MA 266 – 930

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Easiest ODE $\quad \frac{dy}{dx} = f(x)$

$$y = \int f(x)\,dx + C$$

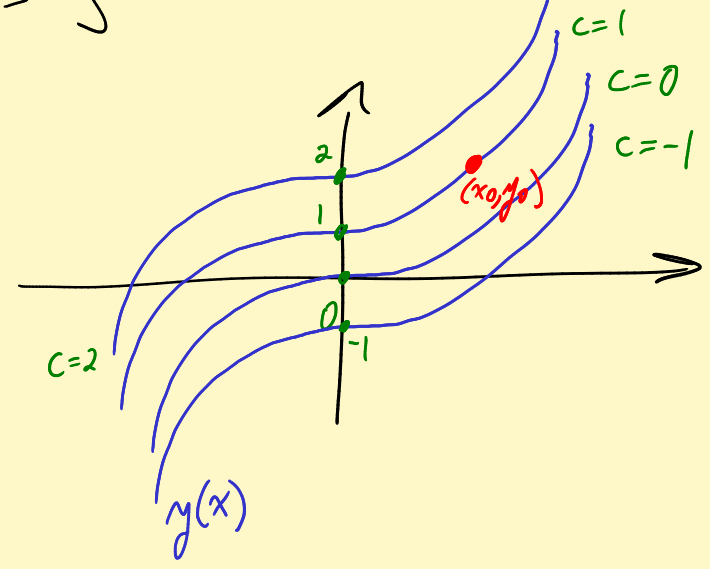
∞ many sol'n's.

any antiderivative of f

Nice one: $F(x) = \int_a^x f(t)\,dt$

EX $\quad \frac{dy}{dx} = 3x^2$

$$y = \int 3x^2\,dx + C = x^3 + C$$



Pin one down:

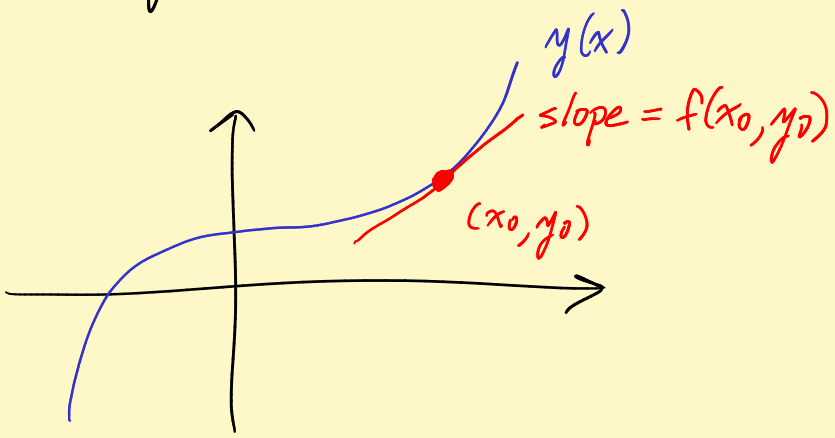
$y(x_0) = y_0$

Think: x = time ← independent var

y = fcn of x ← dependent (on x) = $y(x)$

General ODE $\frac{dy}{dx} = f(x, y)$

Meaning of "sol^n":



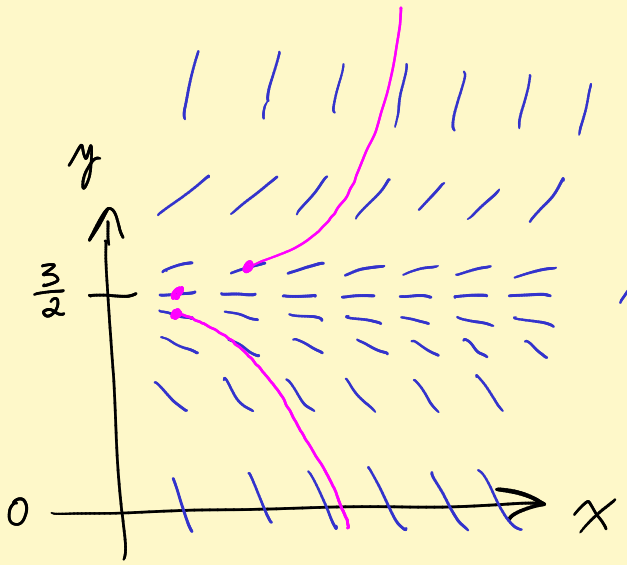
EX $\frac{dy}{dx} = 2y - 3$

$2y - 3 = s$ ← slope s, s=const.

Curve where s = const is called an isocline.

$2y - 3 = s$

$y = \frac{s+3}{2}$ horiz line



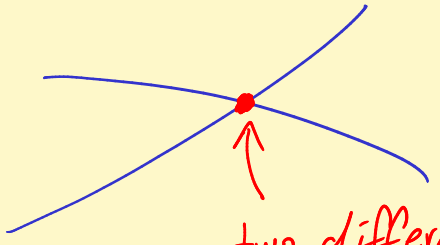
$y = \frac{3}{2}$ isocline: $s = 0$

($y = \frac{3}{2}$ is also a sol^n!)

$y = 0$ isocline: $s = -3$

Think: Sol^n: Drop a cork in the "flow"

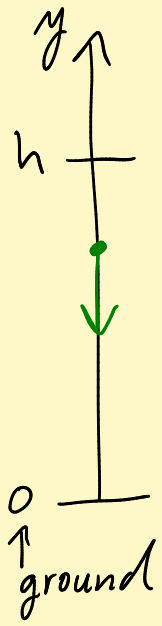
Fact Solutions can't cross

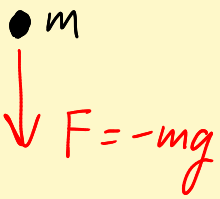


two different slopes at pt!

Newton's law: $F = ma = m\frac{dv}{dt} = m\frac{d^2y}{dt^2}$ ← 2nd order ODE!

EX Drop a rock





$$F = ma$$

$$-mg = m\frac{dv}{dt}$$

ODE $\quad \frac{dv}{dt} = -g$

$$v = \int -g\,dt + C = \boxed{-gt + C}$$

"Dropped" means $v(0) = 0 \leftarrow v_0$

Initial condition: $\quad v(0) = v_0$

$$-g \cdot 0 + C = v_0$$

$$\boxed{C = v_0}$$

$$\boxed{v = -gt + v_0}$$

Next: $\quad \frac{dy}{dt} = v = -gt + v_0$

$$y = \int -gt + v_0\,dt + C_2$$

$$y = -\frac{1}{2}gt^2 + v_0 t + C_2$$

Dropped from height h. $y(0) = h$

$$-\frac{1}{2}g \cdot 0^2 + v_0 \cdot 0 + C_2 = h$$

$$C_2 = h$$

$$y = -\frac{1}{2}gt^2 + v_0 t + h$$

↑ $v_0 = 0$ if dropped