Review 2

Final Exam, Thurs., Dec 16, 7–9pm
in WALC 200 Balcony (entrance on 3rd floor)

Three lowest MyLab online homework dropped, but important to do last assignments on Laplace transform to be ready for the final!

Elementary Laplace Transforms

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\frac{1}{s}, \quad s > 0$
2.	e^{at}	$\frac{1}{s-a}, \quad s > a$
r=0 3.	$t^n, \quad n = \text{positive integer}$	$\frac{n!}{s^{n+1}}, \quad s > 0$
4.	$t^p, \quad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5.	$\sin at$	$\frac{a}{s^2+a^2}, \quad s > 0$
6.	$\cos at$	$\frac{s}{s^2+a^2}, \quad s > 0$
7.	$\sinh at$	$\frac{a}{s^2-a^2}, \quad s > \lvert a\rvert$
8.	$\cosh at$	$\frac{s}{s^2-a^2}, \quad s > \lvert a\rvert$
9.	$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$
10.	$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$
11.	$t^n e^{at}, \quad n = \text{positive integer}$	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
12.	$u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$
13.	$u_c(t)f(t-c)$	$e^{-cs}F(s) \leftarrow$ t-shift rule
14.	$e^{ct}f(t)$	$F(s-c) \leftarrow$ s-shift rule
15.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right), \quad c > 0$
16.	$\boxed{\int_0^t f(t-\tau)g(\tau)\,d\tau}$ $f*g$	$F(s)G(s)$ $\quad \mathcal{L}[f'] = sF - y_0$
17.	$\delta(t-c) \leftarrow \delta_c(t)$	e^{-cs} $\quad \mathcal{L}[f''] = s^2F - sy_0 - y_0'$
18.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
19.	$(-t)^n f(t)$	$F^{(n)}(s)$

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25. $\mathcal{L}\{e^t(1+\cos 2t)\} =$ $\mathcal{L}[e^t] + \mathcal{L}[e^t \cos 2t] = \frac{1}{s-1} + \frac{(s-1)}{(s-1)^2+2^2}$

A. $\dfrac{1}{s-1} + \dfrac{1}{(s-1)^2+4}$ **B.** $\dfrac{1}{s-1} + \dfrac{s-1}{s^2-2s+5}$ **C.** $\left(\dfrac{1}{s-1}\right)\left(\dfrac{1}{s} + \dfrac{s-1}{(s-1)^2+4}\right)$

D. $\left(\dfrac{1}{s-1}\right)\left(\dfrac{s-1}{s^2-2s+5}\right)$ **E.** $\dfrac{1}{s} + \dfrac{s}{(s-1)^2+4}$

26. Find the Laplace transform of $f(t) = \begin{cases} t, & 0 \le t < 1 \\ 0, & 1 \le t < \infty \end{cases}$.

$\int_0^1 t\, e^{-st}\, dt$

A. $e^{-s}\left(\frac{1}{s} + \frac{1}{s-2}\right)$ **B.** $\frac{1}{s^2} + 2e^{-s}\left(\frac{1}{s} + \frac{1}{s^2}\right)$ **C.** $\frac{1}{s^2} - e^{-s}\frac{1}{s^2}$ **D.** $\frac{1}{s^2} - e^{-s}\left(\frac{1}{s} + \frac{1}{s^2}\right)$
E. $e^{-s}\left(\frac{1}{s} + \frac{1}{s^2}\right)$

$$f(t) = t\left[u_0(t) - u_1(t)\right]$$

$$\mathcal{L}\left[u_c(t) f(t-c)\right] = e^{-cs} F(s)$$

$$= \underbrace{u_0(t)\, t}_{\text{Same as } t \text{ for } \mathcal{L}} - u_1(t)\, t$$

wish this was $t-1$

$$= u_0(t) t - u_1(t)\left[\underbrace{(t-1)+1}_{g(t-1)}\right]$$

$g(t-1)$ where $g(\tau) = \tau + 1$

$g(t) = t+1$

$\mathcal{L}[g] = \frac{1}{s^2} + \frac{1}{s}$

$$\mathcal{L}[f] = \frac{1}{s^2} - e^{-1\cdot s}\left(\frac{1}{s^2} + \frac{1}{s}\right)$$

EX: $\sin t = \sin\left((t-\pi) + \pi\right)$

$$t^2 = \left[(t-3)+3\right]^2 = \underbrace{(t-3)^2 + 6(t-3) + 9}_{g(t-3)}$$

where $g(\tau) = \tau^2 + 6\tau + 9$

27. Solve $y'' + 3y' + 2y = 4\,u_1(t),\ y(0) = 0,\ y'(0) = 1$

A. $u_1(t)\left(2 - 4e^{-(t-1)} + 2e^{-2(t-1)}\right)$ **B.** $u_1(t)\left(2 - 4e^{-(t-1)} + 2e^{-2(t-1)}\right) + e^{-t} - e^{-2t}$
C. $u_0(t)\left(2 - 4e^{-(t-1)} + 2e^{-2(t-1)}\right) + e^{-t} - e^{-2t}$ **D.** $\left(2 - 4e^{-(t-1)} + 2e^{-2(t-1)}\right) + e^{-t} - e^{-2t}$
E. $e^{-t} - e^{-2t}$

$$\left[s^2 Y - s\underbrace{y(0)}_{0} - \underbrace{y'(0)}_{1}\right] + 3\left[sY - \underbrace{y(0)}_{0}\right] + 2Y = 4\,\frac{e^{-1\cdot s}}{s}$$

$\delta_\pi(t)$

28. Find the solution of the initial value problem $y'' + y = \delta(t-\pi)$, $y(0) = 0$, $y'(0) = 1$.

A. $y = \sin t + u_0(t)\sin t$ **B.** $y = \sin t + u_\pi(t)\sin \pi t$ **C.** $y = \sin t + u_\pi(t)\sin(t-\pi)$ ✓
D. $y = u_\pi(t)(\sin t + \sin(t-\pi))$ **E.** $y = u_\pi(t)\sin t$

$$\left(s^2 Y - s\cdot 0 - 1\right) + Y = e^{-\pi s}$$

$$\mathcal{L}[u_c(t)f(t-c)] = e^{-cs}F(s)$$

$$Y = \frac{1}{s^2+1} + e^{-\pi s}\underbrace{\frac{1}{s^2+1}}_{F(s)}$$

$F(s)$, so $f(t) = \sin t$

So $$y(t) = \sin t + u_\pi(t)\underbrace{\sin(t-\pi)}_{-\sin t}$$

$$\sin(t-2\pi) = \sin t$$

$$\cos(t-2\pi) = \cos t \qquad \cos(t-\pi) = -\cos t$$

29. The inverse Laplace transform of $F(s) = \dfrac{se^{-s}}{s^2+2s+5}$ is

A. $u_1(t)\left(e^{-t}\cos 2(t-1)\right) - \frac{1}{2}e^{-t}\sin 2(t-1)$ **B.** $\left(e^{-t+1}\cos 2(t-1)\right) - \frac{1}{2}e^{-t+1}\sin 2(t-1)$
C. $u_1(t)\left(e^{t-1}\cos 2(t-1)\right) - \frac{1}{2}e^{t-1}\sin 2(t-1)$ **D.** $u_0(t)\left(e^{-t}\cos 2t\right) - \frac{1}{2}e^{-t}\sin 2t$
E. $u_1(t)\left(e^{-t+1}\cos 2(t-1) - \frac{1}{2}e^{-t+1}\sin 2(t-1)\right)$

$$\frac{as+b}{\text{Quadratic}} = \begin{cases} \dfrac{A}{s-r_1} + \dfrac{B}{s-r_2} & \text{if roots} \neq \text{, real.} \\ \dfrac{A}{(s-r)^2} + \dfrac{B}{s-r} & \text{if real root repeated} \\ \text{Complete square in denom} & \end{cases}$$

$$G(s) = \frac{s}{s^2+\underset{=}{2}s+5} = \frac{s}{(s+\frac{1}{2}\cdot 2)^2+4} = \frac{(s+1)-1}{(s+1)^2+2^2}$$

$$= \frac{(s+1)}{(s+1)^2+2^2} - \frac{1}{2}\cdot\frac{2}{(s+1)^2+2^2}$$

$$= \mathcal{L}\Big[\underbrace{e^{-t}\cos 2t - \tfrac{1}{2}e^{-t}\sin 2t}_{g(t)}\Big]$$

$$F(s) = e^{-s}G(s) \qquad \mathcal{L}\left[u_c(t)f(t-c)\right] = e^{-cs}F(s)$$

<u>Ans</u>: $f(t) = u_1(t)\,g(t-1)$

$$= u_1(t)\left[e^{-(t-1)}\cos 2(t-1) - \tfrac{1}{2}e^{-(t-1)}\sin 2(t-1)\right]$$

$$\underbrace{\quad}^{(\sin 2t) * (\cos 3t)}$$

30. $\mathcal{L}\left\{\int_0^t \sin 2(t-\tau)\cos(3\tau)\,d\tau\right\} = \dfrac{2}{s^2+2^2} \quad \dfrac{s}{s^2+3^2}$

A. $\dfrac{2s}{(s^2+4)(s^2+9)}$ ✓ **B.** $\dfrac{2}{s^2+4}+\dfrac{s}{s^2+9}$ **C.** $\dfrac{1}{s^2+4}+\dfrac{s}{s^2+9}$ **D.** $\dfrac{2}{(s^2+4)(s^2+9)}$
E. $\dfrac{s}{(s^2+4)(s^2+9)}$

$$y'' + 2y' + y = f(t) \qquad y(0)=0 \quad y'(0)=0$$

$$Y(s) = \frac{1}{s^2+2s+1}\cdot F(s)$$

So

$$y(t) = \left(te^{-t}\right) * f(t)$$

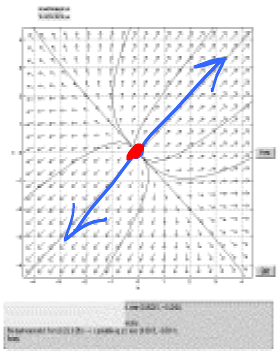
$$= \int_0^t \tau e^{-\tau} f(t-\tau)\,d\tau$$

$$= \int_0^t f(\tau)\,(t-\tau)\,e^{-(t-\tau)}\,d\tau$$

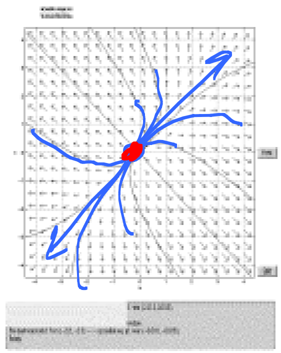
31. The phase portrait of the system $\vec{\mathbf{x}}'(t) = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \vec{\mathbf{x}}(t)$, whose general solution is

$\vec{\mathbf{x}}(t) = C_1 e^t \begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, looks most like : $r = 1, 3$

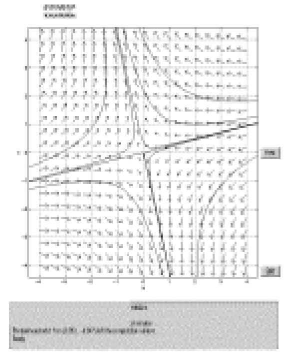
A.



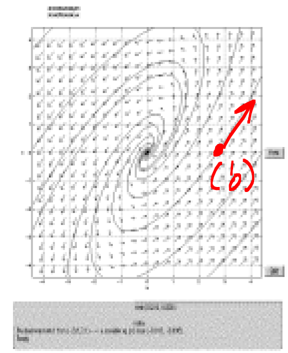
B.



C.



D.



Node rule: Node. Sol'ns near origin hug the e.vect dir corresponding to the e.val closest to zero.

$r = 1$

Answers

1. D
2. C
3. A
4. D
5. C
6. B
7. B
8. D
9. C
10. E
11. A
12. B
13. A
14. D
15. D
16. C
17. E
18. D
19. A
20. A
21. C
22. C
23. A
24. D
25. B
26. D
27. B
28. C
29. E
30. A
31. B