin{cases} y = c_1 e^{-2t} + c_2 e^{-3t} + 2e^t \\ y' = -2c_1 e^{-2t} - 3c_2 e^{-3t} + 2e^t \end{cases}$

want

$\begin{cases} y(0) = c_1 + c_2 + 2 = 0 \\ y'(0) = -2c_1 - 3c_2 + 2 = 0 \end{cases}$

Get c_1, c_2.

$y(1) = c_1 e^{-2} + c_2 e^{-3} + 2e$

15. The differential equation $y'' - \frac{2}{t}y' + \frac{2}{t^2}y = 0$ has solutions $y_1(t) = t$ and $y_2(t) = t^2$. If $y'' - \frac{2}{t}y' + \frac{2}{t^2}y = 2$, $y(1) = 0$ and $y'(1) = 0$, then $y(2) =$

A. 0 **B.** -6 **C.** $8\ln 2$ **D.** $8\ln 2 - 4$ **E.** $8\ln 2 + 4$

$y_p = u_1 y_1 + u_2 y_2$ where $\boxed{u_1' = \frac{-y_2 F}{W} \qquad u_2' = \frac{y_1 F}{W}}$

standard form! $F(t) = 2$

$$W = \det\begin{bmatrix} t & t^2 \\ 1 & 2t \end{bmatrix} = 2t^2 - t^2 = t^2$$

$$u_1' = \frac{-(t^2)(2)}{(t^2)} = -2, \quad \text{So} \quad u_1 = \int -2\,dt = -2t$$

$$u_2' = \frac{(t)(2)}{t^2} = \frac{2}{t}, \quad \text{So} \quad u_2 = \int \frac{2}{t}\,dt = 2\ln t$$

$$y_p = (-2t)(t) + (2\ln t)(t^2)$$

$$= \underbrace{-2t^2}_{\text{a sol}^n \text{ to } L[y]=0!} + 2t^2 \ln t$$

$$2 = L[y_p] = \underbrace{L[-2t^2]}_{=0} + L[\underbrace{2t^2\ln t}_{\text{better } y_p!}]$$

Gen^l Sol^n: $y = c_1 t + c_2 t^2 + 2t^2 \ln t$, etc.

17. A spring-mass system is governed by the initial value problem $x'' + 4x' + 4x = 4\cos\omega t,\ x(0) = 0,\ x'(0) = -2$. For what value(s) of ω will resonance occur ?

A. 0 **B.** 2 **C.** 4 **D.** $2 < \omega < \infty$ **E.** no value of ω

$r^2 + 4r + 4 = 0$

$(r+2)^2 = 0$

$r = -2, -2$

$y_c = c_1 e^{-2t} + c_2 t e^{-2t}$

$y_p = A\cos\omega t + B\sin\omega t$

$= a\cos(\omega t - \varphi)$

Resonace cannot happen!

Friction: $r^2 + cr + 4 = 0$

Underdamped: Roots $r_1, r_2 = \underbrace{-\frac{c}{2}}_{<0} \pm \frac{\sqrt{c^2-16}}{2}$ Complex

real part

$\cos\omega t, \sin\omega t \sim \pm\omega i$ roots.

19. Rewrite the second order equation $2u'' + 3u' + ku = \cos 2t$ as a system of 1^{st} order equations.

A. $\begin{cases} x' = y \\ y' = \frac{1}{2}(-3y - kx + \cos 2t) \end{cases}$

B. $\begin{cases} x' = y \\ y' = \frac{1}{2}(-3x - ky + \cos 2t) \end{cases}$

C. $\begin{cases} x' = x \\ y' = \frac{1}{2}(-3y - kx + \cos 2t) \end{cases}$

D. $\begin{cases} x' = y \\ y' = 2y + kx + \cos 2t \end{cases}$

E. $\begin{cases} x' = 2y + 3x + \cos 2t \\ y' = x \end{cases}$

$\begin{cases} x = u \\ y = u' \end{cases}$

$x_1 = u$

$x_2 = u'$

3 rd order

$x_1 = u$

$x_2 = u'$

$x_3 = u''$

$$\begin{cases} \dfrac{dx_1}{dt} = u' = x_2 \\ \dfrac{dx_2}{dt} = u'' = -\frac{3}{2}u' - \frac{k}{2}u + \frac{1}{2}\cos 2t \end{cases}$$

from 2nd order ODE

$$= -\frac{3}{2}x_2 - \frac{k}{2}x_1 + \frac{1}{2}\cos 2t$$

$$\begin{cases} \dfrac{dx_1}{dt} = x_2 \\ \dfrac{dx_2}{dt} = -\frac{k}{2}x_1 - \frac{3}{2}x_2 + \frac{1}{2}\cos 2t \end{cases}$$

$$\vec{x}\,' = \begin{bmatrix} 0 & 1 \\ -\frac{k}{2} & -\frac{3}{2} \end{bmatrix}\vec{x} + \begin{pmatrix} 0 \\ \frac{1}{2}\cos 2t \end{pmatrix}$$

20. The solution of $X' = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} X, \quad X(0) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ is

A. $2\,e^{3t}\begin{pmatrix} 1 \\ 2 \end{pmatrix} + e^{-t}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ **B.** $2\,e^{3t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + e^{-t}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ **C.** $e^{3t}\begin{pmatrix} 2 \\ 1 \end{pmatrix} + e^{-t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

D. $3\,e^{3t}\begin{pmatrix} 1 \\ -2 \end{pmatrix} - e^{-t}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ **E.** $3\,e^{3t}\begin{pmatrix} 1 \\ 2 \end{pmatrix} + e^{-t}\begin{pmatrix} 0 \\ -4 \end{pmatrix}$

$$\det\begin{bmatrix} 1-r & 1 \\ 4 & 1-r \end{bmatrix} = (1-r)^2 - 4 = 0$$

$$= r^2 - 2r - 3 = 0$$

$$(r-3)(r+1) = 0 \qquad r = -1, 3$$

<u>For $r=-1$:</u>

$$(A - \underset{r=-1}{r}\,\mathbb{I})\vec{a} = 0$$

$$\left[\begin{array}{cc|c} 1-(-1) & 1 & 0 \\ 4 & -1-(-1) & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 2 & 1 & 0 \\ 4 & 2 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} 2 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$\vec{a} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$\left[\begin{array}{cc|c} A & B & 0 \\ \sim & \sim & \sim \end{array}\right]$$

$$\vec{a} = \begin{pmatrix} B \\ -A \end{pmatrix} \text{ or } \begin{pmatrix} -B \\ A \end{pmatrix}$$

<u>Note:</u> If $\vec{a}$ is e.vect, so is $c\vec{a}$ where $c \neq 0$.

21. Solve $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $X(0) = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$.

A. $X(t) = 2\,e^t \begin{pmatrix} \sin t \\ \cos t \end{pmatrix} - e^t \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$ **B.** $X(t) = 2\,e^t \begin{pmatrix} \sin t \\ \cos t \end{pmatrix} + e^t \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$

C. $X(t) = 2\,e^t \begin{pmatrix} \sin t \\ \cos t \end{pmatrix} - e^t \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix}$ **D.** $X(t) = e^t \begin{pmatrix} \sin t \\ \cos t \end{pmatrix} - e^t \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$

E. $X(t) = e^t \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix} - e^t \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$

$$\det\begin{pmatrix} 1-r & 1 \\ -1 & 1-r \end{pmatrix} = (1-r)^2 + 1 = 0$$

$$(1-r) = \pm i$$

$$r = 1 \pm i$$

For $r = 1+i$: $(\underbrace{A - rI}_{r=1+i})\vec{a} = \vec{0}$

$$\left[\begin{array}{cc|c} 1-(1+i) & 1 & 0 \\ -1 & 1-(1+i) & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} -i & 1 & 0 \\ -1 & -i & 0 \end{array}\right]$$

$$\rightsquigarrow \left[\begin{array}{cc|c} -i & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$\vec{a} = \begin{pmatrix} 1 \\ -(-i) \end{pmatrix} = \begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ i \end{pmatrix}(e^t \cos t + i\, e^t \sin t)$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} + i \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t \cos t - \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^t \sin t\right] + i \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t \sin t + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^t \cos t\right]$$

22. Solve the initial value problem $\vec{\mathbf{x}}\,'(t) = A\,\vec{\mathbf{x}}(t),\ \vec{\mathbf{x}}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ where $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$.

A. $e^t \begin{pmatrix} 1 \\ 1 \end{pmatrix} - 2te^t \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ **B.** $e^t \begin{pmatrix} 1 \\ 1 \end{pmatrix} + 2te^t \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ **C.** $e^t \begin{pmatrix} 1 \\ 1 \end{pmatrix} + te^t \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

D. $e^t \begin{pmatrix} 0 \\ 1 \end{pmatrix} + 2te^t \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ **E.** $e^t \begin{pmatrix} 1 \\ 1 \end{pmatrix} - 2te^t \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

($\mathbb{A}$ defective)

$$\vec{x}_2 = \vec{a}\,te^{rt} + \vec{b}\,e^{rt}$$

e.vect for r

$(\mathbb{A} - r\mathbb{I})\vec{b} = \vec{a}$